Effcient Estimation of Critical Transfer Belt Parameters from an Electrical Characterization Fixture

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Abstract

Intermediate transfer belts represent an important technology component of the two-transfer systems commonly found in color laser printers. A number of mechanical and electrical properties contribute to the performance of intermediate transfer belts in these systems. Of these, resistivity and dielectric permittivity are two of the most critical electrical properties and are related to the bulk properties resistance and capacitance, respectively. Accurate measurement and specification of these electrical properties is important to the design of two-transfer systems.

Recently, an electrical characterization method for intermediate transfer belts was published [1], where the resistance and capacitance were determined from amplitude and phase measurements of an applied A.C. voltage. We report on implementation of and improvements to the previously published method. It was found that the previous implementation did not make efficient use of all measured data. We derive a more efficient and robust estimator for the resistance and capacitance of a transfer belt based on measurements of gain and phase. We also performed a gauge repeatability and reliability study of the fixture. Our approach represents a significant refinement of the previously published characterization approach for intermediate transfer belts.

Introduction

Intermediate transfer belts (ITBs) are important components of two-transfer systems typically used in color laser printers. There are several electrical, mechanical, and material parameters that are important to the performance of toner transfer. Since the fundamental physics of transfer relies upon the electric force exerted upon charged toner, the electrical properties resistance and capacitance are two of the more fundamental bulk properties of ITBs.

In this paper, we expand upon the electrical characterization procedure applied by Tsuneo Mizuno and Jun-ichi Hanna to characterize ITBs [1]. The Mizuno fixture is employed to study the conductivity and dielectric properties of both ITBs and paper media. It was found that, while the Mizuno fixture has proven effective, a more detailed analysis of the estimation of resistance and capacitance is needed. In this regard, we have developed a more efficient estimator of the critical ITB electrical parameters.

Experimental Setup

The experimental setup is very similar to that previously described [1]. A material under test is paced between circular metal electrodes with an active measurement diameter of 60 *mm*. The test fixture is pre-programmed to provide either a continuous wave alternating current (AC response) waveform or a step (DC response). An $R_0 = 1\ M\Omega$ resistor is placed in series with the source voltage. The equivalent circuit model of the fixture requires a probe resistance of $R_p \approx 100\ M\Omega$ and probe capacitance of $C_p \approx 3\ pF$ be placed in parallel with the material under test. The purpose of the test fixture is to estimate the belt resistance R_b and capacitance C_b.

A gauge Repeatability & Reproducibility study was conducted to determine the fixture's influence on measurement variation. The test method yielded 270 measurements taken from five different transfer belt materials over a three day period. Both measurements, AC and DC response, were considered in the study. Results show the fixture variation to be approximately 15 % for the AC measurement and approximately 12 % for the DC measurement. Using the commonly accepted threshold of 15 % max variation, the fixture demonstrates good measurement capability.

Estimation of R and C

A non-iterative, closed-form estimator has been developed for the determination of belt capacitance and resistance. There are two steps to this algebraic estimate. First, the gain (Γ) and phase (ϕ) must be determined from raw data. Second, the values of R and C are estimated from Γ and ϕ. Before describing the estimation procedure, a noise model is introduced for the optimization and evaluation of the estimator.

Noise Model

Thermal noise picked up by resistors is commonly modeled as white Gaussian noise [2]. This noise has a flat power spectral density (PSD) over a wide frequency range and is implemented numerically as a random variable with zero mean and nonzero variance σ_n. Independent noise signals $n_i(t)$ and $n_o(t)$ are added to the noiseless input voltage signal $v_i^0(t)$ and noiseless output voltage signal $v_o^0(t)$ to generate synthetic measurement data for the input and output voltages, respectively. The synthetic data is represented mathematically by

$$v_i(t) = v_i^0(t) + n_i(t) = V_i \cos(\omega t + \phi_i) + n_i(t) \quad (1a)$$

$$v_o(t) = v_o^0(t) + n_0(t) = V_o \cos(\omega t + \phi_o) + n_0(t), \quad (1b)$$

where $\omega \equiv 2\pi f$ is the angular frequency, ϕ_i and ϕ_0 are the input and output voltage phases, and V_i and V_o are the input and output voltage magnitudes. In general, V_i is known, but an independent estimate can be used for verification. The output voltage magnitude can be written in terms of the gain (Γ) as $V_o \equiv \Gamma V_i$. Estimation of Γ and the phase $\phi \equiv \phi_o - \phi_i$ is necessary and sufficient for subsequent extraction of the critical ITB electrical parameters.

Estimate of Gain and Phase

The gain and phase of the voltage waveforms are estimated by first writing the waveform of interest as [3]

$$v^0(t) = V \cos(\omega t + \phi) = a \cos(\omega t) + b \sin(\omega t) \quad (2)$$

where V and ϕ are generic variables used here for demonstration. The parameters a and b can be estimated using a least squares fit of the linear equation $\bar{y} = \bar{\bar{M}}\bar{x}$, where $\bar{x} = [ab]^T$ is the unknown parameter vector, $\bar{y} = [v(t_1) \ldots v(t_K)]^T$ is the K by 1 vector of voltage measurements at different times t_k, and

$$\bar{\bar{M}} = \begin{bmatrix} \cos(\omega t_1) & \sin(\omega t_1) \\ \vdots & \vdots \\ \cos(\omega t_K) & \sin(\omega t_K) \end{bmatrix} \tag{3}$$

is a K by 2 matrix, where K is the number of data samples. The system is solved using a QR factorization algorithm to yield the estimates for a and b. The amplitude and phase of a measured waveform is then determined via

$$V = \sqrt{a^2 + b^2} \tag{4a}$$
$$\phi = \tan^{-1}(b/a). \tag{4b}$$

To demonstrate the effectiveness of this method, the amplitude and phase estimate errors have been plotted versus signal-to-noise ratio (SNR) in Figure 1. The $SNR \equiv 10\log_{10}(V^2/\sigma_n)$ gives the ratio of noise powers [2]. The estimate error is given in the same units as the estimated parameter $\hat{x}$ as $RMSE(\hat{x}) = \sqrt{E(\hat{x}-x)^2}$, where $E(\hat{x}-x)^2$ is the expected value of the squared difference between the estimated value $\hat{x}$ and the actual value x of some parameter $x = V, \phi$.

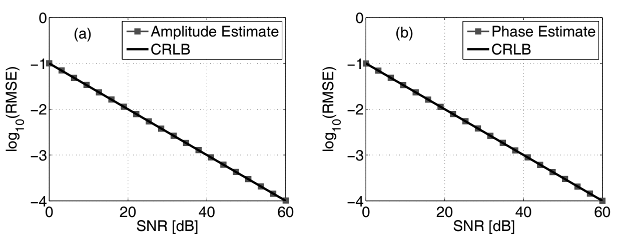

Figure 1. (a) Amplitude and (b) phase estimation error from synthetic data with nominal amplitude $V = 1$ and nominal phase $\phi = -\frac{\pi}{2}$. $K = 10^4$ samples were used in the Monte-Carlo simulation.

It is seen in Figure 1 that the estimates of amplitude and phase approach the Cramer-Rao Lower Bound (CRLB) over the SNR range of interest $SNR \leq 0\ dB$. The CRLB gives the minimum possible variance of an unbiased estimator [4]. It is calculated via

$$CRLB = \sigma_n^2 \left[\bar{\bar{J}}^T \bar{\bar{J}}\right]^{-1}, \tag{5}$$

where the Jacobian is the derivate of Equation (2)

$$\bar{\bar{J}} = \frac{\partial \bar{v}^0}{\partial \bar{x}} \begin{bmatrix} \frac{\partial \bar{v}_0}{\partial V} & \frac{\partial \bar{v}_0}{\partial \phi} \end{bmatrix}. \tag{6}$$

From this analysis, it is expected that the estimates of gain and phase are asymptomatically efficient. That is, they attain the CRLB in low noise.

Estimate of Resistance and Capacitance

In this section, the estimate of resistance and capacitance is given in terms of the gain and phase. Instead of estimating resistance, the conductance $G_b \equiv 1/R_b$ will be estimated for numerical stability and ease of discussion since G_b is more analogous to C_b than is R_b. Likewise, we can define the parameters $G_0 \equiv 1/R_0$ and $G_p \equiv 1/R_p$ as the source conductance and probe conductance, respectively. The algebraic estimator

$$\left[\Gamma_j\right] G = G_0\left(\cos\phi_j - \Gamma_j\right) \tag{7a}$$
$$\left[\omega_j \Gamma_j\right] C = G_0 \sin\phi_j \tag{7b}$$

is derived from the equivalent circuit model given in [1]. Here, the subscript $j \in \{1 \ldots J\}$ spans the number of independent measurements used for a particular estimate. Equation (7) gives estimates of the parallel circuit combinations $G \equiv G_b + G_p$ and $C \equiv C_b + C_p$ in terms of the previously estimated quantities Γ and ϕ. The values of G_b and C_b are deduced simply from the estimates of G and C, which are determined from Equation (7) by QR factorization. This allows for estimation of $G_b = G - G_p$ and/or $C_b = C - C_p$ in either the $J = 1$ critically determined case or the $J > 1$ overdetermined case by least squares solution of a linear system.

There are a few points to make about Equation (7). First, it is numerically robust since it requires only a linear system solution for the parameter estimation. Second, G and C are determined by separate equations. Third, the system is critically determined when only one measurement exists at a single frequency. This last point is an important distinction of our method since the previously published approach requires at least two measurements at different frequencies for a single estimate of G and/or C. This is because we use all of the available data for both gain and phase in our estimate.

In Figure 2 we compare the error induced upon the estimate of G_b and C_b using both the algebraic estimator presented here and the previously published Mizuno estimate [1] which we implemented verbatim. The noise model is implemented and carried through the estimation of gain and phase as described in the previous sections. The figure shows that the algebraic estimate provides a superior estimate of the critical electrical parameters. Furthermore, the estimate error can be highly biased at low SNR. Although not shown, the Mizuno approach can produce significantly higher bias at low SNR, which may severely degrade the subsequent extracted time decay information from DC analysis. Also, the Mizuno estimate is not numerically robust in very low SNR due to the possibility of having negative square roots in the parameter extraction (see for example Equation (9) in [1]).

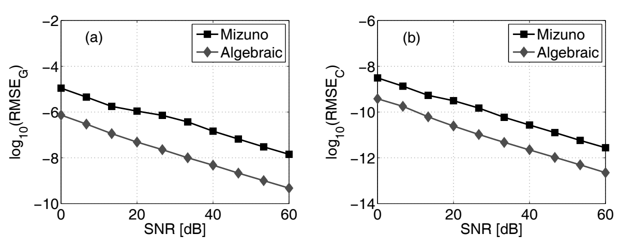

Figure 2. (a) Conductance and (b) capacitance estimation error from synthetic data. $K = 2000$ samples were used in the Monte-Carlo simulation. The actual values used in the generation of synthetic data are $R_b = 10\ M\Omega$ and $C_b = 500\ pF$.

Application to Non-Linear Media

From the previous section, it is known that the estimates of G_b and C_b are more efficient than the previously published method. One point to note from Figure 2, however, is that any desired error tolerance can be achieved using either method by simply increasing the SNR. This can, in principle, be done by increasing the input voltage amplitude V_i. This, of course, assumes that only the white Gaussian noise is present. However, it is well known that all materials exhibit nonlinear response to applied electric field [5]. This nonlinear response can and does introduce error in the estimation of gain and phase.

To label a material as being "linear" requires the specification of an operating range over which the approximation is made. Since no material conductivity remains linear in an applied electric field of arbitrarily large magnitude, we cannot simply increase the source voltage to our material under test to achieve arbitrarily high SNR. The reason is that while the SNR gets larger, the output signal is distorted by the nonlinearity of the material since the conductivity of the material changes as the applied AC voltage signal moves along the sinusoid. Near the zero crossings, the material responds in a linear way, but as the voltage approaches a large magnitude maxima or minima in the waveform, the nonlinear material response deforms the output voltage signal. Fourier transform of the resulting signal should then reveal that higher order harmonics have been excited. That is exactly what is observed in Figure 3. For an applied AC voltage of 50 V magnitude at 250 Hz, no significant generation of high order harmonics is observed. However, as the input voltage is increased to 250 V, 500 V, and 1000 V, the third harmonic at 750 Hz increases to about 30 dB below the primary signal power. Likewise, the SNR goes from approximately 50 dB to greater than 60 dB over the same range. However, the 1000 V input signal produces an output with an *effective* SNR of only about 30 dB due to the aforementioned amplification of the higher harmonics.

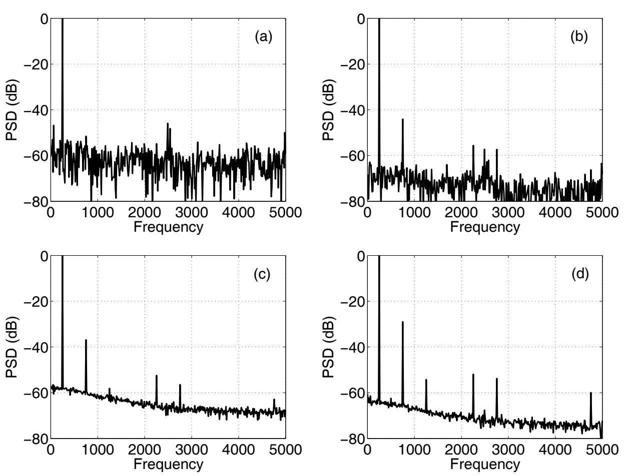

Figure 3. Power spectral density of the AC output voltage measurements for a PI belt. The amplitude of the input voltage is (a) $50\ V$, (b) $250\ V$, (c) $500\ V$, and (d) $1000\ V$. The PSD is computed via FFT in MATLAB ®normalized to the power at the measurement frequency $f = 250\ Hz$. Higher harmonics are generated by nonlinear resistance of the belt at high voltages.

Since it is difficult, if not impossible, to tailor an estimator to a nonlinear system response, we have chosen to optimize our estimator to a linear material response where the signal is corrupted primarily by thermal noise. In this regard, it is necessary to apply the estimation procedure at low SNR to avoid generation of higher order harmonics, hence the need for the more efficient estimator presented in the previous sections.

Application to Frequency Dispersive Media

In a previous publication, the response of capacitance and resistance as a function of voltage was demonstrated [1]. It was shown that while ITB resistance is a strong function of voltage, the capacitance is not. This was done by applying a step response to the belt electrical characterization fixture and measuring the voltage decay. Such a method is useful in mapping the resistance and capacitance versus voltage.

In this section, we ask the question: *Are belt resistance and capacitance temporally dispersive?* Perhaps a more important question to ask is: *Can we measure the frequency response of transfer media?* The answer to that question is *yes* since the estimator derived herein is critically determined for a single measurement. That is, we can extract both R and C at a single frequency and, subsequently, map both $R(f)$ and $C(f)$.

Figure 4 shows the measured response of three media, the PI belt used in the previous section, a sheet of International Paper's 24 # Hammermill ®Laser Print, and Mohawk Paper's 24 # Strathmore Writing ®Paper. These measurements were performed in a controlled humidity chamber at temperature 72° F and relative humidity 50 %. This data shows that both the resistance and capacitance of ITBs and paper media can be functions of frequency. Therefore, both resistivity and dielectric permittivity may be temporally dispersive over the frequency range of interest. The data also demonstrates the ability to extract resistance and capacitance from measurements at single frequencies.

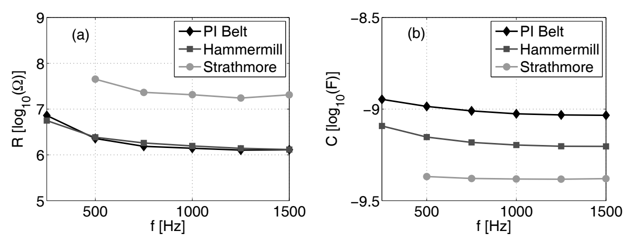

Figure 4. Estimates of (a) resistance and (b) capacitance from measured AC data. The materials under test are a PI transfer belt, Hammermill Laser paper, and Strathmore paper.

Conclusions

An asymptotically efficient estimator has been derived for the estimate of resistance and capacitance from an electrical characterization fixture. The estimator represents an improvement to the method originally published by Mizunno and Hanna for the electrical characterization of transfer belts [1]. Our estimate is needed for three reasons. First, it is numerically robust in that it doesn't suffer from the possibility of negative square roots that can occur in the previous method. Second, the estimates of resistance and capacitance exhibit lower error than the previous method. This is important when dealing with low SNR to avoid corruption of the output voltage signal do to nonlinear response of the material. Third, it allows for the determination of resistance and capacitance at single frequencies, which is necessary to determine the dispersive behavior of the material under test.

We conclude that the belt electrical characterization fixture and modified parameter estimation procedure can be used effectively to predict the critical electrical parameters of transfer belts and transfer media.

References

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Author Biography

Brandon Kemp received his BS in engineering from Arkansas State University (1997), his MSEE from the University of Missouri-Rolla (1998), and his PhD from the Massachusetts Institute of Technology (2007). His work has focused on the application of advanced analytic methods to a variety of technical areas including wave theory, laser printer technology, optical trapping and binding, signal integrity, and electromagnetic compatibility. He presently works as a research and development engineer in color laser development at Lexmark.

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