Theoretical Analysis of Metamer Intersection Locations

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Abstract

There has been a historical assumption that intersection locations of metamers approximately coincide with particular locations in the wavelength range. However, in the existing research, experimental discussions have been performed under the assumption of specific models for spectral radiance distributions. In this paper, from a theoretical point of view, general analytical results not depending on any specific model are described.

Introduction

Metamerism is one of the most fundamental perceptual phenomena of the visual system. Because of their fundamental importance, metamers have received considerable analysis in terms of their spectral and colorimetric properties. Among them, as a classical argument, W. D. Wright proposed and W. A. Thornton and others analyzed the very important assumption that the locations of metamer intersections approximately coincide with particular locations in the wavelength range.[1)2)] Thornton called these wavelengths as the "prime" wavelength and demonstrated their importance in light-source design.[3)] Thornton has analyzed the so-called Bayer set of metamers, and found the statistical peaks of intersections of metamers existing near the above wavelengths.[4)] He also synthesized metameric spectral radiance distributions using Gaussian functions with similar results.

Many researchers[1)-9)] have derived historically important results, but all experimental results depended on specific models of spectral shapes and there are not studies based on theoretical analysis which will be discussed in this paper without any specific model of spectral shapes.

General Formulation of Metameric Black[11)]

A metamer pair of different spectral radiance distributions $s^{(1)}(\lambda)$ and $s^{(2)}(\lambda)$ which cause the same tri-stimulus values can be replaced by a metameric black $s_{mb}(\lambda)=s^{(1)}(\lambda)-s^{(2)}(\lambda)$, where λ indicates the wave length parameter. The discussion in this paper is replaced by the zero-crossing problem of $s_{mb}(\lambda)$. The approach of this paper is that at the first step general formulation of metameric black is derived and at the second step the intersection location problem is discussed using the general formulation. General formulation of metameric black has not been derived up to date.

Although, R-Matrix[11)] decomposes a spectral radiance distribution into the fundamental component and the metameric black component and consequently generates the metameric black, does not describe all of metameric black theoretically existing.

A general formulation of metameric blacks can be defined as the optimum solution of a cost function $f^*\left(s_{mb}(\lambda)\right)$. From various f^* theoretically existing, all of metameric blacks theoretically existing are generated. f^* corresponds the design policy of the metameric black to be designed.

The function f^* can be written including $g(\lambda)$ function explicitly as a parameter function as follows:

$$f^*\left(s_{mb}(\lambda)\right)=f\left(s_{mb}(\lambda),\ g(\lambda)\right). \tag{1}$$

The function f can be approximated by fitting a second-order polynomial expression including cross terms as follows :

$$\begin{aligned} f\left(s_{mb}(\lambda),\ g(\lambda)\right) \cong a_1+a_2 s_{mb}(\lambda)+a_3\left\{s_{mb}(\lambda)\right\}^2 \\ +a_4 g(\lambda)+a_5\left\{g(\lambda)\right\}^2 \\ +a_6 g(\lambda) s_{mb}(\lambda)\ , \end{aligned} \tag{2}$$

where

$a_i(i=1,2,\cdots,6)$: constant coefficients.

When $\dfrac{\partial f}{\partial s_{mb}(\lambda)}=0$ under the constraints of the metameric black conditions of $X=Y=Z=0$, $s_{mb}(\lambda)$ is derived as the solution of the designing problem.

All of metameric blacks can be represented in the universal framework using $g(\lambda)$. $\dfrac{\partial f}{\partial s_{mb}(\lambda)}$ is calculated as follows :

$$\begin{aligned} \frac{\partial f}{\partial s_{mb}(\lambda)} &= a_2+2a_3 s_{mb}(\lambda)+a_6 g(\lambda) \\ &= a\left\{s_{mb}(\lambda)-G(\lambda)\right\}\ , \end{aligned} \tag{3}$$

where

$a=2a_3\ (=1.)$, the scaling factor of the function f normalized,

$$G(\lambda)=-\frac{a_2}{2a_3}-\frac{a_6}{2a_3}g(\lambda) .$$

The Lagrange function for $f\left(s_{mb}(\lambda),\ g(\lambda)\right)$ under the constraint of $X = Y = Z = 0$ derives the following equation.

$$s_{mb}(\lambda) = \left\{k_X \bar{x}(\lambda) + k_Y \bar{y}(\lambda) + k_Z \bar{z}(\lambda)\right\} + G(\lambda)\ , \qquad (4)$$

where

$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$: color matching functions,

k_X , k_Y , k_Z : parameters included in the Lagrange function.

Although, Eq.(4) is the same form with R-Matrix framework, $G(\lambda)$ is permitted to take only positive values on all λ in R-Matrix framework, and $G(\lambda)$ is permitted to take both positive and negative values in our framework describing all of metameric blacks.

By applying Eq.(4) to the metameric black conditions of the X , Y and Z stimulus values, k_X , k_Y , k_Z can be calculated. Eq.(4) is the general formulation of metameric blacks whose design parameter is $G(\lambda)$ $\left(g(\lambda)\right)$. Figure 1 shows an example of $s_{mb}(\lambda)$ whose $G(\lambda)$ has both positive and negative values.

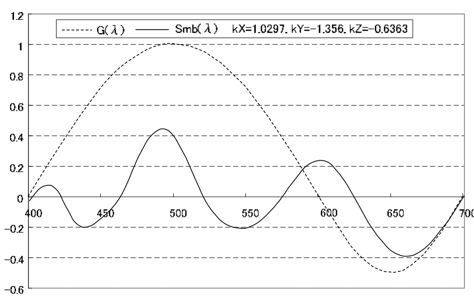

Figure 1 Example of $s_{mb}(\lambda)$ *and* $G(\lambda)$.

The wavelength of intersection locations are calculated using the following equation derived from Eq.(4) with $s_{mb}(\lambda) = 0$.

$$k_X \bar{x}(\lambda) + k_Y \bar{y}(\lambda) + k_Z \bar{z}(\lambda) + G(\lambda) = 0\ . \qquad (5)$$

Characterictics

Five characteristics related to the theoretical model can be provided and proven.[10)]

[Characteristic 1]

For given color matching functions, intersection locations are independent only on $G(\lambda)$ in other words $g(\lambda)$.

[Characteristic 2]

Intersection locations are invariant related to uniform scaling of $G(\lambda)$ by a scaling factor α $(\alpha \neq 0)$ for each wavelength λ .

[Characteristic 3]

$G(\lambda) = const$ $(\rightarrow 0\, and \neq 0)$ is the central of the all variables of $G(\lambda)$. Hereafter, $G(\lambda) = const$ $(\rightarrow 0\, and \neq 0)$ is represented as $G(\lambda) = const_0$.

[Characteristic 4]

The intersection locations of $G(\lambda) = const_0$ which represents all of $G(\lambda)$ counts four points in the wavelength range.

[Characteristic 5]

The four intersection locations of $G(\lambda) = const_0$ have unbiased characteristics.

Considering these Characteristics, the problem should be investigated starting from four intersection cases, though in the past experimental results, three intersection cases have been discussed. In the next numerical illustrations, four intersection cases are discussed.

Numerical Illustrations

In this chapter, the results of numerical calculations based on the results of previous chapters. To eliminate calculation interval effects, all calculations were performed at a $1nm$ wavelength increment. The CIE 1931 2^o and the CIE 1964 10^o color matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ were employed, and basically the wavelength range of $380nm$ to $780nm$ ($1nm$ step) was considered, also the wavelength range of $400nm$ to $700nm$ ($1nm$ step) was considered because there existed both calculations in past studies related to the metamer intersection problems. The intersection wavelengths were calculated using Eq.(5). The maximum value of spectral radiance distributions is normalized to 1.0 .

The unbiased four-intersection locations is derived on the set up of $G(\lambda) = const_0$ (central of all $G(\lambda)$). The results for CIE 1931 (calculated in $380nm$ to $780nm$) is shown in Fig.2. In Fig.2 the intersection locations are $430nm$, $469nm$, $537nm$ and $611nm$ with parameter values of $k_x = -0.549$, $k_Y = -0.914$ and $k_Z = -0.605$. These are the unbiased intersection locations from Characteristic 5. The unbiased intersection locations for CIE 1964 (calculated in $380nm$ to $780nm$) is also shown in Fig.2 whose intersection locations are $430nm$, $468nm$, $533nm$ and $608nm$ with parameter values of $k_x = -0.447$,

$k_Y = -0.953$ and $k_Z = -0.524$. The unbiased intersection locations for CIE 1931 (calculated in $400nm$ to $700nm$) is shown in Fig.3 whose intersection locations are $430nm$, $469nm$, $537nm$ and $610nm$ with parameter values of $k_x = -0.533$, $k_Y = -0.923$ and $k_Z = -0.608$. The unbiased intersection locations for CIE 1964 (calculated in $400nm$ to $700nm$) is also shown in Fig.3 whose intersection locations are $430nm$, $468nm$, $533nm$ and $607nm$ with parameter values of $k_x = -0.434$, $k_Y = -0.960$ and $k_Z = -0.525$.

These are the first results from theoretical point of view. Though four-intersections and three-intersections cannot be compared in the same ring, the following considerations are provided for a reference. In the results of Fig.2 (CIE1931), $611nm$ intersection corresponds to $612nm$, $537nm$ intersection corresponds to $537nm$ derived by Thornton.[2)] In the same way, other results in Fig.2 and Fig.3 can be corresponded to Thornton and others results. The mid point between the two intersection locations of short wavelength is expected to corresponds to the intersection of the shortest wavelength of existing experimental results of three-intersections.

The combination of this section and the previous section has completed the analysis and the discussion related to the aspect.

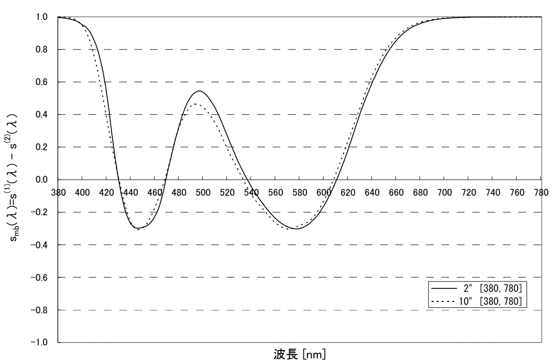

Figure 2 Intersection Locations

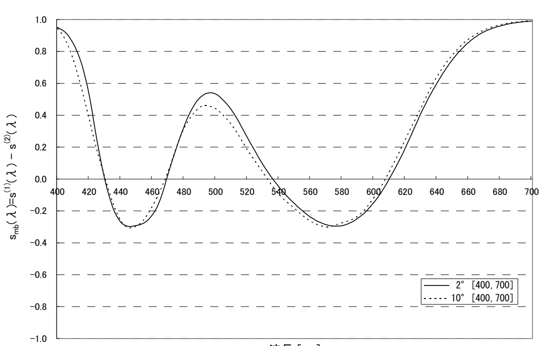

Figure 3 Intersection Locations

Conclusions

The argument that the locations of metamer intersections approximately coincide with particular locations in the wavelength range is a classic argument. Though there were researchers which derived historically important results, we investigated the aspect using a theoretical model.

In the first step, a general formulation of metameric black describing all of metameric blacks has been derived. In the second step, the intersection locations problem has been discussed using the result of the first step.

Here, one of the historical issues is settled regarding the intersection locations using theoretical analysis.

References

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Biography

Nobuhito Matsushiro received his PhD degree (Information engineering) from University of Electro. Communications, Tokyo, Japan, in 1996 and PhD degree (Color science) from Chiba University in 2006. He is a PhD (Neuro and Brain Science) candidate at School of Medicine, Chiba University, Chiba, Japan. He works for Oki Electric. Co. Ltd., Printing Company Oki Data.