# Research on Moiré Fringe in Frequency Conversion Amplitude-Modulated Screening and Its Equation Derivation

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#### **Abstract**

The concept of frequency conversion amplitude modulation (FCAM) screening was proposed in this paper, and the generation and variation of Moiré Fringe in FCAM screening were studied in this paper. We found the original Tollennar Moiré formula was not suitable for FCAM, to the forecast and calculation of Moiré Fringe in FCAM screening, we should establish the new formula. This paper provided the new Moiré Fringe formula based on the curve fitting, it could accurately describe and forecast the rules of Moiré Fringe in FCAM screening; In the meanwhile, this paper revised the mathematical model of Moiré Fringe based on the Fourier optics in the frequency domain, The new model was suitable for the Moiré Fringe calculation of the same frequency and frequency conversion AM screening.

#### Introduction

The AM screening is the most commonly used screening method in current color image printing or hardcopy output. But when any two AM halftone images are overlapped, the Moiré Fringe will occur, its size and shape will affect the visual effects of copy image, so it is an important topic of how to controlling and reducing the effect of Moiré Fringe in image printing output. At present, the researches of Moiré Fringe mainly focus on the same frequency AM screening method, although they solve the Moiré Fringe problem preferably in four-color printing, they are powerless for the screening of high-fidelity printing image more than four-color. Therefore, the author proposed principles and methods of frequency conversion AM screening in reference [1], that is, using AM screen dot of different frequencies to superimpose printing images and variation rule of Moiré Fringe varying with the screening angle. We could achieved the screening of high-fidelity image more than four-color[2]. In our research, in the FCAM screening, the generation and variation of Moiré Fringe were different from the same frequency AM screening, they didn't follow the Tollennaar formula[3]. Therefore, we needed study and summarize the generating phenomena, rules and theory mechanisms of Moiré Fringe in the FCAM screening, revise and derive the Moiré Fringe formula. This paper provided the new Moiré Fringe formula based on the curve fitting and analysis of FCAM screening, it could accurately describe and forecast the sizes and variation rules of Moiré Fringe in FCAM screening. This paper revised the mathematical model of Moiré Fringe based on the Fourier optics in the frequency domain, established the new mathematical model, The new model was suitable for the Moiré Fringe calculation and forecast of various frequencies AM screen superimposing, the Moiré Fringe of same frequency AM screening was a special case of Moiré Fringe in frequency conversion screening.

# Variation rule of Moiré Fringe in frequency conversion AM screening Moiré Fringe phenomenon in same frequency AM screening

Moiré Fringe is the inevitable physical phenomenon in image screening, the size and shape of Moiré Fringe will affect the visual effects of printing image enormously, they are the important parameters and must be restricted in screening image output. In conventional four-color printing, the screening method of image output is generally using the same lpi AM screening. Both the theory and experiment have proved that the Moiré Fringe of two screens overlapping to same frequency AM screening can be expressed by following Tollenaar formula:

$$P_{M} = \frac{P_{1} \cdot P_{2}}{\sqrt{P_{1}^{2} + P_{2}^{2} - 2P_{1}P_{2}\cos\theta}}$$
(1)

Where, P1 is the space of screen 1, P2 is the space of screen 2, PM is the space to Moiré Fringe,  $\theta$  is the overlap angle of two screens[3][4].

For example, the Moiré Fringe occurred when two screens with 175 lpi overlapped, now,P1= P2, according to the formula (1), the theoretical calculations and actual measurements were consistent, we obtained the curve O as shown figure 1. Figure 1 showed that we could obtain the descending curve of fringe spacing when the overlap angle ranged from 0° to 45°, which is infinite at 0° and minimum at 45°. We generally believed that the Moiré Fringe was very small after 30°, visual range was acceptable.

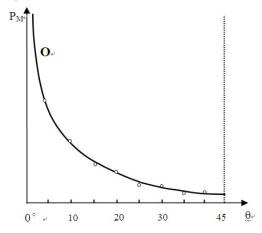


Figure 1. Moiré Fringe curve of same frequency AM screening

## Changes of the FCAM Moiré Fringe

The FCAM is a screening process based on the AM, which adopts varying screening rulings (frequency) to process images (screening) and generates superimposed pictures before or behind output. It has been found that the FCAM is clearly different from the traditional AM with respect to the relationship of Moiré Fringe sizes and screen angles. To simplify research, we began with the case of Moiré Fringe containing two screens. Given that the screening rulings used were 175dpi and 120 dip, respectively, the ruling difference whereof is 55 dpi. The sizes of Moiré Fringe calculated using Eq. (1) change with the screen angles of screening ruling, as shown in Figure 2. However, the sizes of Moiré Fringe experimentally tested are not identical with the calculated values, tested values shown in Figure 3. Regarding to other FCAM cases, where values of screening ruling variation are larger than 25 dpi, the changing trend of Moiré Fringe sizes exhibits similar tendency as Figure 3. All of them are discriminatory to Figure 2.

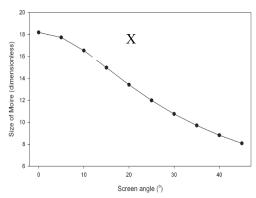


Figure 2. Calculated results of formula (1) of Moiré Fringe in FCAM screenina

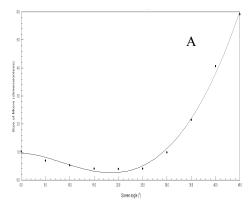


Figure 3. Measured curve of Moiré Fringe in FCAM screening

By referring Figure 3, it can be seen that: a) the size of Moiré Fringe is convergent when the screen angle variation of FCAM approaches to zero; by contrast, it is emanative in the case of traditional Moiré Fringe; b) the screen angle that generates the smallest size of Moiré Fringe is between 20°-30° in the case of FCAM, but it is 45° with respect to the traditional screening; c) the size of FCAM Moiré Fringe reaches its maximal value at the 45 ° screen angle variation, it reaches the smallest value in the case of traditional screening.

## **Development of Moiré Fringe equation aiming** at the FCAM

#### The fitted equation of FCAM Moiré Fringe

The curve (A) in Figure 3 is a reprehensive example about the size of Moiré Fringe of FCAM. The equation that describes the Moiré Fringe size of FCAM is an empirical model derived from the experimental data through use of the least square method, which writes:

$$P_{M} = 1.88\theta^{2.5} - 1.01\theta^{2} + 0.98 \tag{2}$$

where  $P_M$  denotes the size of Moiré Fringe,  $\theta$  demotes the angle variation between two screens.

#### Development of Moiré Fringe equation based on Fourier frequency domain

Supporting that there are two screen dots with same frequency in the space distribution, which are represented by  $D_1(x, y)$  and  $D_2(x, y)$ , respectively. The Moiré Fringe generated by superimposing  $D_1(x, y)$  and  $D_2(x, y)$  is written by [5][6]:

$$D(x, y) = D_1(x, y) \cdot D_2(x, y)$$
 (3)

Where D(x, y) is the Moiré Fringe.

In order to analyze the D(x, y) in the frequency domain quantitatively, it has to do the Fourier transform on the D(x, y):

$$F\left\{D(x,y)\right\} = F\left\{D_1(x,y) \cdot D_2(x,y)\right\} = D(f_x,f_y) = D_1(f_x,f_y) * D_2(f_x,f_y)$$
 where, \* signifies convolution calculation.

$$D(f_x, f_y) = D_1(f_x, f_y) * D_2(f_x, f_y) = \int_{-\infty}^{\infty} D_1(\xi, \eta) D_2(f_x - \xi, f_y - \eta) d\xi d\eta$$
(4)

The frequencies of  $D_1(x, y)$  and  $D_2(x, y)$  have distinguished values, in view of the fact that the screen rulings of FCMA is not identical. Assuming that the period of  $D_1(x, y)$  is T1, and  $D_2(x, y)$  is T2, according to the reference [6], the expression of screen dot can be written as:

$$\begin{split} D_1(f_x, f_y) &= \sum_m \sum_n A_{mn1} \cdot \left( f_x - \frac{m \sin \theta_1 + n \cos \theta_1}{T_1}, f_y - \frac{n \sin \theta_1 - m \cos \theta_1}{T_1} \right) \\ D_2(f_x, f_y) &= \sum_m \sum_n A_{mn2} \cdot \left( f_x - \frac{m \sin \theta_2 + n \cos \theta_2}{T_2}, f_y - \frac{n \sin \theta_2 - m \cos \theta_2}{T_2} \right) \end{split}$$

In terms of the properties of Fourier transform and impulse function, we get:

$$D(f_x, f_y) = D_1(f_x, f_y) * D_2(f_x, f_y) = \sum_{m} \sum_{n} B_{mn} \cdot \delta \begin{bmatrix} f_x - \frac{(m\sin\theta_1 + n\cos\theta_1)T_2 + (m\cos\theta_2 + n\cos\theta_2)T_1}{T_1T_2} \\ f_y - \frac{(n\sin\theta_1 - m\sin\theta_1)T_2 + (n\cos\theta_2 + m\cos\theta_2)T_1}{T_1T_2} \end{bmatrix}$$

(5)

By using Eq. (5), the frequency spectrum of Moiré Fringe can be drawn to assist to analyze the distribution of Moiré Fringe, in other words, the size of Moiré Fringe can be calculated and predicted. For the traditional AM, where T1=T2=T, Eq. (5) can be simplified as:

$$D(f_x, f_y) = D_1(f_x, f_y) * D_2(f_x, f_y) = \sum_{m} \sum_{n} B_m \cdot \delta \begin{bmatrix} f_x - \frac{m(\sin\theta_1 + \sin\theta_2) + n(\cos\theta_1 + \cos\theta_2)}{T} \\ f_y - \frac{n(\sin\theta_1 + \sin\theta_2) - m(\cos\theta_1 + \cos\theta_2)}{T} \end{bmatrix}$$
(6)

It can be seen that the Moiré Fringe of traditional AM is just a special case in the framework of FCMA.

#### Conclusion

The research of this paper demonstrates that the FCAM Moiré Fringe has different production mechanism and variation rule from the traditional AM with same frequency. The existing Tollenaar Moiré Fringe equation is no longer suitable to the FCAM. Based on the mechanism of FCAM Moiré Fringe and the analysis of experimental data, we developed the fitted equation to calculate the size of Moiré Fringe under the FCAM. The empirical equation we developed has desirable ability to describe and predict the Moiré Fringe of FCAM. Moreover, we also developed the mathematical model of computing the size of Moiré Fringe based on the Fourier frequency domain for the FCAM, which can be used to calculate the size of Moiré Fringe of both FCAM and traditional AM. The traditional AM that has same screen rulings is just a special example of the FCAM. Combining the FCAM and traditional AM, it can realize the screening process for printing images that contain color separation plates more than four.

# **Acknowledgement**

This paper has been supported by the fund of the National Natural Science Foundation, Grant No.60972134.

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