Jet Diameters and Velocity Profiles in Continuous Ink-Jets

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Abstract

Theoretical predictions of the diameters of continuous ink-jets downstream of long nozzles are generalized to include the important cases of ink-jet fluids and shorter nozzles where the velocity profile at the nozzle exit is undeveloped (non-parabolic). Comparisons of the new predictions with experiments and simulations are made for fairly long nozzles with tapered profiles and short nozzles with conical profiles; experimental and simulated profiles are also compared downstream of the nozzle exit for both industrial and large scale ink-jet print heads.

Precise measurements of the un-modulated jet diameters downstream of the nozzle exit can set really useful limits to the possible shapes of the flow profile right at the nozzle exit, and in particular allow some assessment of the axial velocity gradients and fluid shear rates at the nozzle exit where direct speed measurement is usually impractical.

Simulations allow further study of the relaxation of the velocity profile downstream of the nozzle exit, and are reported for both un-modulated and modulated CIJ jetting. Implications of this work include speeding up CIJ simulations, absolute calibration of the applied CIJ system modulation, and the likely magnitude of dynamic surface tension effects on observed CIJ satellite speeds.

Relaxation effects in continuous jetting

Jets of Newtonian fluids emerging in laminar flow from very long nozzles are associated with a 'fully developed' parabolic Poiseuille velocity profile at the nozzle exit, and the relaxation of this profile downstream of the nozzle towards a uniform, 'plug', velocity profile. Within shorter nozzles (as shown in Figure 1), the velocity profile does not fully develop, and is not parabolic at the exit. An unmodulated continuous ink-jet (CIJ) relaxes downstream from the nozzle: its diameter changes from that of the nozzle diameter at the exit, towards an equilibrium diameter which depends for Newtonian jets on the values of Reynolds and Weber number. In industrial CIJ, jet diameters are smaller than the nozzle exit diameter. Here we consider theoretical predictions of jet profile and diameter relaxation for long and short nozzles, the application of recent simulation codes to these nozzles, and related experimental results. This provides an insight into conditions within other CIJ nozzles based solely on precise measurements of unmodulated jet diameters, and also allows the prediction of the downstream distances required for full relaxation.

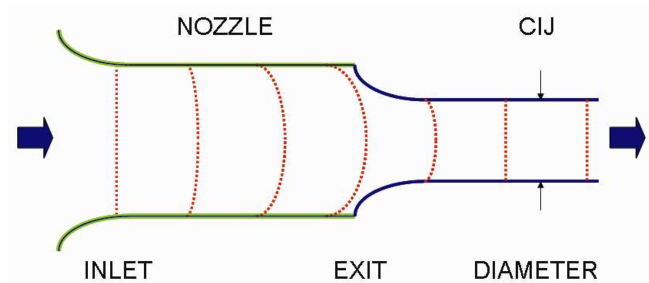

Figure 1: *Idealized CIJ nozzle outline (green) with velocity profiles shown as red broken lines at the shaped inlet and various locations within and downstream from the exit. The velocity profile develops from flat towards the parabolic Poiseuille shape, and back again, as the fluid passes through the nozzle into the free jet (outlined in blue). Typically the free jet may reduce to ~ 92% of the nozzle exit diameter.*

Theoretical treatment of relaxation

Middleman and co-workers published a series of papers [1] that treated the basic equations of continuity, momentum and energy for a liquid jet issuing from a long nozzle. Their axisymmetric approach linked the exit and the downstream jet diameters to radial integrals over various powers (n = 1, 2 and 3) of the fluid velocity profile at these locations. For nozzle exit radius a, and average velocity c, the exit velocity profile was assumed to be that given by Poiseuille's theory, which can be expressed by the power law equation (1) with $n = 2$:

$$u(r) = \frac{n+2}{n} c \left(1 - \frac{r^n}{a^n}\right) \tag{1}$$

Using power law radial profiles with $n = 2$ imposes an unnecessary limitation in the study of relaxation inside real nozzles, since quite adequate theories (e.g. [2]) of the velocity profile developed within shorter nozzles are available. For a power-law profile, a more uniform velocity requires higher n.

We have modeled Langhaar's theory [2] for finite length nozzles in order to link an empirical power law exit velocity profile with the fluid dynamics, the diameter of the nozzle and its length. We then generalized the Middleman approach to handle power law profiles, as power laws fitted to the Langhaar theory, and to other simulation results [3], provide rather simple, good and fast representations of the results obtained by using far more complex theory and lengthy computation times.

Upstream from the nozzle exit

Our study of relaxation effects downstream of the nozzle actually starts upstream, with the relaxation theory for a uniform radial velocity profile at the inlet to a short nozzle. In the literature [1] a set of 'power-law fluid' results for long nozzles were corrected later by Gavis. Those power laws are for the fluid behavior under shear, whereas our approach focuses on equation (1) for a continuous jet. One key implication for the present work from the earlier theory [1, 4] is that surface tension has only a small effect on the free jet diameter, which suggests that the jet diameter can be used to investigate other variables such as the velocity profile, fluid viscosity and (for non-Newtonian fluids) elasticity.

Langhaar's theory [2], which is based on the Navier-Stokes equation for the axial component of fluid velocity in the nozzle (with radius a), can be used to determine the ratio λ between the speed at radius r and the mean axial speed c, at all locations z within the straight nozzle section, with a parameter γ depending on z alone:

$$\lambda = \frac{I_0(\gamma) - I_0(\gamma r / a)}{I_2(\gamma)} \tag{2}$$

The functions $I_0(x)$ and $I_2(x)$ are the modified Bessel functions of the first kind of order 0 and 2 respectively [6]. The axial speed on the nozzle axis ($r = 0$) is defined by $\lambda_0 = I_0(\gamma)/I_2(\gamma)$.

The parameter γ is a function of (dimensionless) axial position s defined by:

$$s = \frac{z}{a\,\mathrm{Re}} \tag{3}$$

where Re is the Reynolds number defined by the mean speed c, fluid density ρ and viscosity η:

$$\mathrm{Re} = \frac{\rho c a}{\eta} \tag{4}$$

The relationship between γ and s given numerically by Langhaar's Table 1 shows that the 'transition length' required for development of the Poiseuille velocity profile is conventionally taken when s is sufficient to produce $\lambda_0 > 1.98$, i.e. within 1% of the value for a truly parabolic profile for which $\lambda = 2$; then $s = 0.23$ [1]. We can therefore use equations (3) and (4) to determine whether the nozzle exit profile is likely to be close to the Poiseuille conditions, and use equation (2) when it is not parabolic.

Figure 2 shows Langhaar's general results; Figure 3 shows the prediction for a short nozzle as typically used for continuous inkjets [7].

Figure 4 shows the profiles for the 2.2 mm long, 2.2mm diameter nozzle used in our large scale CIJ experiments [7] corresponding to the fluid used in Figure 3. The Poiseuille parabolic representation is clearly poor, though a power law, as equation (1) with n=5.0, is rather close to the Langhaar profile, equation (2), when normalized to the same central value λ_0.

Figure 5 shows the radial profile produced by a simulation of the same fluid and nozzle using a code written at Leeds [3], together with the power law fit n=4.9 that produced the same central velocity. Again, we find that the power law representation is a reasonable fit to the simulation, and furthermore the index n is very similar to the fit to the Langhaar profile as shown in Figure 4.

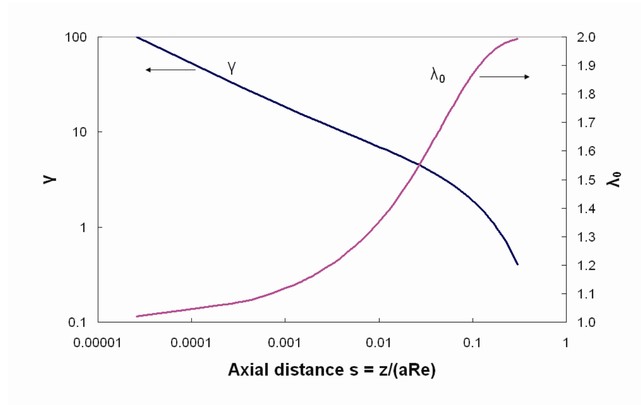

***Figure 2**: Data taken from Langhaar Table 1 [2] to show how the parameter γ (blue curve) and the central axial speed λ_0 (magenta) vary with the dimensionless axial distance s defined by equation (3).*

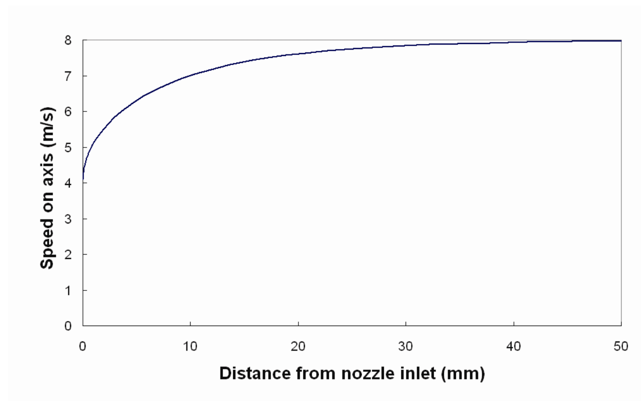

***Figure 3:** Computed values of central axial speed for a large-scale 2.2 mm diameter CIJ nozzle, with average initial jet speed 4 m/s, fluid density 1100 kg/m³ and viscosity 0.032 Pa s. [7] The nozzle length was 2.2 mm, and the radial velocity profile is far from parabolic: here $\gamma \approx 6.25$.*

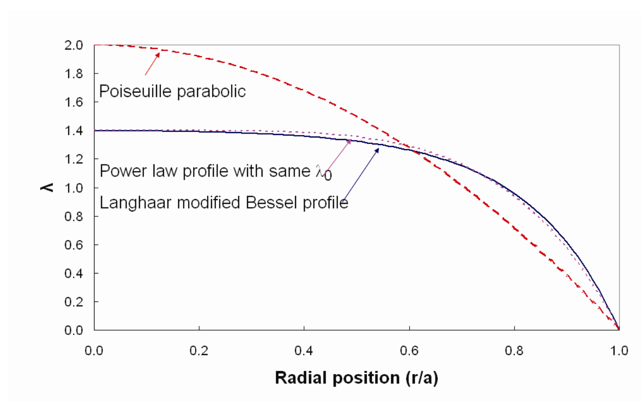

***Figure 4**: λ computed from equation (2) for $\gamma \approx 6.25$ and the same fluid properties and speed as in Figure 3, compared with the Poiseuille parabolic (red broken line) and power law fit from equation (1) with n = 5. Such representation of the results from Langhaar's theory by power law fitting holds for other values of γ, and provides a considerable simplification.*

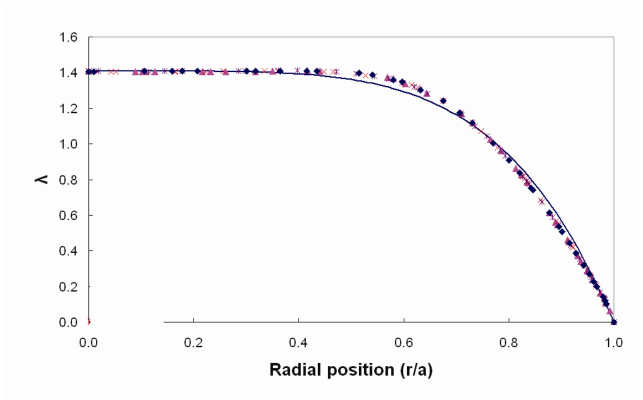

Figure 5: Simulation of the same system as in Figures 3 and 4, also showing a power law fit from equation (1) with n = 4.9 (blue line) compared with the axial speed and radial position derived from the computational model. Near agreement of simulation and power law fits extends to theory of equation (2) and to Figure 4.

Figure 6 shows our general method for linking (D/L)Re to γ and n, with the dotted lines correlating values quite typical for CIJ.

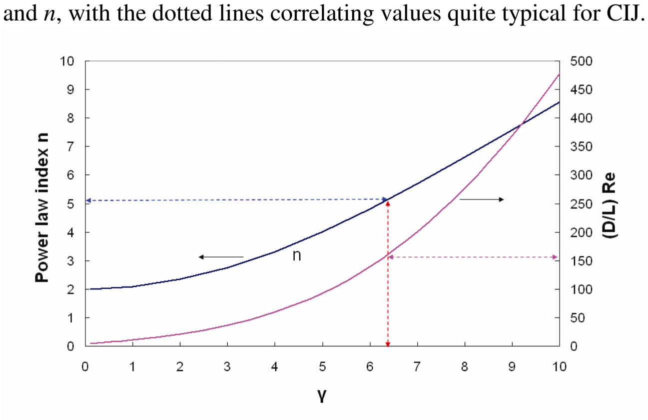

***Figure 6**: Fitted power law index n in equation (1) for distributions following equation (2) as a function of Langhaar's γ and the product Re (D/L). Reynolds number Re is from equation (4) and the nozzle has diameter D and length L. Dashed lines also shown link typical CIJ values of (D/L)Re to γ and n.*

Jet downstream from the nozzle exit

We have shown above that power law representations of the radial variation of axial speeds at the nozzle exit may be quite accurate (within <10% of c), and certainly better than the assumption of either parabolic (Poiseuille) or flat (plug flow) distributions. We can therefore generalize the model applied by Middleman et al [1,4] to relaxation within the jet downstream from the exit, by using the power law representation of exit speed profile given by equation (1).

For fluid flowing from a long nozzle, the ratio X defined by X = (downstream jet diameter/nozzle exit diameter, D) has the value $X = \sqrt{3}/2 = 0.866$, according to the original theory [1] with a parabolic speed profile at the exit. The pressure difference Δp between fluid in the nozzle exit and that downstream where the jet diameter is XD, is determined by the surface tension σ via equation (5):

$$\Delta p = \frac{X-1}{\mathrm{We}} \tag{5}$$

$$\mathrm{We} = \frac{\rho a c^2}{\sigma} \tag{6}$$

The Weber number We defined by equation (6) is usually >>1 for typical CIJ conditions. Equations (5) and (6) show that the pressure difference Δp between the exit and the downstream locations that is due to surface tension effects will be small, since typically $0.866 < X < 1.000$ for CIJ, provided that the surface tension does not vary along the jet.

When the nozzles are shorter, with length L and diameter D, the jet diameter XD can be expressed as:

$$X = X_{long} + f(\frac{D}{L}\mathrm{Re}) \tag{7}$$

Equation (7) has been evaluated here using Langhaar's theory [2] in a similar manner to that used by Gavis and Modan [6], except that we explicitly present the results as our Figure 7, for X as a function of the Reynolds number based on nozzle radius as in equation (4). For L/D =1, Langhaar's γ = 6.353, while the power law exponent n = 5.115. For typical fluids that we have jetted using industrial CIJ print heads with $L/D \approx 1$, Re(D/L) ≈ 160. Figure 7 shows that the predicted value for X is typically very close to 0.920 for un-modulated CIJ.

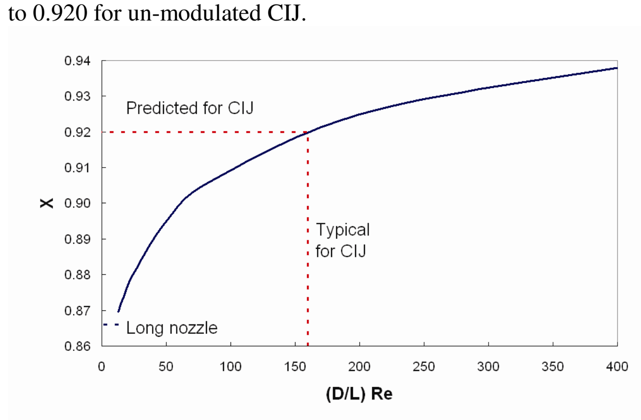

***Figure 7**:Ratio X between downstream jet diameter and nozzle exit diameter ratio for short nozzles plotted against the product Re(D/L). For a typical CIJ fluid jetted from a nozzle with (L/D) ≈ 1, X ≈ 0.920.*

CIJ diameters as a measure of velocity profile

We have explicitly shown in Figure 7 how CIJ diameters depend on the variable (D/L)Re and in Figure 6 how the same variable links to a power law index value that closely mimics the exit velocity profile predicted theoretically by Langhaar for short straight nozzles. Thus measurements of (un-modulated) jet diameter can be closely linked to the exit velocity profile and may provide some further information about the geometry of the nozzle. Such an approach is valuable as the exit velocity profile is normally difficult to access directly by experiments on practical systems, whereas jet diameter can be observed and measured optically to better than 1%.

Simulation of downstream profiles

Numerical simulation code developed at the University of Leeds [3] can be used to predict the velocity distribution and radial profiles at different axial locations ($z \geq 0$) relative to the nozzle exit, and thus allow the relaxation of the velocity profile to be described. A useful measure of this profile is the normalized velocity range at z as measured by

$$Range(z) = \frac{u(0) - u(R)}{< u(r) >} \tag{7}$$

At the nozzle exit, $Range(0) = \lambda_0$ from equation (2). As the profile flattens out, $Range(z)$ tends to 0. The jet diameter also varies, so that radial averages and limits for equation (7) will alter as z increases. Figures 8 and 9 show some predictions of the range from equation (7) and the normalized jet radius for a jet of water emerging at 5.66 m/s from nozzle with the geometry of a 100 µm (exit) diameter Microdrop® nozzle, for an un-modulated jet, and also for a jet with 2% modulation of the fluid flux.

The simulations were performed with and without applied modulation in order to investigate the variation of jet break-up point with the level of modulation, and also whether jet relaxation is influenced by modulation. Comparison of Figures 8 and 9 shows the effects of modulation on the range and radial size. The major relaxation of the velocity profile remains the same until the overall effects produced by the magnitude of the applied modulation, or 'noise', take over at long distances downstream. Further details are beyond the scope of this paper. Figure 10 shows other results for the relaxation of a free jet.

Discussion

For the nozzle exit velocity profile, the good correspondence found between fluid flow theory [2] and the results from the numerical simulation code [3] provide validation of the latter. Representation of the profile by an empirical power law is useful for further computation of downstream relaxation effects in other fluid jetting applications [8-11]. Prediction of the exit velocity profile and diameter for an unmodulated jet directly from the value of the product Re(D/L) is very convenient: a useful 'rule of thumb' for $L/D = 1$ and Re ≈ 150 is that the power law $n \approx 5$ and the jet diameter is ~ 0.920 of the nozzle diameter D.

A summary and numerical investigation of development lengths in laminar flow has been presented by Durst et al [12]. They conclude that most of the treatments of flow development (including that of Langhaar [2]) incorrectly predict a development length L proportional to Re. Their best estimate for development length is [12]:

$$\frac{Length}{Diameter} = (0.619^{1.6} + (0.0567\mathrm{Re})^{1.6})^{1/1.6} \tag{8}$$

Typically Re > 100 for industrial CIJ, so equation (8) would predict that Length/Diameter ≈ Re and we can exploit Langhaar's flow results [2] as a guideline.

If the fluid shows dynamic surface tension differences on the timescale of the downstream distance divided by the axial speed c, then the pressure difference equation (5) would have to be altered to:

$$\Delta p = \frac{X}{We_{downstream}} - \frac{1}{We_{exit}} \tag{9}$$

We have already surmised that dynamic surface tension effects might contribute to changes in satellite speeds in CIJ [8]; equation (9) provides a route for changing the proposed balance between the jet size and the velocity profile for similar Reynolds numbers. It is clear that if the exit nozzle face were wetted, the conditions at the exit and downstream jet would be determined not simply by the nozzle exit diameter but also be influenced by the wetting. In such cases the downstream jet diameter is typically greater than the nozzle diameter. The practical significance of this is that we are currently unable to predict jet diameters from a wetted nozzle. Further work is planned to address this challenge.

The power law relationship at the nozzle exit may even be adapted to mimic the relaxation of the velocity profile downstream. This relaxation has proved important [9] for the treatment of obliquely colliding fluid jets that have a fixed free length before interacting, although until now the assumptions have been that the profiles are uniform, parabolic or modified parabolic [10]. We suggest that the velocity profile downstream at z may be empirically represented by

$$u(r) = u(R) + [u(0) - u(R)](1 - \frac{r^n}{R^n}) \tag{10}$$

This equation (9) matches the exit velocity profile equation (1) at $z = 0$, where parameters $R = a$, $u(R, 0) = 0$ and $u(0, 0) = (n + 2)/n$. Downstream from the exit, the jet diameter = $2R$ and the velocity profile parameters n, $u(R, z)$ and $u(0, z)$ will all vary with distance z. We intend to investigate this representation in order to improve our understanding of measurements on sheets produced by colliding fluid jets [11], following on from the simulation predictions shown in Figure 10.

These and other applications for the general simulation code [13] are now well underpinned by measurement and theory for typical CIJ conditions.

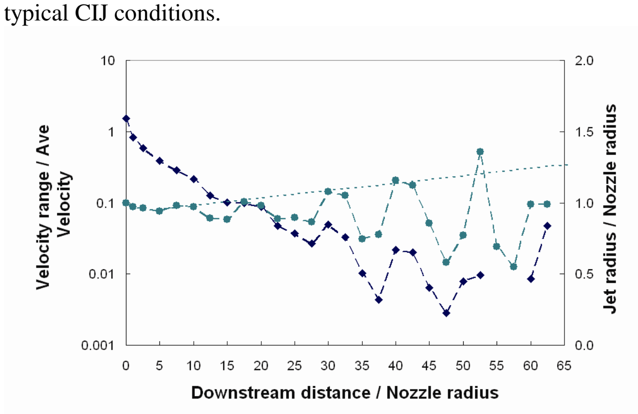

Figure 8*: Results for a 5.66 m/s water jet from a 50 µm radius Microdrop® nozzle at 2% flow modulation; the velocity range in equation (7) is shown by (-♦-) starting from > 1 at the nozzle exit, and the jet radius divided by nozzle radius is shown by (-•-), with a dotted line representing the radial growth trend.*

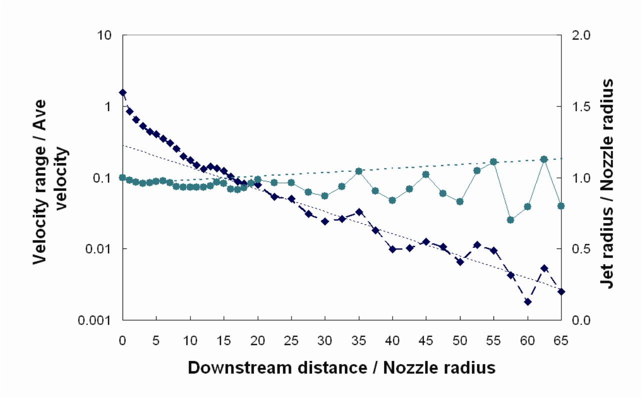

***Figure 9**: Results from the simulation of the same fluid and nozzle as in Figure 8 with 0% flow modulation, showing an additional dotted line representing the trend for the relaxation of the velocity profile downstream. The growth rate for the jet radius is lower than in Figure 8, since the simulation represents the free jet break off.*

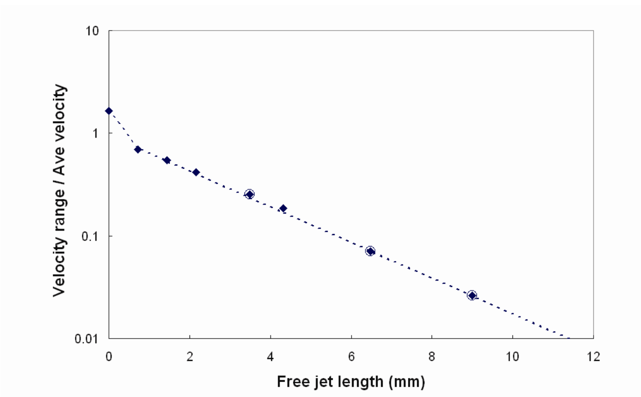

***Figure 10**: Predictions for free jet lengths of 0.011 Pa s glycerol and water mixture from a 0.85 mm diameter nozzle, as used in colliding jet experiments [11] Free jet lengths used are ringed (O), while the simulation results are (-♦-); the dashed line represents a 2-component exponential decay of the velocity range, which is a typical behavior for the relaxation downstream (e.g. as in Figure 9).*

Acknowledgements

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Author Biography

Stephen Hoath received his BA in physics (1972) and then his DPhil in nuclear physics (1977) from the University of Oxford, UK. Lecturer in Physics at the University of Birmingham, UK (1979), he then worked for BOC Edwards High Vacuum (1986) and smaller companies(1997, 2001) before joining the University of Cambridge, UK, Inkjet Research Centre (2005). His inkjet work has focused on jetting. An IS&T and IOP member, he is a Teaching Associate at Gonville & Caius College, Cambridge, UK.