FM Screening Algorithm of Multiplicative Congruence Pseudo-random based on Dot Gathering Model

Xiao Zhou, Ruizhi Shi, Da Li; Zhengzhou Institute of Surveying and Mapping; Zhengzhou; Henan Province/ China

Abstract

In FM screening techniques, using pseudo-random generator to ensure exposure dots is a method to generate screen dot. Dots generated by traditional algorithm scatter greatly which leads to dot gain and cannot accurately reproduce manuscript's color and tone. On the basis of traditional algorithm, this paper brings forward a multiplicative congruence pseudo-random algorithm based on dot gathering model. It firstly puts forward the modeling thought and screening project, and then sets multiplicative congruence pseudo-random algorithm as basic model to ensure pseudo-random number generator. On the analysis of dot data, it sets up the dot gathering magnitude model according to pixel grey level. Then coordinate system on the halftone unit is set up to ensure dot coordinate. Finally, dots are generated. By designing screening experiment, it compares this algorithm with traditional ones and evaluates the quality of halftone images and printed images respectively. Experiment testifies that this algorithm has the advantages of FM screening, whose printing characteristics are better than traditional AM screening and dither screening.

Introduction

Digital halftoning is also called digital screening. In FM screening, other than traditional dither screening, another method to generate FM dot is using pseudo-random function to generate dot number. In the halftone unit, exposure dots and non-exposure dots are ensured by the sequence number of recorder grid. Pseudo-random generator's quality and the sequence number of recorder grid influence dot shape and structure greatly. Therefore, generator quality and the efficiency of generating number should be testified[1][2]. Under the condition of ensuring the pseudo-random generator which can generate all grey level numbers to express the manuscript tone, recorder grid numbers are recombined. Relationship model between pseudo-random function and recorder grid number is an effective way to generate FM dots with optimized structure. On the basis of multiplicative congruence pseudo-random generator, this paper sets up a dot gathering algorithm model which can effectively generate hybrid dots of first order and second order with the combination of dot gathering and scattering. It can control dot gain and optimize FM dot structure[3][4].

Modeling thought and screening project

Modeling thought

In the pseudo-random screening, one pixel corresponds to one halftone unit. Halftone unit is divided into 16×16 recorder grids with numbers to express 256 grey levels. Certain algorithm is used to generate pseudo-random numbers with the same value as pixel grey level, and then they correspond to the recorder grid numbers to ensure the exposure dots. If grey level is K, there will be amount of K recorder grids to expose. The periodicity and statistics characters should meet the needs of generating dot numbers. In order to generate FM dot corresponding to grey level, pseudo-random generator should be invoked no fewer than K times. Number values generated should not exceed the range of grey level. Meanwhile, to avoid grey level loss, repeated numbers should be abandoned and the amount of K different numbers is finally got[3].

1. Foundation of basic model

Multiplicative congruence pseudo-random function has the characters of simple, high efficiency, long periodicity and generated pseudo-random numbers with perfect statistics characters. It meets the need of homogeneity test and independence test, and can also generate dot numbers of 0~255 grey levels. Multiplicative congruence pseudo-random generator is shown as Formula 1.

$$x_{n+1} = x_n \cdot a \pmod M \qquad (1)$$

Parameter a and M have the following relationship. $M= 2^s$， $a = 8k\pm3$， $a= 2t+1$. Pseudo-random numbers are generated by the former pseudo-random number multiplying a and dividing M to get the remainder. Parameter M and X_0 are relatively prime numbers. Parameter X_0 and a are relatively prime numbers. Number s is a positive integer. Parameter a usually process numbers close to $a \cong 2^s/2$ and meets the formula of $a= 8k\pm3$. Number K is a positive integer. In practical application, if the purpose of generating pseudo-random numbers is to ensure dot numbers, the optimized parameters are not confined to this condition[5][6].

2. Dot gathering model

If an exposure dot is seen as a first order screen dot, four exposure dots uniformly gather together and can be seen as a second order screen dot. According to geometry relationship, the more the first order dots in the halftone unit are, the longer the total dot perimeter will be, and the severer the printing condition will need. Therefore, the purpose of improving dot structure is to decrease the amount of independent first order dots and gather first order dots to second order ones, which can decrease total dot perimeter and reduce the printing difficulty. Meanwhile, dot gathering and scattering should be harmonized, for which over gathering may weaken the advantages of FM screening and insufficient gathering may not improve dot structure. According to printing principle, second order dots gathered by 4 or 9 first order dots have perfect printing characteristics and can express FM screening advantages. This paper recombines sequence numbers of recorder grids and sets up dot gathering model under the condition of pseudo-random generator with the thought of 4 or 9 first order dots gathering[7].

Foundation of screening project

1. Confirmation of pseudo-random generator and generating of dot numbers

According to parameters relationship, parameter M of the multiplicative congruence pseudo-random generator can get minimum value 255 in the grey level range of 0~255 while its maximum value is four times of the minimum value. Confirm the parameters by the relationship and figure out the M values. They are 1024, 512 and 256 while the a values are 29, 35, 21, 19 and 13. According to generator's characters, the smaller the a and M values are, the higher the efficiency to generate numbers will be[8]. Basic pseudo-random generator confirmed in the paper is shown as formula 2.

$$x_{n+1} = x_n \cdot 13 \left(\mathrm{mod}\, 256 \right) \quad (2)$$

Set the initial value 1, and invoke the function to generate pseudo-random numbers to meet the grey level 100, as Figure 1 shows.

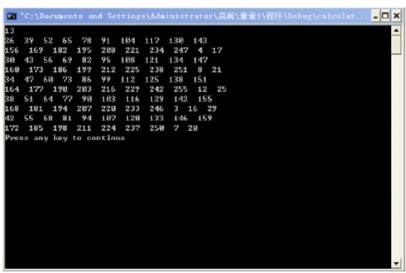

***Figure 1.** Pseudo-random numbers generating*

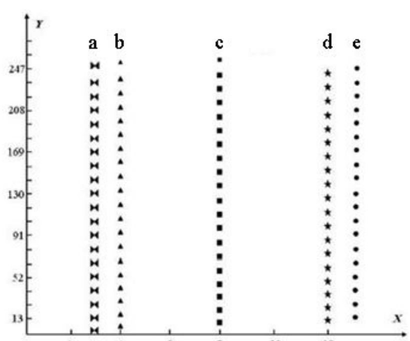

***Figure 2.** Coordinate system of pseudo-random numbers*

2. Dot number analysis and foundation of screening project

Analyze the pseudo-random numbers. All 100 pseudo-random numbers can be grouped into several arrays with different initial values. Pseudo-random numbers in Figure 1 can be grouped into the following arrays.

a: {3，16……237，250，7 ,20}；

b: {4，17……238，251}；

c: {8，21……242，255}；

d: {12，25……233，246}；

e: {13，26……234，247}；

Set each array's initial value as the abscissa and the values of the array as ordinate to set up coordinate system, as Figure 2 shows. In the same array, among the dot numbers with the same abscissa and pattern shape, adjacent pseudo-random numbers have the tolerance 13, which is the parameter a. Different pseudo-random numbers have the discrepancy of 13's integer multiple, meanwhile, corresponding pseudo-random numbers in different arrays have the same tolerance. Pseudo-random arrays and generated dot structure will vary with different pseudo-random functions and different initial values. Recorder grid numbers are sequenced and gathered to generate second order dots according to parameter a. Dot gathering project is as follows.

1) Project one. Gather numbers with 13 or 13's integer multiple.

2) Project two. Gather numbers with the same tolerance in different arrays.

Under the condition of image setter 2400dpi and screen line number 150lpi, 20μm and 30μm have perfect printing characteristics. Therefore, 4 or 9 dot gathering will generate perfect dot structure[9][10]. As generated dot numbers are discrepant greatly, the locations of gathered dots may be discrepant, too. In the halftone unit, this discrepancy directly determines the dot structure. The key to realize FM screening is ensuring all the halftone units to have different dot number gathering structures respectively which can lead to different dot structures.

Algorithm modeling

Confirm the starting point of dot gathering in the halftone unit. Set the left top angular point of halftone unit as the origin and set up XY coordinate system. Initially number the recorder grid 0~255 according to the sequence from top to bottom and from left to right. In the coordinate system of halftone unit, set the recorder grid coordinate as (i, j), the recorder grid number is P. Model one is set up as follows.

$$P = 16i + j + 1 \quad (3)$$

In the model, $i, j \in Z$，$0 \le i, j \le 15$. Parameter i and j are the ranks numbers of recorder grid. Each recorder grid number corresponds to one unique coordinate. Modeling steps are as follows.

1) Step one. Confirm the amount of dots with different size.

First order dots and second order dots have different size. According to the pixel grey level K, the amounts of 4 dots gathered screen dots (*Dots two*) and 9 dots gathered screen dots (*Dots two*) are m and n respectively. Meanwhile, set the amount of surplus exposure dots is s. Model two is set up as follows.

$$K = 4m + 9n + s \quad (4)$$

In the model, $0 \le m \le |P/m|$，$0 \le n \le |P/n|$，$0 \le s \le K$.

2) Step two. Confirm screen dot coordinate.

According to the amount of m and n, the location of *Dots one* and *Dots two* and their coordinates in the XY coordinate system are confirmed. Dots centers are set as the first exposure dot on the left top, as Figure 3 shows. Select m coordinate combinations E: $(i_1, j_1), (i_2, j_2), \dots, (i_m, j_m)$ as the location of *Dots one*. Select another n coordinate combinations F: $(i_1, j_1), (i_2, j_2), \dots, (i_n, j_n)$ as the location of *Dots two*. Meanwhile, as the dot centers are set as the left top exposure dots, the right and the bottom dots should not exceed the boundary of halftone unit. Thus coordinates should meet the condition of $0 \le i, j \le 13$.

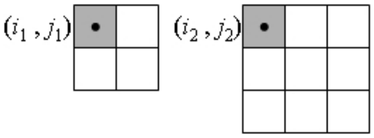

Figure 3. Dot center and dot generating model

3) Step three. Dot gathering and screen dot generating.

Project one. Gather numbers with 13 or 13's integer multiple.

Randomly select one array from arrays a~e, and randomly select 4 or 9 elements form the selected array. Generate one screen dot of *Dot one* or *Dot two* starting from one random coordinate of *E* or *F* group. Elements in each group can merely be selected once. According to the same method, all screen dots are generated and all elements in the arrays are used out. The surplus *s* exposure dots should adhere to the second order dots to control dot perimeter.

Project two. Gather numbers with the same tolerance in different arrays.

Randomly select two or more arrays from arrays a~e, and randomly select 4 or 9 elements form the selected arrays. Generate one screen dot of *Dot one* or *Dot two* starting from one random coordinate of *E* or *F* group. Elements in each group can merely be selected once. According to the same method, all screen dots are generated and all elements in the arrays are used out. The surplus *s* exposure dots should adhere to the second order dots to control dot perimeter.

Project one and project two can both generate hybrid of first order dots and second order dots. Figure 4 and Figure 5 are one type of dot gathering model and dot structure.

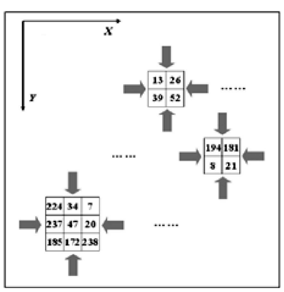

Figure 4. One type of dot gathering model

Figure 5. One type of dot structure

Experiment and analysis

Experiment design

Choose one image with bright color, perfect tone distribution, and excellent expressive force as manuscript image. After four-color separation, M plate image is chosen as screening object, as Figure 6 shows. In order to observe screening effect, low resolution manuscript and output resolution are set as screening conditions. AM screening with square shape dots of 45° screen angle and 150lpi screen line number, Bayer dither screening and Error diffusion screening are used to screen M plate image as control group, and halftone images ①, ② and ③ are obtained. Then dot gathering algorithm is used to screen the same image as experimental group, and halftone image ④ is obtained. Halftone images' quality of control group and experimental group are evaluated.

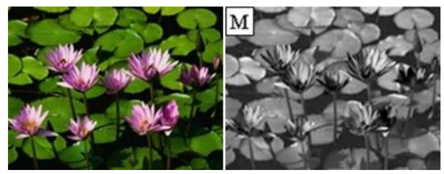

Figure 6. Manuscript image and M plate image

Generating of halftone image

Under the condition of image setter 2400dpi, halftone unit is divided into 16×16 recorder grids. On the computer screen, black pixels are used as exposure dots to display halftone image. Obtain manuscript image's pixel grey level values and elements of $N \times M$ dimension matrix. Halftone image matrix composed by halftone unit corresponding to pixel is set up. Extend each halftone unit to $16N \times 16M$ dimension matrix, as Figure 7 shows.

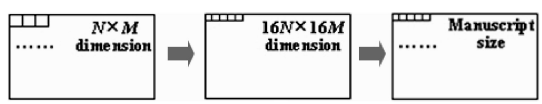

Figure 7. Relationship of manuscript, pixel and halftone image

Use three screening algorithms in control group to screen M plate image and get halftone images ①, ② and ③. Use dot gathering algorithm to screen the same image and get halftone image ④. Amplify the same zone in these halftone images to 300% to observe dot structure, as Figure 8 shows.

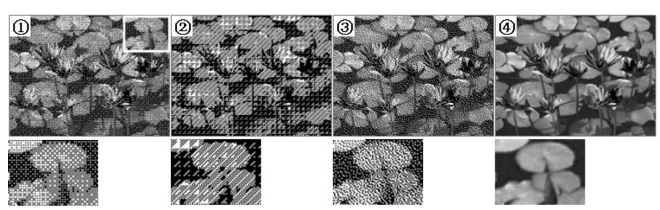

Figure 8. Halftone image and dot amplification image

Image quality evaluation

1. Dot structure evaluation

Observe halftone images ①, ② and ③ in control group. Halftone image ① of AM screening is relatively elaborate, single pixel in manuscript image corresponds to gathered multi-dots.

After amplification, dot shape is square, screen angle is 45° while dots have fixed interval in space. Bayer dither screening aims at single pixel and directly processes it, whose halftone image ② is inferior to AM screening. Halftone image ② has periodical disturb outline which influences visual effect. After amplification, dots are scattered. In the same period, dots distribute randomly. Error diffusion screening also aims at single pixel. Its advantage superior to Bayer dither is that it diffuses the error produced by threshold comparison to pixels around, which reduces the error impact on single pixel. Thus halftone image ③ is more elaborate than AM screening halftone image ① and Bayer screening halftone image ②. After amplification, dots are scattered. In the whole halftone image range, dots are also scattered. Observe experimental group halftone image ④. Halftone image of dot gathering algorithm is far more elaborate than the three halftone images in control group. Single pixel corresponds to multi-dots to generate scattered second order screen dots. Compared with control group, amplified halftone unit of the experimental group's halftone image has more elaborate and refined dot structure, and it has more dots. When it expresses 0~256 grey levels, it has better effect.

2. Image printing quality

Screen the four color plates in control group and experimental group and then overprint them respectively to get printed images. Observe printed images, AM screening printed image in control group inferior to Bayer dither screening and error diffusion screening in color and tone expressive force, which proves that FM screening's printing quality and effect are better than AM screening. Printed image of dot gathering algorithm has the best color and tone expressive force, and the printed image is the most elaborate one. Meanwhile, dot gathering algorithm's dot gain is relatively severe. In order to accurately restore manuscript image, it is necessary to make dot compensation.

Conclusion

Dot gathering algorithm brought forward in this paper is based on multiplicative congruence pseudo-random basic model. It uses random selection method to recombine and gathers pseudo-random numbers to generate hybrid structure of first order dots and second order dots. After analysis of pseudo-random screening, it brings forward dot gathering algorithm and analyzes dot numbers and structure. By modeling, it studies the specific algorithm to generate FM dots and halftone image. Finally, an experiment is designed to compare control group and experimental group to testify that the dot gathering algorithm has FM screening characters and better printing quality than traditional AM screening and dither screening.

References

[1] Kazuhisa Yanaka, Hideo Kasuga and Yasushi Hoshino, Digital Halftoniing Based on a Repulsive Potential Model, NIP17: International Conference on Digital Printing Technologies, 2001

[2] J.C. Stoffel, And J.F. Moreland , A Survey of Electronic Techniques for Pictorial Image Reproduction, IEEE Transactions on Communications, Vol. Com-29, No.12, December 1981

[3] P.W.Wong and N.D.Memon, Image Processing for Halftones, IEEE Signal Processing Magazine, 1053-5888/03 2003IEEE

[4] Marius Pedersen, Fritz Albregtsen and Jon Yngve Hardeberg, Detection of Worms in Error Diffusion Halftoning, Proceedings of Conference on Electronic Imaging, 2009

[5] Keith T. Knox, Error Image in Error Diffusion, 268/SPIE Vol. 1657 Image Processing Algorithms and Techniques III, 1992

[6] Bruno J. Schrappe, Hybrid Screening Algorithm, Proceedings of TAGA, 1998

[7] Daniel L. Lau, Gonzalor. Arce, and Neal C. Gallagher, Green-Noise, Digital Halftoning, Proceedings of the IEEE, Vol.86, No.12 December 1998

[8] J. Sullivan, L. Ray and R. Miller, Design of Minimum Visual Modulation Halftone Pattern, IEEE Trans. on Systems, Man and Cybernetics, Vol.21, No.1 January/February 1991

[9] Garth R. Oliver and Jerry J. Waite, Demystifying the Halftoning Pricess: Conventional, Stochastic, and Hybrid Halftone Dot Structure, The Technology, 2006

[10] Qing Yu and Kevin J. Parker, Adaptive Color Halftoning for Minimum Perceived Error Using the Blue Noise Mask, Proc. SPIE 3018, 1997

Author Biography

Xiao Zhou, born in 1986. He is a doctoral candidate and his main research interest is digital halftoning.

Dr./Prof. Ruizhi Shi, born in 1962. She is a Ph.D. supervisor. Her main research interests include image processing and graphic communication integration.

Da Li, born in 1992. He is a graduate student and his main research interest is digital image processing.