Analytical Modeling of Electrostatic Toner Adhesion

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Abstract

Persistent conflicts between toner adhesion measurements and theory have prompted scientists and engineers to propose a myriad of models for particle adhesion. Adhesion, generally consisting of an electrostatic component and a non-electrostatic component (i.e. van der Waals), is a critical factor in determining transfer efficiency in laser printers. The electrostatic component of adhesion depends upon many factors: toner size, shape, and composition; development and transfer voltages; and design parameters such as component materials, geometry, and process speed. Many of these factors are directly related to the charge distribution and dielectric properties of the particles. In order to account for these factors, different analytical models have been proposed that separately consider dielectric polarization, charge distribution, and neighboring particles. Still, measurements of electrostatic adhesion force tend to be as much as one order of magnitude higher than calculations. We propose a fully analytic model of particle adhesion that incorporates many of these factors including, multiple particle and image force interactions, non-uniform charge distribution, and dielectric polarization. The multi-particle force model consists of a field expansion in the spherical basis followed by integration of the Maxwell stress tensor. Combining these physical mechanisms bring the predicted adhesion on spherical particles closer to measured results.

Introduction

Overcoming charged particle adhesion is critical to the electrophotographic process, particularly development and transfer. However, significant confusion regarding how to model adhesion illustrates the overall lack of understanding. As stated by Schein [6]

> ...experiments have shown that the adhesion is at least one order of magnitude larger than predicted by a model in which the charged object is represented by putting the total charge in the center of the object. Even with modifications to include nonuniform charge distributions and nonelectrostatic forces of adhesion, these models cannot explain the observations.

This represents a real scientific problem in applied physics and a significant issue for the laser print industry. The issue is complicated by the fact that there are both long-range electrostatic forces and short-range mechanical (Van der Waals) forces, as well as interactions between the two [10]. Several models have been advanced to explain the electrostatic component of charged particle adhesion [7].

In this correspondence, we provide a complete analytical model of electrostatic adhesion of spherical toner. The model simultaneously includes the effects of multiple charged particles, dielectric polarization, and non-uniform surface charge distribution. Two main issues are addressed using this model.

1. Toner adhesion measurements are consistently about one order of magnitude larger than predicted by the simple model of a point charge above an image plane [6].
2. Recent measurements of toner adhesion reveal that adhesion force versus mean toner charge varies by both a squared term given by Coulomb interaction with it's image and a linear term not previously understood [10].

Adhesion Models for Spherical Toner

The most basic model for toner adhesion is Coulomb attraction between a point charge and it's image, referred to herein as the point-image force. This model represents a single spherical particle with dielectric constant $\varepsilon_p = 1$ and continuous, uniform surface charge density. For a particle with radius R resting on an image plane, the force acts only in the $\hat{z}$-direction and is given by the equation

$$\bar{F}_0 = -\hat{z}\frac{q^2}{16\pi\varepsilon_0 R^2}, \tag{1}$$

where q is the total charge on the particle and ε_0 is the permittivity of the surrounding air. There are three immediate extensions of this model that have been considered.

First, the multiple point-image model considers J point charges interacting with all corresponding images. Like the point image model, it assumes uniformly charged spherical particles all with dielectric constants $\varepsilon_p = 1$. The mathematical generalization is

$$\bar{F}_j = \frac{q_j}{4\pi\varepsilon_0}\left[\sum_{k\neq j} q_k \frac{\bar{r}_j - \bar{r}_k}{|\bar{r}_j - \bar{r}_k|^3} - \sum_k q_k \frac{\bar{r}_j - \bar{r}'_k}{|\bar{r}_j - \bar{r}'_k|^3}\right], \tag{2}$$

where the first term in the brackets gives the interactions between the j^{th} particle and all other particles and the second term in the brackets gives the force between particle j and all images. Because of all the interactions between toner charge and all images, this model produces a larger adhesion force than the point-image model.

Second, a uniformly charged dielectric particle with arbitrary dielectric constant ε_p has been proposed as a more accurate model for toner adhesion than the point-image model [1]. The system can be solved analytically in spherical coordinates with the appropriate translational theorem [8]. Alternately, the bi-spherical coordinates [12] can be used for a particle above an image plane, or the degenerative bi-spherical coordinates [5] apply to a particle resting on a plane. The dielectric polarization produces additional force between the induced dipoles and the image. Thus, the adhesion of a uniformly charged dielectric sphere resting on an image plane results in a net increase over the point-image force.

Third, discrete, uniform charge distribution on a single spherical toner has also been proposed as a suitable model for toner adhesion [2]. The interesting consequence of the discrete charge surface charge model is the so-called proximity force. The proximity force for a single particle with dielectric constant $\varepsilon_p = 1$

$$\bar{F}_{proximity} = (1+4/\pi)\bar{F}_0 \tag{3}$$

results in about twice the adhesion as the point image model. The enhancement is due to the assumption of discrete charges closest to the image plane.

Each of these established adhesion theories are plotted versus particle charge in Figure 1. All models assume uniformly distributed surface charge. The multiple point-image model produces the largest force of greater than 5*x* the point-image force for the case of $J = 9$ (*i.e.* a particle and 8 nearest touching neighbors). The proximity force produces a force $> 2x$ the point-image force, while the dielectric particle with $\varepsilon_p = 3$ provides only about a 50 % enhancement. Furthermore, each of the curves is represent a function of Q^2 only. The remainder of this paper will explore two hypotheses: (*i*) if the aforementioned models are combined into one general theoretical model, the resulting enhancement should produce the 10*x* enhancement in adhesion force necessary to compare with measured results, and (*ii*) non-uniform surface charge may explain the Q dependence of adhesion force seen in experimental results [10].

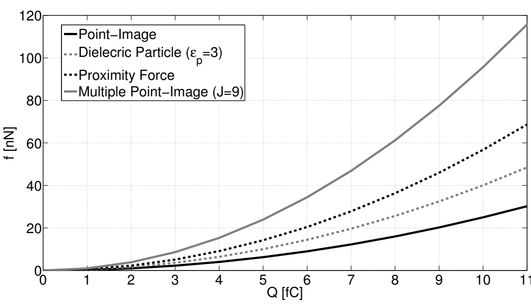

Figure 1. *Adhesion force versus total particle charge for a few existing models of uniformly, charged spherical toner. A dielectric particle with* $\varepsilon_p = 3$*, the proximity force, and multiple point-image model with* $J = 9$ *total particles give enhancements over the point-image model of about* $1.5x$*,* $2x$*, and* $5x$*, respectively.*

Analytical Multiple Particle Model

The analytical multiple particle model presented here consists of expansion of the spherical modes with particle-to-particle translations for application of the boundary conditions [8]. We have previously applied similar analytical models to The electrostatic potential external to particle j is

$$\psi^{(j)} = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}\left[B_{nm}^{(j)} r_j^{-(n+1)} + W_{nm}^{(j)} r_j^{n}\right] P_n^{|m|}\left(\cos\theta_j\right) e^{im\phi_j}, \tag{4}$$

where (r_j, θ_j, ϕ_j) are the spherical coordinates with the center of particle j taken as the origin and $P_n^m(\cdot)$ represents the associated Legendre function. A similar expansion is applied for the potential internal to the particle. The unknown coefficients $B_{nm}^{(j)}$ and $W_{nm}^{(j)}$ provide the magnitudes of the modes for particle j and all other particles and images, respectively. The surface charge density is also expanded in the orthogonal spherical basis,

$$\rho^{(j)} = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \alpha_{nm}^{(j)} P_n^{|m|}\left(\cos\theta_j\right) e^{im\phi_j}, \tag{5}$$

and the unknown coefficients $\alpha_{nm}^{(j)}$ are determined by least squares fit to arbitrary surface charge distribution. Once the $\alpha_{nm}^{(j)}$ are defined, the coefficients $B_{nm}^{(j)}$ and $W_{nm}^{(j)}$ are determined from the boundary conditions. First, the translational theorem contained in Ref. [8] is used to express all other particles and images besides j in the (r_j, θ_j, ϕ_j) system. The $W_{nm}^{(j)}$ coefficients are related to the $B_{nm}^{(j)}$ coefficients for all other particles by

$$W_{nm}^{(j)} = \sum_k \sum_{\nu=0}^{\infty}\sum_{\mu=-\nu}^{\nu} T_{nm\nu\mu}^{(jk)} B_{\nu\mu}^{(k)}, \tag{6}$$

where $T_{nm\nu\mu}^{(jk)}$ give the elements of a 6-rank tensor and both $B_{nm}^{(j)}$ and $W_{nm}^{(j)}$ give the elements of tensors of rank 3. The boundary conditions yield the additional relation between the coefficients

$$B_{nm}^{(j)} = \frac{\left[(n+1)\varepsilon_b + n\varepsilon_j\right] R_j^{-(n+2)} + W_{nm}^{(j)} n(\varepsilon_j - \varepsilon_b) R_j^{n-1}}{\alpha_{nm}^{(j)}\varepsilon_0^{-1}}. \tag{7}$$

Substituting Eq. (6) into Eq. (7) yields a linear equation with respect to $B_{nm}^{(j)}$, which is solved via matrix inversion. Finally, the electrostatic adhesion force on particle j (*i.e.* the force in the $-\hat{z}$ direction) is given by a simple summation of mode coefficients [4]

$$\begin{aligned}\bar{F}_E^{(j)} &= -\hat{z}4\pi\varepsilon_0\varepsilon_f\left[\sum_{n=0}^{\infty} B_{n,0}^{(j)} W_{n+1,0}^{(j)}(n+1)\right.\\ &\left.+\sum_{n=1}^{\infty}\sum_{m=1}^{n} 2\Re\left\{B_{n,m}^{(j)} W_{n+1,-m}^{(j)}\right\}\sqrt{(n+1)^2 - m^2}\right].\end{aligned} \tag{8}$$

Discussion

Non-uniform charge distributions on toner particles have long been suspected as being responsible for increased adhesion force [3]. It is known that partial charge distributions on the bottom of the particle closest to the image plane result in significant enhancements to adhesion force. In printer systems, recharging typically occurs to the top of the particle, for example during the passage of toner on an intermediate transfer belt through additional first transfer nips. However, it has been pointed out that such charge imbalances will result in a net moment on the particle causing the particle to rotate so that the partial distribution will be oriented closest to the image plane [9]. In the model implemented here, any arbitrary charge distribution can be incorporated. We choose a model that mimics the operation of toner. We assume that additional charge is deposited onto the top half of the particle. Because the particle is spherical, we assume a cosine function for the additional charge distribution. If the particle then rotates under the additional force imbalance, the charge on the bottom of the particle ($\pi/2 < \theta < \pi$) is be described by

$$\rho^{(j)} = \rho_0^{(j)} - \Delta\rho^j \cos(\theta) \tag{9}$$

and $\rho_0^{(j)}$ on the top of the particle ($0 < \theta < \pi/2$). This charge distribution is shown in Fig. 2 along with the spherical mode expansion used to represent the charge density. In the case shown, the total charge is $Q = 10$ fC, obtained by integrating the charge density over the surface of the sphere.

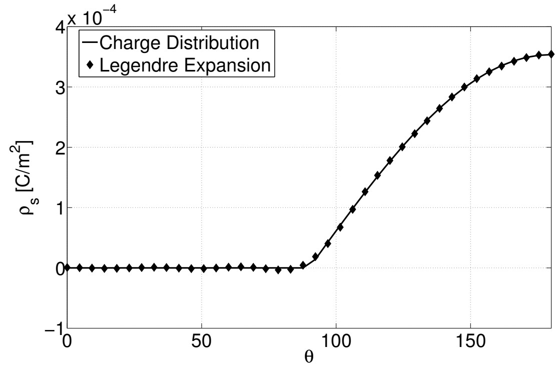

Figure 2. Charge distribution used in the model along with the spherical mode representation $N = 10$ using a summation of Legendre functions. For this case, $\rho_0 = 0$ and the total charge per particle is $Q = 10$ fC.

Equation (8) provides the most complete analytical model of electrostatic toner adhesion to date. It allows for all of the models previously discussed to be incorporated within the same calculation. Figure 3 shows a plot of F_E for the case of a collection of $J = 9$ dielectric particles ($\varepsilon_p = 3$) with the particle of interest in the center surrounded by 8 nearest neighbors touching on a square grid. Both cases of uniform and nonuniform charge distributions are shown along with the point-image force F_0 for comparison. The uniformly charged particles result in about 7x enhancement over the charge-image model, while the non-uniformly charged particles yield a 10x increase.

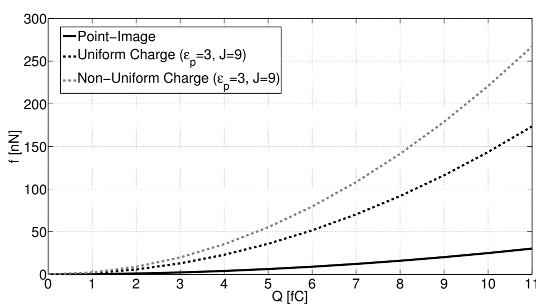

Figure 3. Electrostatic force from the presented analytic model of adhesion versus charge. The dielectric constant of all the particles is $\varepsilon_p = 3$ and the 9 particles are touching on a square lattice. The forces shown are for the center particle under uniform and non-uniform charging. The adhesion force plotted is the force acting toward the image plane in the $-\hat{z}$ direction.

An additional observation when studying non-uniformly charge distributions is the existence of a linear Q term which is not present for uniform charge distributions. To illustrate why this is, refer to Fig. 4. Each point along the force versus charge plots (*e.g.* Fig. 1 and Fig. 3) represent a separate experiment where downstream 1^{st} transfer stations are typically used to provide additional charge [11, 10]. After reorientation, the toner rests with additional charge on the bottom closest to the image plane. Consider the simple case where a particle has discrete charges q_1 and q_2 on the top and bottom, respectively. The total charge is $Q = q_1 + q_2$. The total force is obtained by summing forces between the charges and the images

$$\bar{F} = -\hat{z}\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1^2}{(4R)^2} + 2\frac{q_1 q_2}{(2R)^2} + \frac{q_2^2}{(2\delta)^2}\right]. \quad (10)$$

Successive experiments tend to increase q_2 while maintaining q_1 constant. Therefore, $q_2(Q) = Q - q_1$ and the terms in the brackets can be written as

$$\left[\left(\frac{q_1^2}{4\delta^2} - \frac{7q_1^2}{16R^2}\right) + Q\left(\frac{q_1}{2R^2} - \frac{q_1}{2\delta^2}\right) + Q^2\left(\frac{1}{4\delta^2}\right)\right], \quad (11)$$

and a plot of force versus charge will include a linear term and a constant term (q_2 is constant).

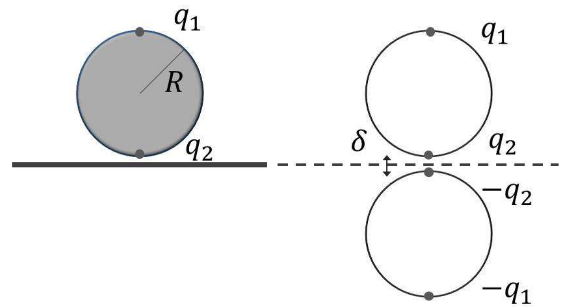

Figure 4. Example non-uniform charge distribution for illustration. (left) A particle above an image plane with dielectric constant $\varepsilon_p = 1$ and radius R has discrete charges q_1 and q_2 on the top and bottom, respectively. (right) Image theory equivalent of the charge distribution.

As an example, Fig. 5 shows the plot of adhesion force versus charge for a single dielectric particle with base uniform charge of 3 fC and additional charging up to a total of 10 fC using the previously described charge distribution. The polynomial $f = 2.01 - 2.87Q + 1.13Q^2$ fits the model output exactly, with the Q and constant terms arising from the non-uniform surface charge distribution. Therefore, the general dependence of particle adhesion force on charge is

$$f_{adhesion} = A_0 + A_1 Q + A_2 Q^2 + A_3 Q^3, \quad (12)$$

where A_0 comes both from mechanical adhesion and interaction between the base uniform charge density and the superimposed non-uniform charge distribution , A_1 gives the proportion of force due to non-uniform charging, A_2 gives the proportion of electrostatic force due to all Coulombic interactions, and A_3 is due to mechanical and electrostatic force interactions [10]. Since the $\rho_0 = 0$ in the calculations in Fig. 3, those plots are quadratic in Q with $A_0 = 0$ and $A_1 = 0$.

Conclusions

An analytical model of electrostatic particle adhesion has been presented which incorporates multiple particles, arbitrary dielectric constants, and non-uniform surface charge densities. Including the effects of dielectric constant, multiple particles, and

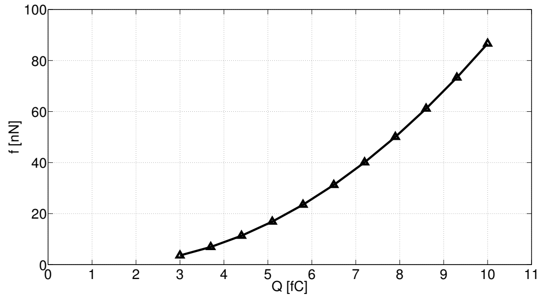

Figure 5. *Force versus charge for a single dielectric particle with* $\varepsilon_p = 3$. *The particle has a uniform base charge density* ρ_0 *providing* 3 *fC of charge plus an additional charge according to Eq.* (9) *to bring the total charge to the value indicated on the graph. The resulting curve has both linear* Q *and squared* Q^2 *dependence. The markers show the output of the analytical adhesion model and the line indicate the best fit curve.*

non-uniform charge density can provide more than $10x$ enhancement over the standard model of a point charge above a ground plane. Furthermore, the non-uniform charge density adds a linear term and a constant term when plotting force verses charge. Because the mechanical (*i.e.* van der Waals) force is also independent of charge, separating electrostatic and non-electrostatic forces from experiments involving non-uniform charge distributions may be very difficult. Recent experimental work indicate that an additional factor proportional to Q^3 is due to interaction between electrostatic and non-electrostatic forces. Therefore, attempts to separate the electrostatic and non-electrostatic mechanical adhesion contributions by experimentally varying charge remains a challenge.

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Author Biography

Brandon Kemp received his PhD from the Massachusetts Institute of Technology (2007), his MS from the University of Missouri-Rolla (1998), and his BS from Arkansas State University (1997). He is presently a member of the Center for Efficient and Sustainable Use of Resources (CESUR) and a faculty member in the College of Engineering at Arkansas State University.

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