Improved Models for Drop-on-demand Ink-jet Shortening

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Abstract

A previous 1-D model for the shortening of an unbroken drop-on-demand ink-jet ligament has been extended to the case of an arbitrary attached tail mass, and can also include extensional viscosity (which has ~ 2% effect) as well as linear elasticity in the fluid. Predictions from the improved model are shown to be very similar to results from 2-D axisymmetric numerical simulations of DoD ink-jet ligaments and also to the results of recent experiments on Newtonian fluids jetted without satellite formation.

Introduction

In a previous NIP paper [1] we based a model for drop-on-demand ligament shortening on modification of the classical Taylor model for the shortening of a long cylindrical ligament after break-off. The Taylor speed v_T for a fluid thread with diameter D, surface tension σ and density ρ, as depicted in Figure 1(a), is given by:

$$v_T = 2\sqrt{\frac{\sigma}{\rho D}} \tag{1}$$

We have now removed limitations affecting computational results in the earlier paper [1] and have extended our model to include the effects of extensional viscosity. In order to explore further whether elasticity does indeed lead to significantly faster ligament shortening, as discussed previously [2], we have formulated a new version of the original model. In the present work we concentrate on modeling the behavior before the ligament breaks up into satellite drops [3]. Additional tests of the model have been made by comparing it with experimental data for a Newtonian fluid jet forming main drops and ligaments which do not break up, and also the results of more recent 2-D simulations.

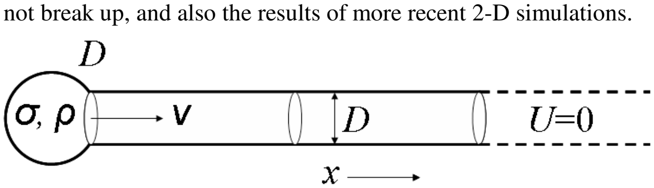

Figure 1a. *Taylor model for shortening of a long static fluid ligament [1].*

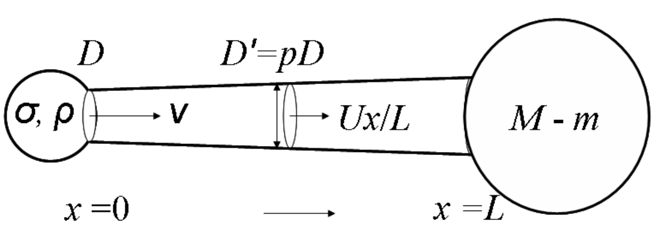

Figure 1b. *Improved model for shortening of a finite ligament [1].*

The model shown in Figure 1(b) (from [1]) treats a conically-shaped ligament with an initial finite length L and a linear internal velocity gradient between the tail and the head ends, which represents the head speed and the initial stretching of the fluid jet at the break-off time. The head end of the jet had a mass (M-m), where M is the total mass of the jet and m is the tail end mass. At a time t after break-off, the instantaneous length was defined as the sum of all the axial fluid lengths [1].

We developed this approach after studying the behavior of both Newtonian and slightly elastic fluids: having found a constant but higher ligament shortening speed for some fluids than that predicted from the Taylor model for a Newtonian fluid, we tried to correlate this anomalous behavior with fluid elasticity [2].

Extended model of ligament shortening

In the previous analysis the initial acceleration of the tail end was finite, due to the inertia of the volume of fluid at the end of the cone, which was assumed to be hemispherical and of diameter D [1]. We now extend the earlier treatment to incorporate a tail end which has β times the mass of a hemisphere of diameter D. This leads to a value for the cylindrical tail end acceleration dv/dt, at time t given by:

$$\frac{dv}{dt} = 3\frac{{v_T}^2 - v^2}{\beta D + 3x} \tag{2}$$

Here x is the length of the original ligament that is accreted after time t into the tail end mass due to its relative motion; this tail acceleration always directs the tail speed towards the Taylor result, but at a rate that reduces with time as x and the tail volume grow. For β = 1, equation (2) reduces to the original result. For β = 0, equation (2) gives infinite acceleration when x = 0, unless $v = v_T$ at that time, which is most unlikely. Ink-jet fluid motion is usually rather slow close to the break-off location: i.e. with initial speed $v(0) < v_T$. Figure 2 shows the increasing tail speed for β = 1 and $v(0)$ = 0 as predicted from equation (2) for the fluid jet with the measured profile shown in Figure 3 [from 1].
The timescale required to approach the equilibrium depends on the mass attached to the tail. We estimate this time from equation (3):

$$\text{timescale} \approx \frac{\beta D}{v_T} \tag{3}$$

Applying equation (3) to a massive jet head, characterized by β'' for head end size, ligament diameter D'' and with Taylor speed v_T'' from equation (1), the timescale to approach equilibrium is found to increase as β'' and also as $(D'')^{3/2}$. For $\beta'' \approx 100\beta$ and $D'' \approx 2D$, typical of the jet shown in Figure 3, the head takes >300 times as long as the tail to attain its (lower) Taylor speed and can thus play no significant role in jet shortening. Conversely, if the head and tail are similar, jet shortening occurs at a speed of 2 v_T after a time given approximately by equation (3).

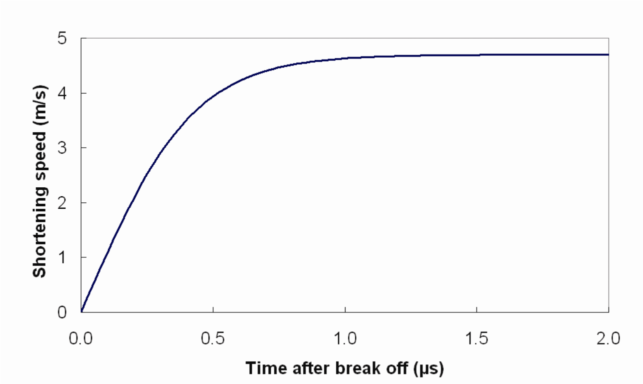

***Figure 2**: Increase in tail speed after break-off (for v(0)= 0 and β = 1) towards the Taylor speed of 4.7 m/s as predicted from equation (2) for the ligament of Figure 3. The timescale predicted from equation (3) is ~1.3 µs for D= 6 µm.*

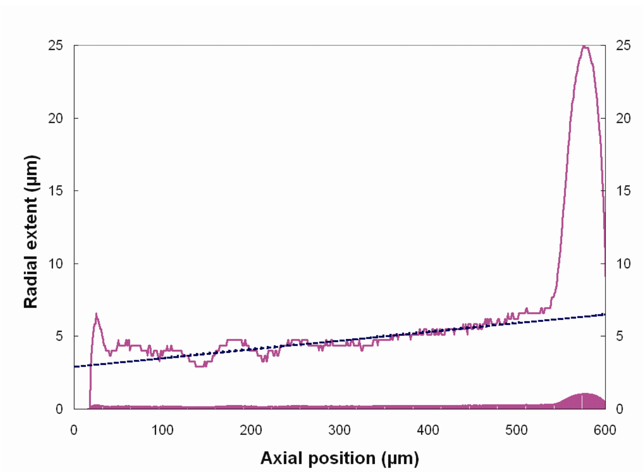

***Figure 3:** Radial profile of a viscous (η = 10 mPa s) fluid jet just after break-off [1]. Fluid at the tail end of the ligament (at axial position 0) approaches the average Taylor speed while the head end travels at ~ 6 m/s. (Image of jet also shown to scale.)*

The ligament shown in Figure 3, for a Newtonian fluid with viscosity ~0.01 Pa s and a drop speed of ~6 m/s, has an almost conical shape, in contrast to the near-cylindrical forms reported [4] for viscosity ~1 – 5 mPa s and also observed [5] for a fluid with viscosity of 10 mPa s jetted from another print-head design.

To check our viscosity modeling we have performed experiments with the same fluid in another print-head with smaller nozzles [5], producing under certain conditions a main drop speed of ~ 6 m/s with no satellites. The fluid ligament in this condition exhibited a narrowing neck prior to detaching close to the nozzle, in such a way as to allow the whole jet to reach the head end speed, thereby eliminating any velocity gradient along the ligament length. The ligament shortening speed is the difference between the tail speed (~11.0 m/s) and the head speed (~ 6.2 m/s): i.e. ~ 4.8 m/s. The ligament radius of ~ 2.4 µm implies a Taylor speed (for an inviscid fluid) of ~ 5.3 m/s: the speed discrepancy of ~ 0.5 m/s provides an upper limit for the effect of fluid viscosity of ~10 %.

We can incorporate Newtonian extensional viscosity $\eta_e = 3\eta_0$, for shear viscosity η_0 and density ρ, by extending equation (2) to:

$$\frac{dv}{dt} = 3\frac{v_T^2 - v^2}{\beta D + 3x} + 3\frac{\eta_e}{\rho L}\frac{dl/dt}{\beta D + 3x} \qquad (4)$$

Here the rate of change of the ligament length (dl/dt) = – v, so that the influence of viscosity is to damp the tail acceleration. When the viscous shortening speed becomes constant, equation (4) shows that the Taylor speed has a lower value $v_{T\eta}$ given by the solution of the quadratic equation

$$v_{T\eta}^2 = v_T^2 - \frac{\eta_e}{\rho L} v_{T\eta} \qquad (5)$$

We find that the effect of extensional viscosity for typical ink-jets is ~ 2% of the Taylor speed given by equation (1). The use of simple models, without viscosity, is probably consistent with the experimental reduction of < ~10% noted above.

We have also modeled fluid elasticity by using a linear spring equation by modifying equation (2) to:

$$\frac{dv}{dt} = 3\frac{v_T^2 - v^2}{\beta D + 3x} + 12k\frac{f(x)}{\beta D + 3x} \qquad (6)$$

Here the function $f(x)$ is either a constant L, for the case where the ligament retains its tension throughout the collapse, or L-x where the tension reduces during the collapse of ligament. The constant $k \geq 0$ is proportional to the usual linear spring constant (but also depends on the actual values of the constants L, D and ρ), and has dimensions of acceleration as the function $f(x)$ is a length.

The form of $f(x)$ chosen to represent elastic ligament tension determines the evolution of acceleration given by equation (5). For the constant tension scenario $f(x) = L$, and the Taylor speed is raised to a higher value v_{Tk} given by the quadratic equation

$$v_{Tk}^2 = v_T^2 + 4kL \qquad (7)$$

This scenario provides a clear indication that higher, constant shortening speeds can be produced for unbroken elastic ligaments. The timescale for achieving this higher speed is correspondingly shorter than predicted from equation (3): the elastic "snap-back".

To incorporate the finite length L, the conical shape and the initial stretching of the ligament due to the velocity gradient (U/L) between the tail and head ends, equation (2) for an initial attached tail with mass β times that of a hemisphere of diameter D becomes:

$$\frac{dv}{dt} = 3\frac{v_T^2 p - vp^2(v - Ux/L)/(1 + Ut/L)}{\beta D + (1 + p + p^2)x} \qquad (8)$$

The variable $p=D'/D$ defines the conical shape at position x; for p = 1 at every x, equation (8) relates to a stretching cylindrical ligament, while for U = 0 there is no stretching. Equation (4) reverts to equation (2) for p = 1 and U = 0. Variable t represents elapsed time, which enters due to the persistent stretching of the ligament fluid by the massive head at speed U. In our experiments, $p > 1$ for $x > 0$, so in the absence of stretching the tail acceleration is systematically reduced compared with that for a cylinder.

Equation (8) can be used to treat the head motion, although we actually used a different balance in our previous work [1], such that the combined effects of the tail and head end accelerations were used to compute the length shortening speed due to surface tension of the ligament by time integration over the accelerations.

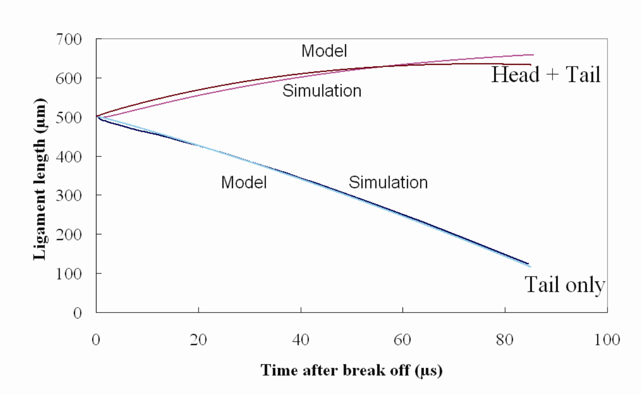

***Figure 4**: Results from 2-D numerical simulation of a Newtonian fluid ligament and from the improved 1-D model for identical conditions at break-off*

The initial conditions for our original computations of Newtonian ligament length shortening [1] used a tapering form and the tail to head end velocity difference of ~ 6 m/s just after the break-off to form a constant velocity gradient in the fluid along the ligament length. Therefore the 'Tail only' computations shown in Figures 3(a)-(c) of [1] are still valid. However, the shortening speed for 'Head + Tail' computations in [1] was incorrect, as the initial condition for the head speed U for the head forward motion was set at zero, which is inconsistent with a finite gradient.

As in our experiments [2] the Taylor speed for the tail end was slightly smaller than the drop (head) speed, the physical distance between the tail end and the head end must initially increase and not shorten because there is insufficient time for the massive head to have gained backward speed. In fact the small difference between the drop speed and the Taylor speed, together with the ligament break-off length, can control how far the last satellite drop is positioned behind the main drop. The overall ligament length cannot decrease even as fast as the Taylor speed, unless either elasticity is present and/or breaks occur along the ligament length, with each ligament rupture providing relatively low mass ($\beta \approx 1$) tails attached to shortening fluid ligaments.

Figure 4 shows the results for 'Tail-only' and 'Head + Tail' ligament lengths generated by 2-D simulation codes written by Harlen and Morrison [6, 7], together with the improved 1-D model predictions for the same break-off conditions: a conical jet shape, an initial head speed U of ~ 6.5 m/s and finite internal gradient at break-off for ~ 500 μm ligament length. Very similar ligament shortening behaviors are produced by these 2-D and 1-D models for this fluid with viscosity 0.01 Pa s and surface tension 0.037 N/m, when identical average ligament widths and head speeds are used.

Further 1-D modeling (not shown here) reveals sensitivities of the unbroken ligament length evolution, in comparison with the 2-D simulations, to the head speed and the average ligament width. The axial velocity gradients in the 1-D model depend on the head speed U assumed, whereas the 2-D code and the original images [1] reveal that the fluid piles up behind the head at a faster speed. Likewise, the geometric 'Tail only' behavior appears to mimic the dynamic 2-D simulation results only by using the average ligament width, rather than the conical width profile seen at the break off.

Conclusions

We have shown by comparison with axisymmetric simulations and with experiments involving no ligament break-up that our improved model gives a good representation of average jet shortening for Newtonian ink-jets. The inclusion of extensional viscosity within the 1-D model equations lowers ligament length shortening speeds for Newtonian ink-jet fluids by only 2%. The improved 1-D model yields very similar results to 2-D numerical simulation results for fluids having low shear viscosities typical of DoD ink-jet fluids.

Inclusion of linear elasticity in the 1-D model can, under constant tension scenarios, result in a higher "effective" Taylor speed with a correspondingly faster timescale to attaining this. Other elastic scenarios produce shortening speeds that are not constant but decay with time, in contrast to the observations [2].

Fragmenting Newtonian ligaments also shorten far faster than Taylor speed, which still requires proper quantitative explanation. We will continue to research the underlying cause(s) of the high shortening speeds for fragmenting ink-jet ligaments [2].

Acknowledgements

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Author Biography

Stephen Hoath received his BA in physics (1972) and then his DPhil in nuclear physics (1977) from the University of Oxford, UK.A Lecturer in Physics at the University of Birmingham, UK (1979), he then worked for BOC Edwards High Vacuum (1986) and smaller companies(1997, 2001) before joining the University of Cambridge, UK, Inkjet Research Centre (2005). His inkjet work has focused on jetting. An IS&T and IOP member, he is a Teaching Associate at Gonville & Caius College, Cambridge, UK.