# Dependence of Drop Speed on Nozzle Diameter, Viscosity and Drive Amplitude in Drop-on-Demand Ink-jet Printing

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## Abstract

*Results of recent experiments and numerical simulations are presented, which have been used to establish empirical rules for the dependence of drop speed on nozzle diameter and drive amplitude for Newtonian and non-Newtonian fluids printed with a range of different ink-jet print-head technologies.*

*Experiments were carried out with Xaar, MicroFab and Spectra Dimatix print heads and with solutions of polystyrene in diethyl phthalate as model fluids. These results are compared with predictions from recent numerical codes developed by collaborators in the University of Leeds, and from simple models for drop-on-demand fluid jetting resulting from physical laws.*

## Introduction

Drop-on-demand (DoD) ink-jet printing successes in a widening range of industrial applications have continued to spur efforts to provide manufacturers of print heads, ink-jet fluids and printing systems with working rules, as well as a deeper understanding of jetting processes [1], that can lead to improvements. As there are many contributing, and sometimes conflicting, factors in DoD printing, the approach taken in our Inkjet Research Centre [2] has been to use model fluids, jetted from single print heads, in order to build up a better picture of key features of the problems. Most of our results confirmed the expectations of industrial DoD practitioners, but new insights were gained from some of them [3]. DoD simulations have been performed using the numerical code developed by Harlen and Morrison at the University of Leeds, UK, for viscoelastic polymer additives to Newtonian solvents [4].

## Empirical modeling of jetting speed

In our previous studies [3] of DoD jetting we have reported that the jet tip position ($s$) and the speed ($u$) beyond the nozzle exit, at time t after emergence, can be well represented by simple empirical functions of the form

$$s = vt + (v_0 - v)t_0 \exp(-t/t_0) - \frac{1}{2}at^2 \tag{1}$$

$$u = v + (v_0 - v)\exp(-t/t_0) - at \tag{2}$$

Here $v$ is the target tip speed ignoring the slowing down term characterized by the constant $a$, the jet velocity at emergence from the nozzle exit is $v_0$ and $t_0$ is a characteristic timescale for the exponential decay of the tip speed towards to the target tip speed $v$. The deceleration $a$ is 1000's of times larger than gravity, and opposes it for conventionally oriented DoD print heads. For some fluids $a$ can be neglected whilst for others their larger $a$ values are associated with "bungees" that will never jet or produce drops [4].

## Simple Models

For an incompressible inviscid liquid of density $\rho$ flowing at an instantaneous volume flow rate $Q$ through a nozzle exit of area $A$; the fluid speed is linked to the conservation of volume by:

$$Speed = Q/A \tag{3}$$

For a given DoD waveform shape applied to the nozzle to drive $Q$ there will be an average value of $Q$ that is proportional to the peak value; for example the average/peak is $2/\pi$ if the waveform is a half-sinusoid, 2/3 for parabolic (quadratic) and 1/2 for triangular. So the peak value of $Q$ can represent the DoD waveform. It is well known that if the flow rate $Q$ is too low or does not persist long enough then the surface tension $\sigma$ acting at the nozzle exit will tend to prevent drops from either forming or leaving the nozzle region with a usable outwards speed. The existence of a finite threshold value is clearly inconsistent with equation (3). The drop that does form is typically as wide as the nozzle exit diameter $2R$ for DoD model fluids [5]: can simple models based on equation (3) ever incorporate such well-known physical behavior and features? Flowing viscous fluids have a radially-dependent velocity profile across the nozzle that alters the relationship between $Q$ and fluid tip speed $v$ (<< velocity of sound in fluid). Physical analysis [6] for fully developed viscous flow with an average speed $U$ at Reynolds number $Re = \rho RU/\eta$ in a pipe of radius $R$ due to a pressure difference p* across length $L$ reveals two dimensionless groups: ($p^*A^2/\rho Q^2$), which contains the ratio Q/A of equation (3), and ($L/RRe$), which depends on viscosity. These may imply that

$$U \sim 1/\eta \tag{4}$$

When comparing inkjet drop speed with the peak drive amplitude for different nozzle exit areas, viscous DoD drop speeds should retain behavior arising from volume conservation suitably modified by the viscous flow constraints. We can check this by performing experiments and using numerical simulations of the print head nozzle and the fluid jets; while real experiments require accurate assessments of nozzle sizes and may suffer from differences between nozzle drive couplings, simulations also need the nozzle shapes and sizes as inputs, will use a drive waveform that is not known and necessarily assume the rupture of real fluid threads takes place at a finite radial width (which influences the number of satellites formed by simulations). When a CIJ fluid is modeled [7] the formation of a liquid jet requires the creation of extra surface and hence introduces a surface energy penalty against the kinetic energy of the jet produced by the drive waveform. This energy penalty scales as surface tension times the area of the drop. For DoD, the viscosity of the fluid produces forces that depend on fluid shear rates across the size of the droplet, but are often neglected as they are expected to be smallest for the slowest drops. The extra surface energy (constant $k \sim 1$) for the DoD drop diameter $D$ reduces the kinetic energy of the main drop according to equation (5):

$$Final\_kinetic\_energy = Initial\_kinetic\_energy - k\pi\sigma D^2 \quad (5)$$

Thus the threshold for drop production ignoring viscosity is given by an exact balance of the 2 terms on the RHS of equation (4). The initial kinetic energy ($\frac{1}{2}mv_0^2$) is that corresponding to the speed in equation (3) and the mass contained in the diameter $D$ of the drop. Rearranging the balance for zero final drop speed requires

$$v_0{}^2 = \frac{12k\sigma}{\rho D} = 3kv_T{}^2 \quad (6)$$

where the velocity $v_T$ is the Taylor retraction speed for a fluid ligament of diameter $D$, which we have discussed previously [5]:

$$v_T = 2\sqrt{\frac{\sigma}{\rho D}} \quad (7)$$

The threshold value of drop speed produced by the drive waveform needs to exceed the Taylor retraction velocity for the fluid, which depends inversely on the (square root of) drop diameter $D$. The consequence of this physical threshold mechanism for outward release of drops with a finite speed is that the drop diameter for should be as large as possible: the nozzle diameter presents a likely largest drop diameter while producing slowly moving DoD drops. (Non-circular nozzles have been numerically designed to reduce (by ~ 20%) the drop volume [8], while far smaller drops can be generated using higher radial modes across the nozzle [1, 9].) So we have clearly established that the surface energy argument will result in DoD drops of comparable size to the nozzle diameter, and that there is a threshold value for the speed in equation (3), as given by equation (6) in terms of a known fluid parameter $v_T$, from equation (7) with $D$ set to the size of the nozzle diameter. Rearranging our various equations to include the threshold leads to the final model. This has modified the linear dependence of the speed in equation (3) to a behavior written in terms of $v_0$ and the magnitude of the volume flow $Q$ due to the waveform drive, for given fluid properties and nozzle Area:

$$Final\_Speed = \sqrt{[Q / Area]^2 - v_0{}^2} \quad (8)$$

Ignoring the dependence of the drop diameter on the volume flow $Q$ fixes $v_T$ for a given nozzle *Area*: thus the curve of Final-Speed vs. the flow rate $Q$ due to the drive waveform amplitude can be simply understood in terms of fluid properties and the nozzle *Area*. Although the speed curve is not linear, straight-line fits over a equally spaced range of points up to $4v_0$ above the threshold speed $v_0$ lie within 10% of unit slope (no effects due to surface tension). This modeling implies that the final drop speed above threshold will increase approximately linearly with the average flow rate $Q$. As the relationship between $Q$ and the drive amplitude is usually assumed to be linear, although the power losses determined for a piezoelectric actuator were quadratic in DoD drive amplitude [10], this means that the model predicts a linear rise of drop speed with drive amplitude above a threshold, where this threshold depends on the Taylor speed $v_T$ of equation (7) and hence inversely with the square root of the drop (nozzle) diameter $D$. The nozzle exit *Area* increases as the square of diameter, so the output speeds for the same fluid ejected from different nozzle diameters (but the same actuation coupling and channel dimensions) should have very similar increases with parameter $Q/D^2$ ~ drive amplitude/$D^2$, although the slopes and the threshold for each nozzle size will modified slightly by the $D$ dependence of the Taylor speed term $v_0$.

Model predictions for nozzles jetting fluid with the physical properties of DEP (diethyl phthalate) are shown in Figure 1, assuming the drop diameter matches the nozzle diameter, and the flow rate in the nozzle is normalized by the drop threshold value. In this model the final drop speed climbs above this threshold towards that of the drop speed expected without surface tension.

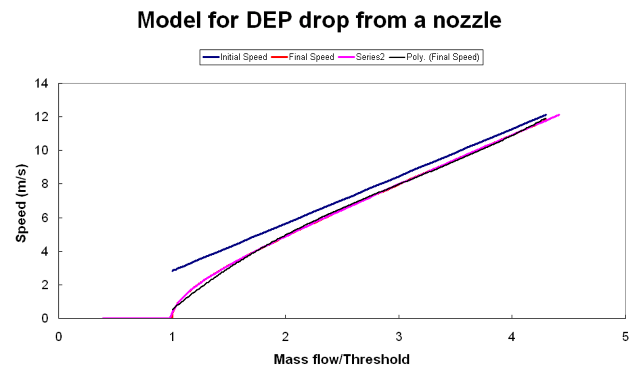


***Figure 1**: Predictions of speed for a "low" viscosity DoD fluid (DEP) jetting from a nozzle. An empirical fit extending well beyond the threshold is roughly linear. The "low" viscosity limit reaches typically ~ 0.020 Pa.s for fast DoD jetting [11].*

Equation (8) at the drop production threshold for low-viscosity fluids from a nozzle, whereas equation (4) applies above the jetting threshold for viscous fluids flowing through the same nozzle, but the effects of viscosity $\eta$ on the DoD drop speed outside the nozzle have been ignored so far. For DoD printing it is known that viscosity plays a key role [11], because the parameter $\eta R/\sigma$ is a controlling timescale for a fluid jet radius $R$ to radially pinch off. In addition, the stretching of the ligament prior to break off causes the extensional viscosity $\eta_E$ to be raised above the low shear value $\eta_0$, to values 3 times greater (the Trouton ratio) for Newtonian fluids, and even 100's of times higher for viscoelastic fluids [12]. One theoretical model of the effect of viscosity [13] predicts the drop speed is reduced by $\eta_E/(\rho L)$ where $L$ is the short length of ligament at the time when the kinetic and surface energies first balance (after the maximum of the drive voltage, but before any stretching), assuming also that the velocity profile in the nozzle is parabolic. For fast jetted DEP, this model predicts that stretching ligaments will slow the drop by ~ 0.75-3 m/s for nozzle $R$ = 40-10 μm and assuming the value of the initial (unstretched) fluid length $L \sim R$. Our empirical finding for ~ 6 m/s jetting of PS+DEP fluids is that the DoD jet tip velocity changes by a factor of 2-3 after emergence, which is not inconsistent with predictions using this model [13]. Viscosity, in the simple drop formation picture represented by equation (4), should enter only indirectly through the velocity profile in the nozzle rather than directly in the energy balance [13]. The effects with intermediate velocity profiles (between pure viscous parabolic and inviscid plug flow) should lie between the limits from the model [13] for the long jets we see [3].

## Results of numerical simulations

Newtonian fluids with similar Weber number but different viscosities typical of the range encountered in DoD printing were chosen for simulation: DEP (0.010 Pas) and DOP (0.050 Pas). We simulated MicroFab drop speed variation with nozzle exit diameter because this provided a link with some of our experimental data. Each MicroFab nozzle shape pre-measured [14] for the simulation.

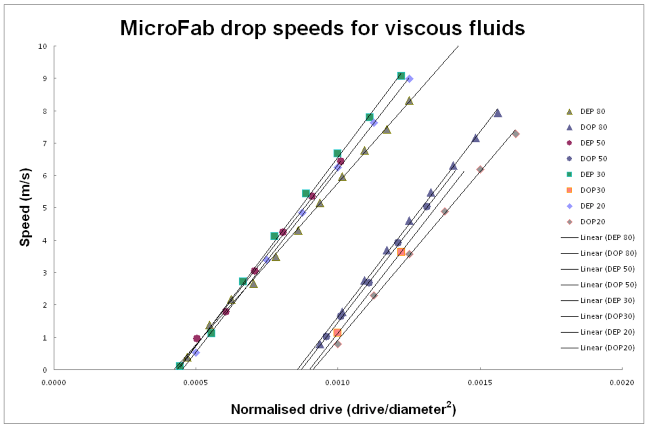


*Figure 2: Results of our numerical simulations of various MicroFab nozzles. Final drop speed is plotted against the normalized drive, which is the simulation drive voltage setting divided by the square of the nozzle exit diameter. See text.*

Figure 2 shows results from the numerical simulations of various MicroFab nozzles. Final drop speed is shown (up to at least 6 m/s) to rise roughly linearly above a threshold value for the normalized drive, which is the simulation drive voltage setting (amplitude applied to a common waveform) divided by the square of the nozzle exit diameter. The results cluster around normalized drive thresholds which depend significantly on viscosity and therefore do not follow limits from CIJ [6]. (Our simulation results for 50µm nozzle were corrected for a ~ 10% shift of measured nozzle bore compared with other nozzles.) The values of the extrapolated drive threshold of DOP and DEP were found by other simulations to lie on a nearly linear curve above a low viscosity limit (not shown) that is determined by the fluid surface tension. The common waveform used throughout these sets of simulations was deduced from PIV measurements on an 80µm diameter nozzle reported by our group elsewhere at this conference [14]. Other waveform profiles, which were based on the applied voltage set by MicroFab JetDrive III controller, produced a similar pattern of speeds results near threshold against an appropriate drive voltage setting.

## Experiments

Experiments were carried out with Xaar, MicroFab and Spectra Dimatix print heads and with dilute solutions of polystyrene (PS) in diethyl phthalate (DEP) as model fluids. Shadowgraph images of DoD jets and drops were obtained using several experimental set-ups in the Inkjet Research Centre, as appropriate to the print head technology used (Xaar, Spectra Dimatix or MicroFab), the light source type (20ns NanoLight, a Xenon flash, 2ms high power flash) and cameras used (Nikon D40, Prosilica CCD, Shimadzu HyperVision 1,000,000 fps). Some of the experimental set-ups are reported elsewhere [14, 15]. Images sequences showing evolution of jets into drops are later analysed in time (< 1µs) and calibrated (< 1 µm) to determine the drop (or the jet tip) velocity at a specified (1.0 mm) stand-off distance from the nozzle exit.

## Experimental Results

Some representative fluid jetting speed data obtained from our ongoing collaborative studies using Xaar, Spectra Dimatix and MicroFab print heads are shown in Figures 3-5 respectively.

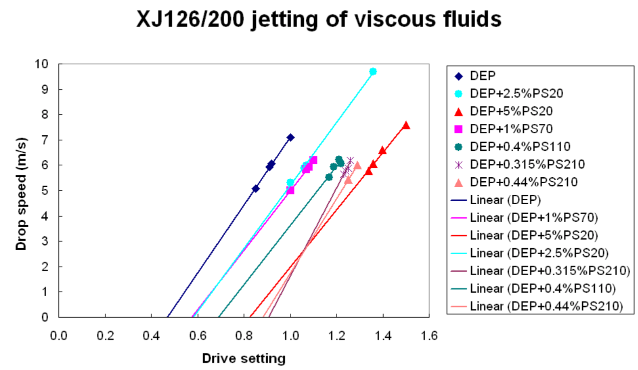


*Figure 3: Xaar XJ126-200 nozzle jetting dilute PS+DEP (polystyrene) fluids.*

Figure 3 shows jetting from a Xaar XJ126-200 (50 µm diameter) print head nozzle has very similar gradients of drop speed versus drive setting for some viscous PS+DEP fluids. The two exceptions correspond to long ligaments for 0.315% and 0.44% PS210+DEP fluids above the limits of jettability found previously [3].

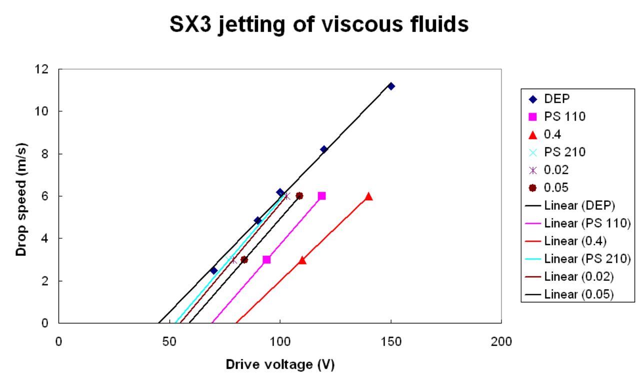


*Figure 4: Spectra Dimatix SX3 nozzle jetting PS+DEP fluids. The solvent DEP jetting speed between 3-12 m/s is close to linear above a threshold drive. The PS (polystyrene) additives of PS110,000 (0.2wt%) and (0.4wt%) and PS210,00 (0.01wt%), (0.02wt%) and (0.05wt%) jetted at 3 m/s and 6 m/s show similar gradients but an increased threshold voltage that depends on fluid viscosity.*

Figure 4 shows jetting from a Spectra Dimatix SX3 print head. Despite having a square 27µm nozzle exit diagonal, rather than circular nozzle exits like the other print heads used in the present study, the linear jetting speed regularity persists, with the threshold increasing as the added PS concentration raises the fluid viscosity.

 

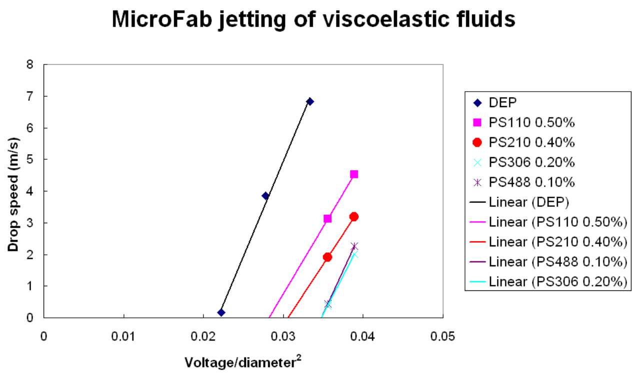


***Figure 5**: MicroFab nozzle jetting viscoelastic fluids. The PS series, chosen to have similar linear viscoelasticity and viscosity, have similar jetting thresholds [12]; the solvent DEP has a somewhat lower jetting threshold.*

Figure 5 shows jetting speed curves obtained using a 30 µm diameter MicroFab nozzle for the comparison of jetting of viscoelastic fluid samples with measured rheology [15]. The DEP fluid and the PS+DEP series was jetted using a 30 µm diameter nozzle. There is a clear difference between the threshold values of drive/(diameter)² for the DEP and PS series, although both appear consistent with a similar gradient in the drop speed above the threshold. The PS110 (0.5%) fluid jets well with a raised viscosity relative to pure DEP, while the other PS fluids, prepared for similar linear elasticity to the PS110 (0.5%) fluid, are increasingly closer to their jettability limits due to effects of non-linear viscoelasticity [15], and so show even higher jetting thresholds.

## Comments

We have demonstrated by a combination of experimental results, across different DoD nozzles and manufacturing technologies, and numerical simulations of fluid jetting, that some regularities in the jet speed should be expected and are predictable whenever nozzle diameters, fluid viscosities and drive amplitudes are changed. This knowledge could be helpful whenever such changes are necessary. The simulation results are consistent with measurements of drop speed measured with PS+DEP fluids that are weakly viscoelastic, i.e. non-Newtonian and dominated by their extra viscous content.

## Conclusions

Some very simple guidelines, due to the fundamental fluid dynamics behavior of inviscid CIJ jets from nozzles, do not explain DoD jetting of fluid with higher viscosity. The general results for DoD jetting of Newtonian fluids with a given waveform is that the drop speed is linear in normalized drive (being the drive setting divided by the square of the exit diameter) above a threshold that is independent of the nozzle exit diameter but which does depend significantly on the viscosity of the fluid. For the viscosity range simulated, the drive thresholds for a given nozzle vary linearly viscosity above a low viscosity value determined by surface tension. Both this linear viscosity dependence and the normalization of Q by the exit area are consistent with a dimensional analysis for fully developed viscous flows in pipes, and should apply even better for the higher viscosity DoD fluids.

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