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Generalized Computational Halftone

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Abstract

A basic premise of forming a halftone image pattern on a two-dimensional substrate receiver in a traditional printing process is that there exists an identical spatial mapping relationship between the original image and its reproduced counterpart. As the printing technology being adopted to become a manufacturing process on flexible substrates as well as three-dimensional objects, this inherent property is no longer valid. Furthermore, similar to all manufacturing processes with mass customization capabilities, the natural evolution for the existing digital printing technology is to become an autonomous process with minimal operator interference. Therefore, a new paradigm of a general halftoning algorithm is proposed where the final image formation is controlled by the local geometry, the intended exposure and instantaneous feedback compensation signals.

Introduction

Halftoning algorithms, such as area-modulation, frequency-modulation and error-diffusion, have been used in existing printing processes to create perception of smooth tone scale gradation virtually indistinguishable from the rendition by a continuous tone process [1]. Although overlaying multiple halftone images to reproduce intended color encounters the fundamental problem of inducing signal interference patterns, denoted as the Morié artifact, it provides a significant advantage in terms of spatial and temporal process stability. Furthermore, a basic premise of forming a halftone image pattern on a two-dimensional flat substrate receiver in a traditional printing process is that there exists an identical spatial mapping relationship between the original image and its reproduced counterpart. As the printing technology being adopted to be a manufacturing process on flexible substrates as well as three-dimensional objects, this inherent identity correspondence is no longer valid [2, 3, 4].

An interesting development in the field of additive manufacturing is the concept of *digital material* [5, 6, 7]. The characteristics of traditional three dimensional fabrication technologies, such as powder bed fusion, material extrusion and direction energy deposition, are still inherent analog, where material is continuously added and bonded isotropically to the unfinished part. Conversely, the digital material consists of a collection of unique discrete components serving as fasteners and index fixtures [6, 7]. There is only very limited number of possible connections between two discrete components. This physical constraint greatly reduces the number of possible shapes of the final product, which, consequently, minimizes the fabrication process variability. As a result, the advancement of digital material might open the door for the next generation 3D additive manufacturing process without expensive high-precision calibration and monitoring systems. Evidently, the concept of the digital material appears abundantly in nature as well as construction toys such as *LEGO*$^{\text{TM}}$. For example, the atom bonding geometries among polymers and deoxyribonucleic acid (*DNA*) are highly constrained to allow them to form a final structure without external guidance. Furthermore, similar to all manufacturing processes with mass-customization capabilities, the natural evolution for the existing digital printing technology is to become an autonomous process with minimal interference. Therefore, a new paradigm of a general halftoning algorithm is needed to actively construct the target object/surface based on steerable halftone structures where the final image formation is controlled by the local geometry, the intended exposure and instantaneous feedback compensation signals simultaneously.

As the image deposition surface begins to deviate from a planar surface, the classical Euclidean geometry inherent to the existing halftoning processes is inadequate to provide a unified solution space to meet future challenges. This is analogous to the revolutionary revelation of the general relativity that, unlike the classical Newton mechanics where the *space* and *time* are independent from each other in a absolute space, the *spacetime* continuum of each individual object curved by celestial bodies is best described within the mathematical construct of the differential geometry [8], Thus, during the image formation process on a three dimensional regular surface **S**, a general halftoning algorithm should be associated with a local coordinate system at the neighborhood of the intended imaging location **p** according to the principles of differential geometry:

1. The intended halftone structure should be parallel to the tangent plane $T_p(S)$ of the predefined regular surface **S** at the location **p**, which is determined by the intended color/shape specification.
2. The normal vector is responsible to describe how each halftone dot grows in the local three dimensional space **W**.

The proposed general halftoning algorithm will be able to emulate the heterogeneous biological membrane architectures between different layers through an affine transformation of the local coordinate system [9, 10]. One immediate derivation from the differential geometry is that the first fundamental forms of the cylinder and the plane are equivalent even though their surfaces appear to be distinct [8]. This provides the mathematical explanation why using the successive cylinder roller transfer architecture in existing commercial printing technologies works properly through two-dimensional halftone matrices. Furthermore, any deviation from the perfect cylindrical surface will result in spatial and temporal variations in the unit vector spanning the tangent plane $T_p(S)$, which often result in perceivable color inconsistency. Therefore, the general halftoning algorithm has the capability to minimize the color inconsistency artifact using the destructive interference principle with a precise surface sensing mechanism.

Theoretical Background

The primary objective of a halftone image formation process is focused on achieving proper tone scale with minimal perceivable image artifacts, such as *moiré* and color contour [1]. This basic operation can be summarized in Figure 1, where the image pixel location $(x_i, y_i, 0)$ relative to the image origin is unchanged and the intended color value C_i is mapped to the intended halftone space H_i via the selected halftone process. Thus, there exists a one-to-one and isometric transformation between an image pixel and the associated halftone tile element.

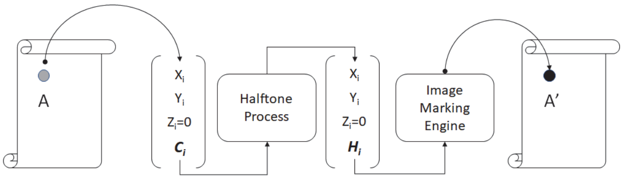

Figure 1. *Basic Process*

In reality, the final reproduced image is always corrupted by geometric and tonal disturbances in various degree, such as misregistration and nonuniformity artifacts, which is illustrated in Figure 2. While traditional printing processes overcome the problem by tightening the manufacturing specification requirement of each imaging component, researchers have proposed different active signal/image correction algorithms to minimize the perceivable artifacts in the digital printing industry [11, 12, 13]. Several digital press optimization architectures have been successfully adopted in commercial equipments and shown their effectiveness against specific image artifacts. Unlike the ideal basic process, the inherent effect of geometric distortion dislocates the spatial location of final reproduced image pixel relative to the associated halftone tile element, which is usually compensated after the traditional halftone process as shown in Figure 2.

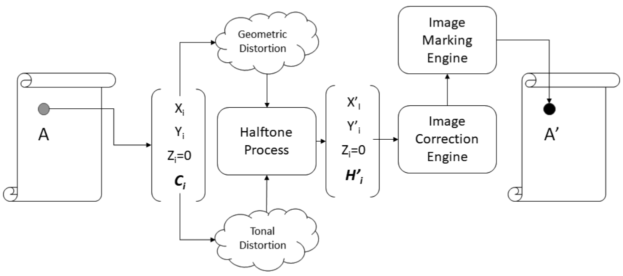

Figure 2. *Perturbed Process*

As the digital printing technology continues to evolve and takes on the role of digital fabrication, not only the production accuracy specification is no longer bounded by human visual acuity but also the intended substrate surface is not restricted to a two-dimensional planar surface with height $Z_i = 0$ as indicated in Figure 1 and Figure 2. Consequently, it is very difficult to adopt the standard predefined halftone tile architecture on a regular three-dimensional surface. Furthermore, similar to the digital printing technology, combining digital halftone algorithm with active noise cancellation capability will benefit significantly the fabrication accuracy, robustness and manufacturing yield rate [14]. Let u be the unit vector representing the substrate moving direction and v be the unit vector parallel to the digital printhead, a regular surface $x(u,v)$ can be parameterized as follows[8]:

$$\mathbf{x}(u,v) = (x(u,v), y(u,v), z(u,v)), \tag{1}$$

where

1. $x(u,v)$, $y(u,v)$, and $z(u,v)$ have continuous partial derivatives of all orders.
2. $\mathbf{x}$ is a homeomorphism and there exists a continuous inverse mapping $\mathbf{x}^{-1} : \mathbf{R}^3 \to \mathbf{R}^2$.
3. $d\mathbf{x} : \mathbf{R}^2 \to \mathbf{R}^3$ is one-to-one.

Therefore, a generalized computational halftone algorithm is proposed in Figure 3. The *Correction algorithm* first converts a local neighborhood on a regular surface to the image domain (u,v). The ensuing *Computational Halftone process* numerically fabricates the targeted regular surface in a halftone formation before sending to the associated image marking engine.

Generalized Computational Halftone

Assuming the intended imaging surface is a regular surface S, we can use the following proposition to further simplify the proposed computational halftone algorithm [8]:

Theorem Let $S \in \mathbf{R}^3$ be a regular surface and $\forall p \in S$. There exists a neighborhood $V \subset S$ of p such that V is a graph of a differentiable function with one of the following three forms: $z = f(x,y)$, $y = g(x,z)$ or $x = h(z,y)$ where $Oxyz$ is a coordinate system in $\mathbf{R}^3$.

Without loss of generality, we can assume that there exist linear mapping function $L : (u,v) \to (x,y)$ and the neighborhood v of current location $p \in S$ is represented as $z = f(x,y)$. Hence, Equation 1 can be rewritten as follows:

$$\mathbf{x}(u,v) = (x(u,v), y(u,v), f(x(u,v), y(u,v))). \tag{2}$$

Figure 4 illustrates the mathematical relationship between the imaging plane in the uv coordinate system to the pattern fabrication surface in the $Oxyz$ coordinate system. The vector basis defining the tangent plane $T_p(S)$ at any $p \in S$ is $\phi_1 = (1,0,f_x)$ and $\phi_2 = (0,1,f_y)$ in the $Oxyz$ coordinate system, where $f_x = \frac{\partial f}{\partial x}$ and $f_y = \frac{\partial f}{\partial y}$. To map from xy surface plane to uv image plane, it is necessary compute the Jacobian matrix $\frac{\partial(x,y)}{\partial(u,v)}$:

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix}. \tag{3}$$

When $(u,v) = (x,y)$, the Jacobian matrix $\frac{\partial(x,y)}{\partial(u,v)}$ becomes a 2×2 identity matrix. Furthermore, the L_2 norm of a basis vectors e_u and e_v on the imaging plane after mapped to the regular surface

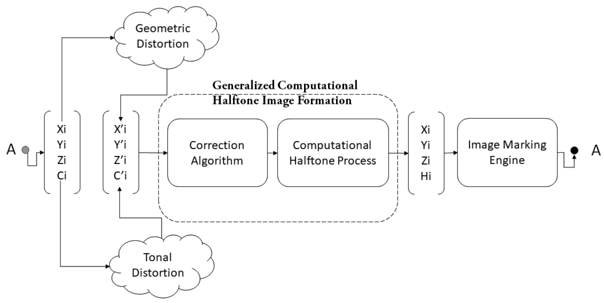

Figure 3. *Generalized Computational Halftone Process*

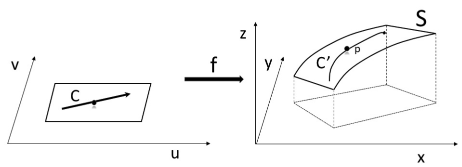

Figure 4. *Image surface mapping*

S are amplified by $\sqrt{1+f_u^2}$ and $\sqrt{1+f_v^2}$ respectively:

$$\begin{aligned} \tilde{e_u} &= \sqrt{1+f_u^2}e_u \\ \tilde{e_v} &= \sqrt{1+f_v^2}e_v. \end{aligned} \quad (4)$$

Thus, the halftone screen frequencies generated by the digital writing system along the in-track orientation, u, and cross-track orientation, v, have to increase by the same factor in order to maintain the consistent halftone screen frequency at the imaging location p on the regular surface S. When the surface is perfect cylinder, the imaging location p is normally at a critical point where the first derivative $df(x_p, y_p) = \mathbf{0}$. Therefore, we demonstrate that there is a identical correspondence between the digital halftone image and the final reproduced image on the cylindrical roller.

We can readily deduce from the previous analysis that forming a amplitude-modulated halftone pattern on a regular three dimensional surface requires the generalized halftone algorithm to locally adjusting the halftone dot area, average exposure and spatial halftone frequency according to the requested image code value, the gradient at the imaging location and local geometric and/or tonal distortion. The proposed computational halftoning process is illustrated in Figure 3 and the steps are summarized as follows [14]:

1. Compute the local gradient at the intended imaging location.
2. Scale the intended AM halftone frequency and screen angle into the coordinate basis $W = \{\omega_1, \omega_2\}$ based on Equation 4.
3. Convert W for the halftone image space, uv.
4. Define the intended tone scale for the associated halftone screen.
5. Upsample each image pixel p to $\{p_{ij} | i = 1 \cdots n_1, j = 1 \cdots {}_n2\}$ and transform each subpixel to the halftone domain via W.
6. Compute the distance to the closest halftone dot and normalize between 0 and 1.
7. Compute the halftone code value for each subpixel based on the halftone dot formation function and the input image code value, which can be modified to compensate for potential static or dynamic image nonuniformaity artifact.
8. Compute the image pixel code value associated with the intended halftone screen by averaging all subpixels.

Experimental Results

Four simulation experiments were conducted to verify the validity of the proposed generalized computational halftone algorithm:

1. Planar surfaces with constant gradient including 0.
2. Increasing gradient along x axis.
3. Increasing gradient in x axis and decreasing gradient in y axis.
4. Varying gradient in the xy domain.

For a locally smooth surface S, it is possible to approximate S by piecewise linear functions. Thus, it is of interest to demonstrate the performance of the proposed algorithm with constant gradients, which is shown in Figure 5.

Figure 6, 7 and 8 illustrate the resulted computational halftone patterns when the approximation resolution space is expanded to quadratic functionals. The proposed computational halftoning algorithm is able to morph each halftone dot continuously at each imaging location p according to the tangent plane $T_p(S)$ on the regular surface $S \in \mathbf{R}^3$.

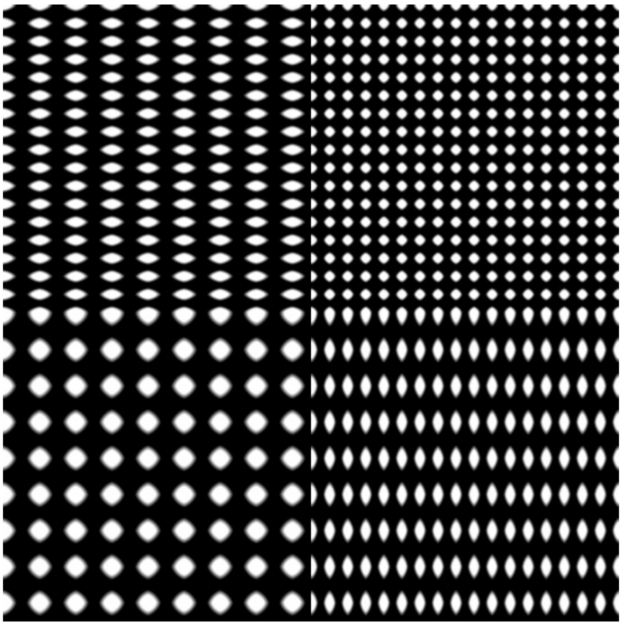

Figure 5. *Halftone screen structure on planar surfaces*

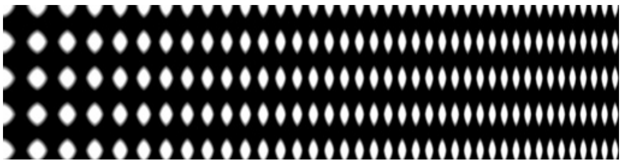

Figure 6. *Halftone screen structure on curved surface with increasing gradient in x-axis*

Figure 7. *Halftone screen structure on curved surface with increasing gradient in x-axis and y-axis*

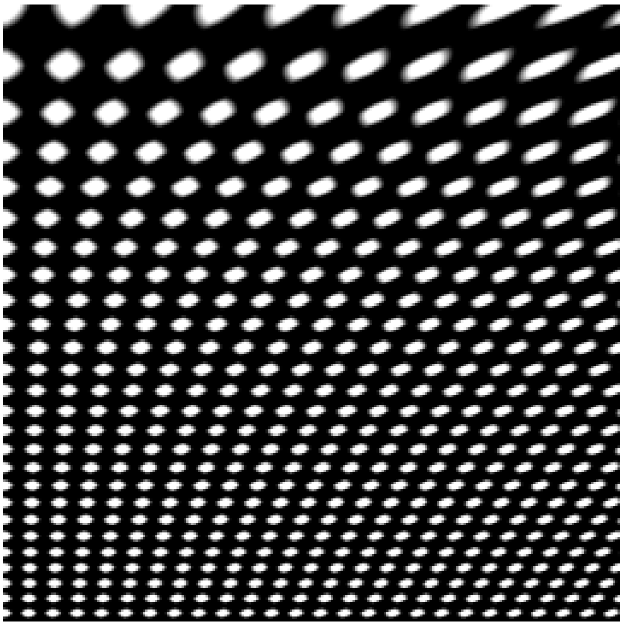

Figure 8. Halftone screen structure on curved surface with varying gradient in the xy imaging domain

Conclusion and Future Works

A generalized computation halftoning algorithm is proposed to form an AM halftone screen image on a regular surface $S \in \mathbf{R}^3$ based on the differential geometry, where the proposed algorithm is simplified to a standard halftoning process when S is a perfect cylindrical or planar surface. Experimental results based on computer simulation demonstrate the effectiveness of the algorithm in continuously following a piecewise quadratic surface function. We plan to extend the computer simulation experiment to use actual computer 3D color models.

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Author Biography

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