Quantified Evaluation of Various Spectral Encoding Techniques

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Abstract

The goal of spectral encoding is to provide a limited dimensional space that preserves as much information about the original reflectance spectrum as possible. Recently, several spectral encoding techniques have been suggested by Nakaya and Ohta, as a simplified improvement over the LabPQR spectral encoding technique. To directly compare the reproduction accuracy between the new techniques and LabPQR a technique was developed to quantify the accuracy of three spectral encoding algorithms. Thousands of random spectra were generated and converted using the three spectral encoding techniques into six dimensions. The six spectral encoded values were then used to regenerate the target spectra. Simple RMS difference, Delta E 2000, and Delta E 2000 under a different illumination was used to evaluate reproduction quality. It was found that the LabPQR technique maintains higher reproduction capabilities than the alternate techniques.

Introduction

Human color vision is based on three sets of color receivers, or cones that are sensitive to different ranges of wavelengths in the visual spectrum. This trichromacy of human vision allows for all colors to be defined as three separate signals to the three different cones. This leads to the natural utilization of three dimensional color spaces to describe all visible colors.[1] Typical color spaces are defined by three dimensions related to a specific illumination. For example, two samples that are a nice blue might be defined under D65 lighting as L*=25 a*=65 b*=-90. Yet, depending on the specific spectral reflection of the samples, the colors might look quite different under a different illumination. These two samples are known as metamers, they look the same under one illumination, but quite different under another. Metemeric imaging is the foundation of the image reproduction industry. A photograph does not reproduce the spectral information of a scene, it reproduces the color of the scene under a specific illumination. The same basic assumptions control almost all color reproduction processes.

The limitations of metameric imaging based on three dimensional color spaces have been well documented. König and Herzog[2] found that the mean color difference of a large collection of spectra imaged under different illuminations can be as large as 0.8 ΔE_{94} and the max difference can be as large as 6 ΔE_{94}. A solution to this problem might be to define a color sample by its spectrum using many more dimensions. Using the reflectance value every 10nm from 400 to 700nm would yield a fairly accurate reproduction of the original spectrum, but would require a 31 dimension color space. Utilizing a 31 dimensional color space may work for an individual spectrum, but becomes unwieldy in the standard color reproduction process. Rosen and Ohta[3] find that using only the standard three dimensional color space utilizes a look-up table on the order of 30KB, yet a 31 dimensional color space would require a look-up table of $8x10^{27}$GB. The goal of spectral color spaces is to utilize a reasonably dimensioned space, yet still be able to transfer more knowledge of the original spectrum.

Overview of three algorithms

TrW6

The first algorithm explored is TrW6, as defined by Nakaya and Ohta.[4] Six sinusoidal functions are utilized as eigenvectors as defined in equation 1.

$$\begin{aligned}
e_1(\lambda) &= \sin\left(\frac{1}{2}\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_2(\lambda) &= \cos\left(\frac{1}{2}\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_3(\lambda) &= \sin\left(\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_4(\lambda) &= \cos\left(\frac{3}{2}\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_5(\lambda) &= \sin\left(2\pi\frac{\lambda-\lambda_{\min}}{\lambda_{\max}-\lambda_{\min}}\right)\\
e_6(\lambda) &= \cos\left(\frac{5}{2}\pi\frac{\lambda}{\lambda_{\max}-\lambda_{\min}}\right)
\end{aligned} \tag{1}$$

The six eigenvectors are used to generate six weighting factors that solve equation 2,

$$p(\lambda) = \sum_{i=1}^{6} w_i e_i(\lambda) \tag{2}$$

where $p(\lambda)$ is the original reflectance spectrum. Solving for w_i for any $p(\lambda)$ will generate a six number description of a complete spectrum. Simply multiplying the weighting factors against the original eigenvectors will generate a reasonable facsimile of the original spectrum in question.

LabRGB

The second algorithm from Nakaya and Ohta explored was LabRGB. The goal here is that the final six encoding values represent a combination of two recognizable three dimensional color spaces. The L*a*b* coordinates of the original reflectance sample are found through standard calculations, under a given illumination. Equations 1 and 2 are utilized again to find the six weighting functions. It is assumed that w_1, w_2, and w_3 roughly represent R, G, B components based on the sinusoidal patterns. Therefore, the spectrum in question can be represented by six values of L*a*b*RGB and an approximate color can be referenced

simply from the six numbers. Since the information from weighting factor 4,5 and 6 is lost, the reconstruction of the spectra contains some added complexity. First, estimated XYZ values are calculated based on the RGB values (w_1, w_2, and w_3) given, as shown in equation 3,

$$\begin{aligned}\hat{X} &= \sum_{i=1}^{3} w_i \int e_i(\lambda)E(\lambda)\bar{x}(\lambda)d\lambda \\ \hat{Y} &= \sum_{i=1}^{3} w_i \int e_i(\lambda)E(\lambda)\bar{y}(\lambda)d\lambda \\ \hat{Z} &= \sum_{i=1}^{3} w_i \int e_i(\lambda)E(\lambda)\bar{z}(\lambda)d\lambda\end{aligned} \tag{3}$$

where $E(\lambda)$ is the spectrum of the illumination in question. The original L*a*b* values are converted to XYZ and the unknown weighting factors (w_4, w_5, and w_6) are calculated utilizing equation 4.

$$\begin{aligned}X - \hat{X} &= \sum_{i=4}^{6} w_i \int e_i(\lambda)E(\lambda)\bar{x}(\lambda)d\lambda \\ Y - \hat{Y} &= \sum_{i=4}^{6} w_i \int e_i(\lambda)E(\lambda)\bar{y}(\lambda)d\lambda \\ Z - \hat{Z} &= \sum_{i=4}^{6} w_i \int e_i(\lambda)E(\lambda)\bar{z}(\lambda)d\lambda\end{aligned} \tag{4}$$

The six weighting factors are again multiplied with the six original eigenvectors shown in equation 1, and the approximate spectrum generated as the sum.

LabPQR

LabPQR[5] utilizes the original L*a*b* coordinates and additional dimensions that describe the metameric black. Several different strategies could be utilized to define the metameric black space. For this analysis, a single strategy described by Derhak and Rosen that utilized three dimensions for the metameric space is used.

For this approach, a large collection of spectra S is required. A reference illuminant is used along with the standard tristimulus functions to generate a distilling matrix D, defined by equation 5,

$$D = \begin{bmatrix} 1/X_{illum} & 0 & 0 \\ 0 & 1/Y_{illum} & 0 \\ 0 & 0 & 1/Z_{illum} \end{bmatrix} \begin{bmatrix} \bar{x}_{\lambda=400nm} & \bar{x}_{\lambda=410nm} & \cdots & \bar{x}_{\lambda=690nm} & \bar{x}_{\lambda=700nm} \\ \bar{y}_{\lambda=400nm} & \bar{y}_{\lambda=410nm} & \cdots & \bar{y}_{\lambda=690nm} & \bar{y}_{\lambda=700nm} \\ \bar{z}_{\lambda=400nm} & \bar{z}_{\lambda=410nm} & \cdots & \bar{z}_{\lambda=690nm} & \bar{z}_{\lambda=700nm} \end{bmatrix} \cdot \begin{bmatrix} i_{\lambda=400nm} & 0 & 0 & 0 & 0 \\ 0 & i_{\lambda=410nm} & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & i_{\lambda=690nm} & 0 \\ 0 & 0 & 0 & 0 & i_{\lambda=700nm} \end{bmatrix} \tag{5}$$

where, X_{illum}, Y_{illum}, and Z_{illum} are the XYZ coordinates of the reference illuminant, i_λ is the spectrum of the reference illuminant and $\bar{x}$, $\bar{y}$, $\bar{z}$ are the tristimulus functions. This distilling matrix can be used to find the normalized tristimulus values for all spectra using equation 6.

$$C = DS \tag{6}$$

An estimation matrix is calculated using equation 7.

$$R = SC^T (CC^T)^{-1} \tag{7}$$

The set of spectral differences of the colorimetry to spectra estimation can be calculated using equation 8,

$$E = (I - RD)S \tag{8}$$

where I is simply the identity matrix. The covariance of E is found using equation 9, and the eigenvectors and eigenvalues can be found by solving equation 10,

$$W = EE^T \tag{9}$$

$$Wv_i = \lambda_i v_i \tag{10}$$

where v_i are the eigenvectors and λ_i are the associated eigenvalues. The 3 eigenvectors associated with the three largest eigenvalues are utilized as matrix V as defined by equation 11.

$$V = \begin{bmatrix} v_0^T \\ v_1^T \\ v_2^T \end{bmatrix} \tag{11}$$

The final PQR values are found for a specific spectrum, s, with equation 12, and combined with the original L*a*b* coordinates give the LabPQR values sought.

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = V(I - RD)s \tag{12}$$

The reconstructed spectrum is created with equation 13, where c is the normalized XYZ coordinates converted from the original L*a*b* values using standard calculations.

$$s' = Rc + V^T \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{13}$$

Results

Spectra Generation

To test the effectiveness of the three spectral encoding techniques, a set of random spectra was initially utilized. This set of random spectra was generated using a modified version of Ohta's simplified method for formulating pseudo object colors.[6] The goal is to produce a random spectrum defined at 10nm intervals that simulates the smoothness of common spectra seen in nature. The general equation used is shown in equation 14,

$$\rho_{i+1} = 2\rho_i - \rho_{i-1} + 2r_{i-1}\Delta \; (i = 2,3...31) \tag{14}$$

where ρ_i is the reflectance from 0 to 1 for 31 points at 10nm intervals from 400 to 700nm. r_i is a random number from -1 to 1 and Δ is a weighting factor of how much the spectrum is allowed to vary from 10nm point to 10nm point. Ohta found that from real world spectra a Δ of 0.03 is reasonable. Ohta states that the first two points of the spectrum should be generated randomly, but simply inserting ρ_1 = 0.001 and ρ_2 = 0.9 into equation 14 shows that this assumption will lead to invalid spectra quite often. To limit this failure it was decided that ρ_1 would be a random number between 0 and 1, and ρ_2 would be generated using equation 15.

$$\rho_2 = \rho_1 + r\Delta \tag{15}$$

There are still many cases where the spectra generated will fail because of values greater than 1 or less than 0. This was dealt with simply by brute force, throwing out all spectra with invalid reflectance and generating a new one.

Spectra Reproduction

The primary goal of all these algorithms is to reproduce an original spectrum as faithfully as possible utilizing only six numbers as the input to the reconstruction algorithm. Figure 1 shows a randomly generated spectrum, and the round trip reproduction produced utilizing all three algorithms.

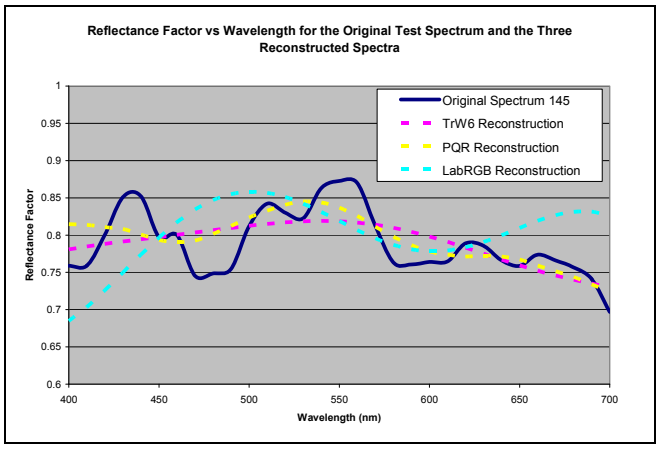

Figure 1. *A random spectrum, and the three reconstructions generated by the chosen algorithms. The solid line represents the original spectrum. The three dotted lines represent the reconstructions created by all three algorithms based on the six numbers generated.*

There are several ways to quantify the differences between the original spectrum and the reconstructed spectrum. One way is to simply calculate the RMS difference between the curves as shown in equation 16.

$$RMS = \sqrt{\frac{\sum_{i=1}^{31}(s_i - x_i)^2}{31}} \tag{16}$$

Where s is the original spectrum, and x is the reconstructed spectrum based on 10nm points between 400 and 700 nm. Obviously an RMS of zero would be a perfect match. For the test spectrum showed in figure 1, the RMS values for the LabPQR, TrW6 and LabRGB algorithms are 0.0299, 0.0386 and 0.0653 respectively. Therefore, for the one test spectrum, it could be stated that LabPQR is the best match to the original spectrum. The algorithms were applied to 1000 random spectra and the RMS difference calculated for each reconstruction. The data is plotted in a histogram shown in figure 2.

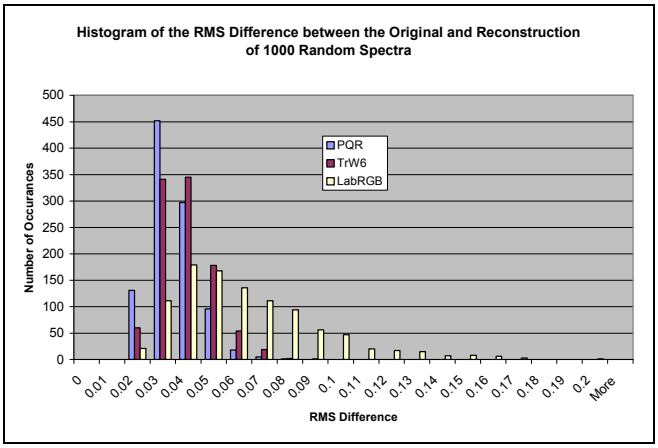

Figure 2. *A visual representation of the accuracy of reproduction of the original spectra as performed by the three algorithms. An RMS difference of 0 is a perfect match. LabPQR performs slightly better than TrW6. LabRGB shows a significant amount of information is lost in an attempt to utilize a combination of two existing three dimensional color spaces.*

Figure 2 clearly shows that for these 1000 randomly created spectra, the LabPQR algorithm generates reconstructed curves that more closely represent the original spectra.

Color Constancy

The real test of a reconstructed spectrum is how well it reproduces the same visual color response as the original when the illumination varies. The reconstructions for all algorithms were carried out utilizing D65 illuminant. For TrW6, this is irrelevant, because the weighting factors generated and used for the reconstruction are simply based on curve fitting and not visual perception of color. Both LabRGB and LabPQR utilize the L*a*b* color descriptors, and by definition, the reconstructed curve and the original spectra have the same color coordinates under the original illumination. The L*a*b* of the reconstructed spectra from all three algorithms was calculated under illuminant A, and compared to the original spectra also under illuminant A. Figure 3 shows the a* and b* coordinates of four randomly chosen spectra in the same quadrant of L*a*b* space.

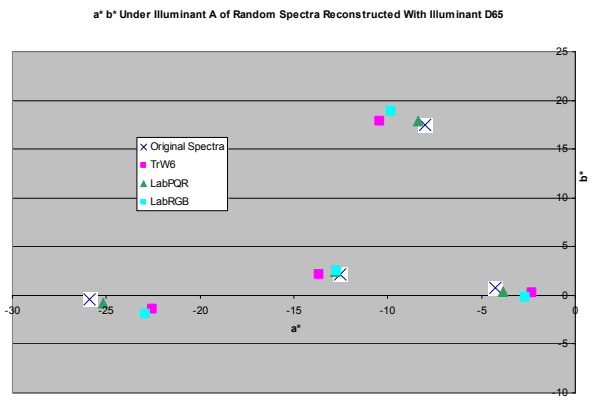

Figure 3. *The original spectra are compared to the algorithm spectra under illuminant A. All algorithm spectra were reconstructed using D65. The L*a*b* coordinates were calculated using the reconstructed spectra and illuminant A. Again, LabPQR tends to be much closer in a* b* to the original spectra.*

The particular implementation of LabPQR tested here utilizes a reference illumination as shown in equation 5. A new random set of 100 spectra was generated, and the roundtrip Delta E_{2000} under illuminant A was recalculated for the LabPQR algorithm. The results as a percentage of frequency is shown in figure 4.

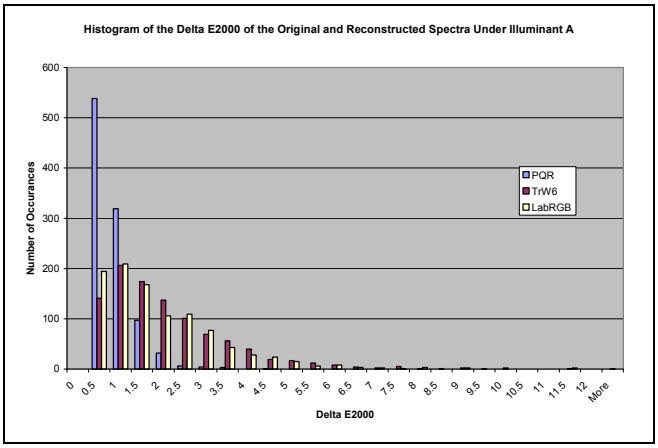

Figure 4. *The Delta E_{2000} between the original and the reconstructed spectra under illuminant A for all 1000 random spectra. A Delta E_{2000} of 0 is a perfect match. LabPQR is clearly a much better match. TrW6 and LabRGB are much closer in Delta E_{2000} than in RMS difference.*

Conclusion

The key difference between the LabPQR and the TrW6, LabRGB techniques is the source of the eigenvectors used. The TrW6 strategy is simply a curve fitting strategy that requires no information about illumination or visual perception at all. The strategy could be employed to generate six dimensional descriptors for any curves. The results show a large proportion of reconstructed spectra to have a Delta E_{2000} of greater than 1 when shifting from D65 to illuminant A. The LabRGB technique utilizes the same strategy, but sacrifices accuracy simply to provide a six number sequence from which an estimate of general color can be obtained from the RGB coordinates. The RMS difference and the Delta E_{2000} both show that this is a large sacrifice of information, as the original TrW6 gives a much more faithful reproduction of the original spectral intent. LabPQR also allows for the estimation of original color based on the L*a*b* coordinates, but shows much higher reproduction accuracy than the TrW6 algorithm. While the TrW6 and LabRGB algorithms may be mathematically simpler, they lead to a decrease in spectral accuracy.

References

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