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Surface tension driven meniscus oscillations and the effects on droplet formation

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Abstract
In a multi-nozzle piezo-electrically driven print head a large number of miniature and valveless pumps are integrated. In order to have a design with the smallest native nozzle pitch possible the pumps are placed as closely as possible next to each other. This implies that the length of the pump chamber has to be long compared to its cross-sectional dimensions in order to enable the piezoelectric actuator to generate enough volume displacement. The layout of such a pump is of the waveguide type and upon actuation waves start to travel back and forth through the pump chamber. The evolution of these waves in course of time depends on the reflection properties at the beginning and the end of the pump chamber, the beginning being the connection to the main ink supply and the end being the nozzle. The attenuation depends on the viscosity of the ink used. At the end of the nozzle a meniscus is formed. In the case the meniscus retracts over a small distance into the nozzle its curvature increases and the capillary pressure increases. This effect forces the meniscus to move back to its original position. During outflow over a small distance the same happens. With increasing outflow the curvature increases and the capillary force opposing the motion increases accordingly. The capillarity builds a kind of mechanical spring action. This spring action together with the mass of fluid in the pump forms a mass-spring system with its own oscillatory behavior. The resonance phenomenon is the so-called slosh-mode, all the fluid contained in the pump moves in phase against the surface tension spring. For higher order meniscus modes, however, the fluid motion is confined to the very close environment of the meniscus. When the print head and the pulse are designed such that an overtone of the waveguide coincides with an axisymmetric higher order oscillation of the meniscus it is possible to make droplets that are much smaller than the standard droplet metered by the nozzle diameter. When such an overtone coincides with a non-axisymmetric mode, straightness errors may be induced. The paper will discuss an enhanced theory on higher order axisymmetric and non-axisymmetric meniscus oscillations and their possible effects on droplet formation and straightness errors.

Introduction

In a multi-nozzle piezo-electrically driven print head a large number of miniature and valveless pumps are integrated [1]. In order to have a design with the smallest native nozzle pitch possible the pumps are placed as closely as possible next to each other. This implies that the length of the pump chamber has to be long compared to its cross-sectional dimensions in order to enable the piezoelectric actuator to generate enough volume displacement. One end of the pump chamber is connected to the nozzle, the other end connected to the main supply channel either directly or via a throttle. The layout of such a pump of the waveguide type is shown schematically in figure 1.

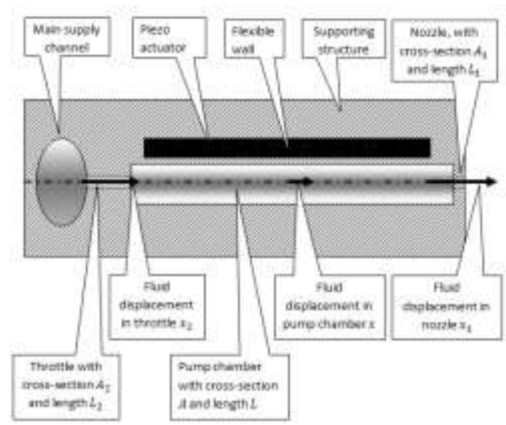

Figure 1: Schematic of one pump out of a print head of the waveguide type. Shown is the closed end-closed end arrangement. For the open end-closed end design there is no throttle. Note that the pump chamber is much longer than the nozzle and the throttle.

Upon actuation waves start to travel back and forth. The evolution of these waves in course of time depends on the reflection properties at the connection to the main ink supply and the connection to the nozzle [2]. The attenuation depends on the viscosity of the ink used and non-linear effects associated with partly filling of the nozzle. At the end of the nozzle a meniscus is formed between ink and air. This meniscus is supposed to be attached to the rim of the nozzle. When the meniscus retracts over a small distance (small means small compared to the radius of the nozzle) further into the nozzle, its curvature increases and the capillary pressure increases. This effect forces the meniscus to move back to its original position. During outflow over a small distance the same happens. With increasing outflow the curvature increases and the capillary force opposing the motion increases accordingly. The capillarity builds a kind of mechanical spring action. This spring action together with the mass of fluid in the pump forms a mass-spring system with its own oscillatory behavior. This resonance phenomenon is referred to as the so-called slosh mode, all the fluid contained in the pump moves in phase against the surface tension spring. For higher order meniscus modes, however, the fluid motion is confined to the very close environment of the meniscus. The response of a waveguide type of print head in the frequency domain is characterized by its key note and its spectrum of overtones. When the pulse is designed such that in its spectrum an overtone of the waveguide coincides with an axisymmetric higher order oscillation mode of the meniscus it is possible to make droplets that are much smaller than the standard droplet sized by the nozzle diameter [3]. When such an overtone coincides with a non-axisymmetric mode, straightness errors can be induced. The higher order meniscus wave forms resemble the wave forms of the surface of a drop sitting on a harmonically excited horizontal surface with pinned contact line [4]. The present paper will discuss an enhanced theory on higher order axisymmetric and non-axisymmetric surface tension controlled meniscus oscillations and their possible effects on droplet formation and straightness errors.

Slosh-mode frequency of a wave guide type print head

The slosh mode kinematics is characterized by the fact that the fluid portions in the pump chamber (with length L and cross-section A), nozzle (radius R_1, cross-section $A_1 = \pi R_1^2$ and length L_1) and throttle (cross-section A_2 and L_2) are moving in phase (see for a schematic of such a print head figure 1) . For the calculation of the surface tension spring generated by the deforming meniscus it is assumed that the meniscus is spherically shaped. Gravity causes large menisci to deform; this effect does not take place as long as the size of meniscus is much smaller than the capillary length defined by [5]:

$$R_1 \ll \frac{1}{\kappa} = \sqrt{\frac{\gamma}{\rho g}} \quad (1)$$

with γ being the surface tension, ρ the density and g the gravitational acceleration.

When considering the window of operation for inkjet printing with regard to the material properties, the capillary length measures 1.4 – 2.7 mm (1.4 mm for $\gamma = 0.02$ N/m and 2.7 mm for $\gamma = 0.07$ N/m, $\rho = 1000$ kg/m^3). For inkjet printing the nozzle and droplets measure up to 100 μm across, so gravity effects can be ruled out.

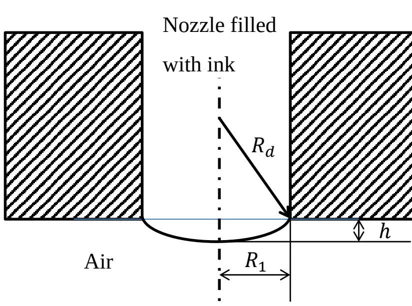

Figure 2: Geometry of meniscus.

In case the meniscus displaces only slightly ($|x_1| \ll R_1$, x_1 being the mean fluid displacement in the nozzle) the following geometrical relations hold true [6] (for negative x_1, similar relations can be derived):

$$V = A_1 x_1 = \frac{1}{6}\pi h\left(3{R_1}^2 + h^2\right), \quad R_1^2 + (R_d - h)^2 = {R_d}^2 \quad (2)$$

The height of the dome shaped meniscus is given by h and the radius of curvature by R_d as shown in figure 2. The outflowed volume is indicated by V. From (2) the radius of curvature can be derived in terms of the fixed parameter R_1 and the variable h:

$$R_d = \frac{{R_1}^2 + h^2}{2h} \quad (3)$$

For the case that $h \ll R_1$ the expressions (2) and (3) can be simplified to:

$$R_d \approx \frac{{R_1}^2}{2h}, \qquad x_1 \approx \frac{1}{2}h \quad (4)$$

The capillary pressure for small displacements ($|x_1| \ll R_1$) is given by the Young-Laplace equation [7]:

$$p_{cap} = \frac{2\gamma}{R_d} \approx 8\gamma \frac{x_1}{R_1^2} = 8\pi\gamma \frac{x_1}{A_1} \quad (5)$$

The force associated with the capillary pressure follows from:

$$F_{cap} = p_{cap} A_1 = 8\pi\gamma x_1 \quad (6)$$

Note that F_{cap} depends linearly on the displacement x_1, similar to a mechanical spring [8]. For the slosh mode the fluid in the print head is assumed to be incompressible, so the total volume of fluid displacements in nozzle, pump section and throttle must be equal (x means mean fluid displacement in pump chamber and x_2 mean fluid displacement in throttle):

$$A_1 x_1 = Ax = A_2 x_2 \quad (7)$$

Applying Newton's law to the fluid mass contained in the nozzle the following result is obtained:

$$F_{cap} = 8\pi\gamma x_1 = M\ddot{x}_1 = \rho\left(A_1 L_1 + AL\frac{A_1^2}{A^2} + A_2 L_2 \frac{A_1^2}{A_2^2}\right)\ddot{x}_1 \quad (8)$$

Solving this equation for harmonic motion the resonance frequency is given by:

$$f_{slosh} = \frac{1}{2\pi}\sqrt{\frac{8\pi\gamma}{\rho\left(A_1 L_1 + AL\frac{A_1^2}{A^2} + A_2 L_2\frac{A_1^2}{A_2^2}\right)}} \quad (9)$$

Following the general design rules of an inkjet print head:

$$A \gg A_1, A_2, \quad A_2 \approx> A_1, \quad L >\gg L_1, \quad L_2 \gg L_1 \quad (10)$$

The middle term between the brackets of the denominator under the square root dominates and the slosh mode frequency for a wave guide type of inkjet print head is given approximately by:

$$f_{slosh} \approx \frac{1}{2\pi}\sqrt{\frac{8\pi\gamma A}{\rho L A_1^2}} \quad (11)$$

Higher order symmetric meniscus oscillations

For higher order meniscus motions only the fluid close to the meniscus is involved, this in contrast to the slosh mode for which all fluid in the system must be considered. The analysis is confined to frictionless fluids (dynamic viscosity $\mu = 0$). To describe the flow a cylindrical co-ordinate system will be used with z the co-ordinate measuring the distance from the meniscus upwards into the nozzle and r the radial co-ordinate measuring the distance from the center line of the nozzle. For the fundamental motion of the meniscus the following ansatz will be posed:

$$w_1(r,t) = B[1+\lambda J_0(kr)]\sin\omega t, \quad w_1 \ll R_1 \tag{12}$$

The small amplitude of the harmonic motion with radian frequency ω is given by B. $J_0(kr)$ is a zeroth order Bessel function of the first kind [9]. The constant λ follows from the argument that at the nozzle wall the displacement of the meniscus is zero:

$$\lambda = -\frac{1}{J_0(kR_1)} \tag{13}$$

The constant k is given by the fact that the fluid motion is local and that there is no net volume displacement (V volume):

$$V = 2\pi B\int_0^{R_1}[1+\lambda J_0(kr)]rdr = 2\pi B R_1^2\left[\frac{1}{2} - \frac{J_1(kR_1)}{kR_1 J_0(kR_1)}\right] \tag{14}$$

The condition $V = 0$ is fulfilled for the first time for (see figure 3, index refers to first mode):

$$k_1 R_1 = 5.135622, \quad \lambda_1 = 7.559745 \tag{15}$$

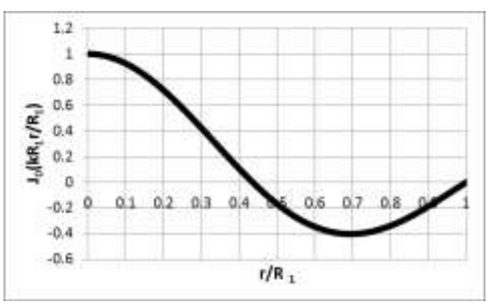

Figure 3: Amplitude of meniscus motion according to (12) for $k_1 R_1 = 5.135622$ and $\lambda_1 = 7.559745$ (first mode).

The fluid velocity at the meniscus is given by:

$$\frac{dw_1}{dt} = B\omega[1+\lambda_1 J_0(k_1 r)]\cos\omega t \tag{16}$$

Suppose the component of the velocity vector in the fluid in axial direction (z-direction) away from the meniscus is:

$$v_z(r,z,t) = B\omega F(z)[1+\lambda_1 J_0(k_1 r)]\cos\omega t, \quad F(0)=1 \tag{17}$$

Using the equation of continuity [10] the component of the velocity vector in r-direction can be determined:

$$v_r(r,z,t) = -B\omega\frac{dF(z)}{dz} r\left[\frac{1}{2} + \frac{\lambda_1}{k_1 r}J_1(k_1 r)\right]\cos\omega t \tag{18}$$

Substitution of the expressions for the components of the velocity vector into the components of the equation of motion yields [10] (skipping the convective terms, this can be justified by the fact that only small deviations from the equilibrium meniscus position are considered, B small):

$$-\rho B\omega^2\frac{dF(z)}{dz} r\left[\frac{1}{2} + \frac{\lambda_1}{k_1 r}J_1(k_1 r)\right]\sin\omega t = \frac{\partial p}{\partial r}$$

$$\rho B\omega^2 F(z)[1+\lambda_1 J_0(k_1 r)]\sin\omega t = \frac{\partial p}{\partial z} \tag{19}$$

In both components of the equation of motion for most values of r in the expressions between brackets the terms involving Bessel functions dominate. Retaining only the Bessel functions, differentiation of the r-component of the equation of motion with respect to z and differentiation of the z-component of the equation of motion with respect to r results in an equality that can only be fulfilled as long as the function $F(z)$ obeys:

$$\frac{d^2F}{dz^2} - k_1^2 F = 0$$

$$F = e^{-k_1 z} \tag{20}$$

This solution for F describes a penetration phenomenon, its value decays to zero for large z. The distance, $z_{penetration}$, at which the exponential has decreased to 5% is given by:

$$k_1 z_{penetration} = \pi, \qquad z_{penetration} = \frac{\pi}{k_1 R_1}R_1 = 0.612\, R_1 \tag{21}$$

Indeed the fluid motion belonging to the first axisymmetric higher order oscillatory mode has a limited reach. The components of the velocity vector are:

$$v_z(r,z,t) \approx B\omega\, e^{-k_1 z}[1+\lambda_1 J_0(k_1 r)]\cos\omega t$$

$$v_r(r,z,t) \approx B\omega\, e^{-k_1 z}\left[\frac{1}{2}k_1 r + \lambda_1 J_1(k_1 r)\right]\cos\omega t \tag{22}$$

The resonance frequency will be calculated using Rayleigh's principle [8,12]. This principle states that for a harmonically moving non-damped system the sum of the potential energy and the kinetic energy stays constant (no losses). The potential energy is maximal at maximum displacement (velocity zero). The kinetic energy is maximal at maximum velocity (displacement zero).

The maximum potential energy equals the increase in surface energy at maximum displacement ($\omega t = \pi/2 + n\pi$, $n = 1,2,\ldots$):

$$U_{max} = \gamma\Delta A_{max} = \gamma\left[2\pi\int_0^{R_1} rdr\sqrt{1+\left(\frac{dw}{dr}\right)^2} - \pi R_1^2\right]$$

$$= \gamma\left\{2\pi\int_0^{R_1} rdr\sqrt{1+[B\lambda_1 k_1 J_1(k_1 r)]^2} - \pi R_1^2\right\}$$

$$= \gamma \left\{ 2\pi \int_0^{R_1} rdr \left(1 + \frac{1}{2} [B\lambda_1 k_1 J_1(k_1 r)]^2 \right) - \pi R_1^2 \right\}$$

$$= \gamma \pi B^2 \lambda_1^2 k_1^2 \int_0^{R_1} rdr J_1^2(k_1 r)$$

$$= 263.169\, \gamma B^2 \tag{23}$$

The maximum kinetic energy (meniscus is flat, $\omega t = 0 + n\pi$, $n = 1,2,3, ...$) is given by (skipping writing down all the operations involving Bessel functions [11]):

$$T_{max} = \frac{1}{2} \rho 2\pi \int_0^{R_1} \int_0^{\infty} rdrdz (v_2^2 + v_z^2)$$

$$= 2.52094\, \rho R_1^3 \omega^2 B^2 \tag{24}$$

As the maximum potential energy must be equal to the maximum kinetic energy the first resonance frequency must be:

$$U_{max} = T_{max} \rightarrow \quad f_1 = 1.66 \sqrt{\frac{\gamma}{\rho R_1^3}} \tag{25}$$

The numerical value in front of the square root is about equal to the value reported in [3] (1.46 instead of 1.66) and in [1] (1.68 instead of 1.66). For a nozzle of 50 μm diameter the resonance frequency calculated with (25) is 94 kHz, for a nozzle of 25 μm diameter 266 kHz ($\gamma = 0.05$ N/m and $\rho = 1000$ kg/m^3).

As explained in [3] the higher order mode reduces the effective nozzle radius for droplet formation. The effective nozzle radius can be estimated by considering the circle for which the amplitude of the meniscus motion is zero: $r = 0.52129 R_1$, reducing the droplet volume by a factor of about 7.

The method outlined in this section can be used to calculate higher order modes as well. The second mode is given by (see figure 4):

$$k_2 R_1 = 8.41725, \;\; \lambda_2 = -15.5085 \tag{26}$$

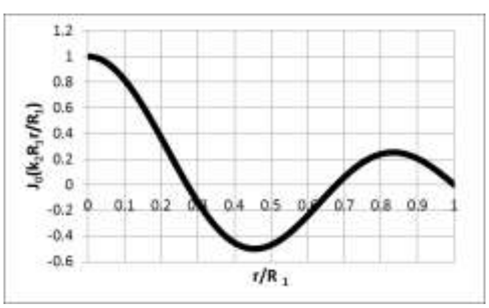

Figure 4: Amplitude of meniscus motion for the second axisymmetric mode according to formula (12) with: $k_2 R_1 = 8.41725$ and $\lambda_2 = -15.5085$.

The penetration depth of the second axisymmetric mode is:

$$k_2 z_{penetration} = \pi, z_{penetration} = \frac{\pi}{k_2 R_1} R_1 = 0.3732\, R_1 \tag{27}$$

The reach of the second mode has been reduced by a factor of roughly two compared to the reach of the first mode and is even more locally. The resonance frequency can be calculated following the same procedure as given for the first mode, with result:

$$f_2 = 3.476 \sqrt{\frac{\gamma}{\rho R_1^3}} \tag{28}$$

Non-axisymmetric meniscus oscillations

At any higher order non-axisymmetric meniscus oscillation it is about the local motion of fluid against higher order non-axisymmetric distortions of the meniscus. The fundamental one is an asymmetric mode, schematically shown in figure 5.

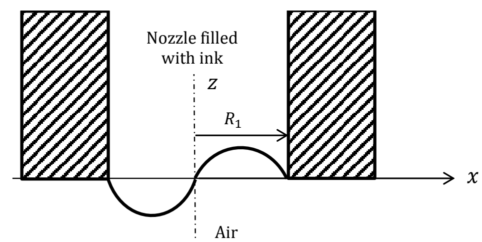

Figure 5: Schematic of non-axisymmetric meniscus distortion.

This problem is essentially three dimensional, but a guess about the kinematics and dynamics can be obtained by considering a 2-D problem of a slot of infinite length and width $2R_1$ filled with ink in which a meniscus is moving harmonically with radian frequency ω up and down as shown in figure 5.

The displacement of the meniscus can be described by (the index $na1$ stands for non-axisymmetric first mode):

$$w_{na1} = B \sin \frac{\pi x}{R_1} \sin \omega t \tag{29}$$

The velocity distribution along the meniscus is given by:

$$\frac{dw_{na1}}{dt} = B\omega \sin \frac{\pi x}{R_1} \cos \omega t \tag{30}$$

The non-zero components of the velocity vector inside the fluid are v_x and v_z, defined with respect to a Cartesian co-ordinate system Oxz (z measures the distance from the meniscus upwards and x gives the distance from the center plane as depicted in figure 5).

Away from the meniscus for the velocity component v_z the following ansatz will be used:

$$v_z = B\omega F(z) \sin \frac{\pi x}{R_1} \cos \omega t \,, \quad F(0) = 1 \tag{31}$$

With the equation of continuity [10] v_x can be found:

$$v_x = B\omega \frac{dF}{dz}\frac{R_1}{\pi}\cos\frac{\pi x}{R_1}\cos\omega t \tag{32}$$

Substitution of (31) and (32) into the components of the equation of motion [10] and neglecting the convective terms (small of order B^2):

$$\rho B\omega^2 \frac{dF}{dz}\frac{R_1}{\pi}\cos\frac{\pi x}{R_1}\sin\omega t = \frac{\partial p}{\partial x}$$
$$\rho B\omega^2 F(z)\sin\frac{\pi x}{R_1}\sin\omega t = \frac{\partial p}{\partial z} \tag{33}$$

Differentiating the x-component of the equation of motion with respect to z and the z-component with respect to x delivers a differential equation for $F(z)$ with solution:

$$\frac{d^2F}{dz^2} - \frac{\pi^2}{R_1^2}F = 0, \quad F = e^{-\frac{\pi z}{R_1}} \tag{34}$$

This solution describes a penetration problem. The negative exponential decays fast to zero. The distance over which the exponential has decreased to 5 %, $z_{penetration}$, is given by:

$$\frac{\pi z}{R_1} = \pi, \quad z_{penetration} = R_1 \tag{35}$$

The motion of the fluid belonging to the first non-axisymmetric mode is limited to a depth of order R_1.

The components of the velocity vector read:

$$v_z = B\omega\, e^{-\frac{\pi z}{R_1}}\sin\frac{\pi x}{R_1}\cos\omega t$$
$$v_x = -B\omega e^{-\frac{\pi z}{R_1}}\cos\frac{\pi x}{R_1}\cos\omega t \tag{36}$$

Again the resonance frequency will be calculated using Rayleigh's principle [8,12]. For a harmonically moving non-damped system the sum of the potential energy and the kinetic energy stays constant (no losses). The potential energy is maximal at maximum displacement (velocity zero and kinetic energy zero). The kinetic energy is maximal at maximum velocity (displacement zero and potential energy zero).

The maximum kinetic energy per unit length equals:

$$T_{max} = 2\frac{1}{2}\rho B^2\omega^2 \int_0^\infty \int_0^{R_1} e^{-2\frac{\pi z}{R_1}}(\sin^2\frac{\pi x}{R_1} + \cos^2\frac{\pi x}{R_1})dxdz$$
$$= \frac{1}{2\pi}\rho B^2 R_1^2\omega^2 \tag{37}$$

The maximum potential energy per unit length follows from the maximum increase in free surface per unit length:

$$U_{max} = \gamma\Delta A = \gamma\left[2\int_0^{R_1} dx\sqrt{1 + \left(\frac{dw}{dx}\right)^2} - 2R_1\right]$$
$$\approx \gamma\frac{\pi^2}{2}R_1\left(\frac{B}{R_1}\right)^2 \tag{38}$$

From the condition that the maximum potential energy equals the maximum kinetic energy the resonance frequency of the first non-axisymmetric oscillatory mode is found [13]:

$$f_{na1} = \frac{1}{2\pi}\sqrt{\pi^3\frac{\gamma}{\rho R_1^3}} = 0.886\sqrt{\frac{\gamma}{\rho R_1^3}} \tag{39}$$

The pre-factor of 0.886 is close to the value reported in [13] (0.596 compared to 0.886 using a different guess of the displacement field).

Discussion

In this paper expressions have been derived for surface tension related resonance phenomena in inkjet print heads. The first resonance phenomenon discussed is called the slosh mode. For this mode all fluid in the print head moves in phase against the surface tension spring in the nozzle. The slosh mode frequency is usually much lower than the key note and overtone frequencies associated to wave effects in the long pump chamber. Such low frequency motions interfere with the droplet motion because at the moment of pulsing at high frequency the position of the meniscus follows the slosh mode motion and can be either slightly retracted or somewhat outside the nozzle. Droplets from a retracted meniscus are smaller and faster compared to droplets jetted from an outside meniscus. It should be noted that the slosh mode frequency depends on the design of pump.

For higher order meniscus oscillations the motion of the fluid is confined to the close neighborhood of the meniscus. The extent over which the meniscus oscillation penetrates into the fluid contained in the nozzle is of the order of magnitude of the radius of the nozzle (see (21) and (34)). Much less mass is moving back and forth and therefore the associated resonance frequencies are much higher. As explained in [12] axisymmetric higher order meniscus oscillations can be used to generate droplets of much smaller size than the nozzle diameter. If in the spectrum of the pulse there is a frequency that coincides with the resonance frequency of an axisymmetric mode, the meniscus is set in motion and small droplets can be jetted [3].

When in the spectrum of the pulse a frequency is present that coincides with the resonance frequency of a non-axisymmetric mode, upon actuation straightness errors may occur. A non-axisymmetric mode can be triggered by a small imperfection of the rim of the nozzle.

Higher order modes are generated by the leading and trailing edges of the pulses used for jetting droplets. So changing the steepness of these edges, the strength of higher order modes can be controlled [1].

The resonance frequencies of the higher order surface tension driven meniscus oscillation modes do not depend on the design of the print head as a whole, but only depends on the radius of the cylindrically shaped nozzle considered in this paper.

References

[1] H. Wijshoff. Structure- and Fluid-Dynamics in Piezo Inkjet Printheads, Thesis University of Twente, The Netherlands 2008.

I.M. Hutchings and G.D. Martin (Editors), Inkjet Technology for Digital Fabrication, Wiley 2013.

S.D. Hoath (editor), Fundamentals of Inkjet Printing, Wiley VCH, 2016.

W. Zapka (editor), Handbook of Industrial Printing, Two Volumes, Wiley-VCH, 2018.

J.F. Dijksman, Design of Piezo Inkjet Print Heads, Wiley-VCH, 2018, Chapter 1.

[2] J.F. Dijksman, Design of Piezo Inkjet Print Heads, Wiley-VCH, 2018, Chapter 5.

[3] P.C. Duineveld and J.F. Dijksman, "Ultra small droplet generation in inkjet printing by higher order meniscus oscillations", in Printing for Fabrication Conference Dresden 2018.

P.C. Duineveld and J.F. Dijksman, "Ultra-small droplet generation in inkjet printing by higher order meniscus oscillations", in The 32nd International Congress on High-speed Imaging and Photonics, Twente, Netherlands, 2018.

[4] A.I.Fedorchenko, "Effect of capillary perturbations on the dynamics of a droplet spreading over a surface", Russ. J. Eng. Thermophys. 10(1), pp. 1-11 (2000).

J.B. Bostwickand and P. H. Steen "Dynamics of sessile drops. Part 1. Inviscid theory", J. Fluid Mech., vol. 760, pp. 5-38, (2014).

C-T. Chang, J.B .Bostwick, S. Daniel and P.H. Steen, "Dynamics of sessile drops. Part 2. Experiment", J. Fluid Mech., vol. 768, pp. 442-467, (2015).

C-T. Chang, J.B. Bostwick, P.H. Steen, and S. Dan iel, "Substrate constraint modifies the Rayleigh spectrum of vibrating sessile drops", PHYSICAL REVIEW E 88, pp 1-17, (2013).

P.H. Steen, C-T. Chnag and J.B. Bostwick, "Droplet motions fill periodic table", PNAS, vol 116, no 11, pp 4849-4854, 2019.

[5] P-G. de Gennes, F. Brochart-Wyart and D. Quéré, Capillarity and Wetting Phenomena, Springer, 2002.

[6] J.F. Dijksman and A. Pierik, Dynamics of Piezoelectric Print-Heads, Chapter 3 in I.M. Hutchings and G.D. Martin (Editors), Inkjet Technology for Digital Fabrication, Wiley 2013.

[7] P-G. de Gennes, F. Brochart-Wyart and D. Quéré, Capillarity and Wetting Phenomena, Springer, 2002.

[8] W.T. Thomson, Theory of Vibrations with Applications, Second Edition, Prentice-Hall, 1981.

[9] M. Abramowitz and I.A. Stegun, Handboek of Mathematical Functions, Dover 1965.

[10] R.B. Bird, W.E. Stewart and E.N. Lightfoot, Transport Phenomena, Second Edition, John Wiley & Sons, 2002.

[11] W. Rosenheinrich, TABLES OF SOME INDEFINITE INTEGRALS OF BESSEL FUNCTIONS, Ernst Abbe Hochschule, 2015.

[12] J.W.S. Strutt, Baron Rayleigh, Scientific Papers, Vol 1, Cambridge at the University Press, 1899, pp 170-181.

[13] J.F. Dijksman, Design of Piezo Inkjet Print Heads, Wiley-VCH, 2018, page 256 (note there is a misprint in formula 5.125, under the square sign a factor of 2 is missing).

Author Biography

Paul C. Duineveld *graduated and did his PhD at Twente University under supervision of Leen van Wijngaarden on bubble dynamics. After his service as Navy officer he started at Philips Research working, together with Frits Dijksman, mainly on inkjet printing for display and bio-sensor applications. In 2004 he moved to Philips Consumer Lifestyle where he started working on what is now Philips AirFloss. In 2007 he became a director of Engineering in the field of Fluid Dynamics and in 2008 a DFSS BB. He is the principal in a team of 8 people working on fluid dynamic applications in household and medical appliances from sleep apneu, vacuum cleaners, irons, air purifiers, baby bottles, toothbrushes, fruit juicers, coffeemakers etc.*

J. Frits Dijksman *obtained his master's degree in mechanical engineering at the Technical University of Delft in The Netherlands in 1973. He finished his PhD within the groups of Professor. D. de Jong and Professor W.T. Koiter (Technical University of Delft, The Netherlands, 1978) focussing on the engineering mechanics of leaf spring mechanisms. He worked with Philips Research Laboratories in Eindhoven, The Netherlands for 32.5 years, mainly on inkjet printing, rheology of polymers and medical devices. After his retirement he continued his work as part time professor at the University of Twente, The Netherlands. The topics include inkjet printing of viscoelastic inks, design of inkjet print heads and printed biosensors. He is now emeritus professor.*