Model Predictive Control Methods for Toner Concentration Control System

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Abstract

In this paper, we propose a new control algorithm based on model predictive control that is targeted to control the TC in a two-component toner dispense system. MPC is extremely effective in controlling systems with nonlinear cost function, time delay, constraints on the actuator and states. We also present a methodology to make the control loop robust to sensor noise.

Introduction

In an electrophotographic printing process, an electrostatic image formed on the surface of a drum or a photoreceptor belt is developed by the application of finely divided toner particles to form an image. For good quality image reproduction, it is essential to closely monitor the development process to insure the color consistency in prints. For example it is very critical to make sure that the four color components (*C*, *M*, *Y*, and *K*) are combined at the correct toner density since any deviation will be visible in the final print, especially in half toned images. Developability is directly related to the rate at which development takes place. The development rate is a function of the toner concentration (TC) in the developer housing. The developer used inside the development system is a mixture of carrier particles and toner particles. In order to produce good quality prints it is necessary to maintain the TC of this mixture to within limits. TC is defined as the ratio of the number of toner particles to the number of carrier particles in the developer mixture. TC is an important parameter that can change during production run (unless efficiently controlled) and can contribute to color drift over time. Overall color stability of the closed loop TC control system depends on how well the system components are optimized.

To improve overall TC performance, at least three major system components must be optimized: (1) open loop toner replenishment (dispense) and mixing system (i.e., process plant itself), (2) a TC measurement system (i.e., TC sensor), and (3) a closed loop control algorithm to maintain the TC level close to a predefined target. Optimizations of the toner replenishment system and measurement system have been described in papers and patents sited in Chapter9 of the book [1]. There is still an opportunity to further improve the TC tracking performance by optimizing the controllers which is the subject of this paper.

In this paper, we propose a new Model Predictive Control (MPC) based feedback control algorithm that is targeted to control the TC in a two-component toner dispense system. MPC method is selected in order to minimize a nonlinear optimization function and yet reach tracking in the presence of additional constraints such as time-delay in the toner dispense, dispense rate limits etc. Linear controllers described in Reference 1 are not suitable to optimize under these constraints. We use the state space representation of the TC control system of Reference 1 to further build on a more complex control algorithm. As a result, conventional Smith Predictor, anti-windup compensators are not required.

Toner Concentration Control

The proper development of a latent electrostatic image on a photoreceptor by the toner particles is directly tied to the correct TC in the developer. Excess toner concentration will result in too much background in the developed image. This means that a white background may appear as a colored background. On the other hand, if the toner concentration is much less than the desired target value, it will result in a lack of toner coverage in the image. Therefore, to insure high quality printing, TC must be continuously monitored and adjusted. A TC control loop normally contains a feed forward and a feedback component, in which the TC level is sensed by a sensor placed in the development housing and control adjustments are made to the toner dispense motor using TC sensor and the image pixels data.

Open Loop TC System Model

Toner concentration is the ratio of toner mass to the carrier mass. The toner mass (tm) at time k (TC cycle) is modeled by the following difference equation

$$tm(k+1) = tm(k) + masdis(k-\mu) - masdev(k) \qquad (1)$$

where $tm(k) =$ mass of the toner at print cycle k in grams. $masdis(k) =$ mass of toner dispensed from the dispenser at print cycle k in grams. $masdev(k) =$ mass of toner which is used in the image based on the consumption profile at print cycle k in grams. $\mu =$ dispense delay (in number of cycles). It includes the delay in the mixing system. $k =$ print number. The mass dispensed is calculated by using the duty cycle obtained by the feedback controller. The TC is defined as the ratio of the toner mass to the carrier mass. Rewriting Equation (1) in terms of TC results in

$$tc(k+1) = tc(k) + g[u(k-\mu) - v(k)] \qquad (2)$$

where $u(k)$ is the dispensed mass and $v(k)$ is the developed mass and g is a scale factor defined as: $g = (carriermass)^{-1}$. The block diagram of the system in Equation (2) is shown in Figure1.

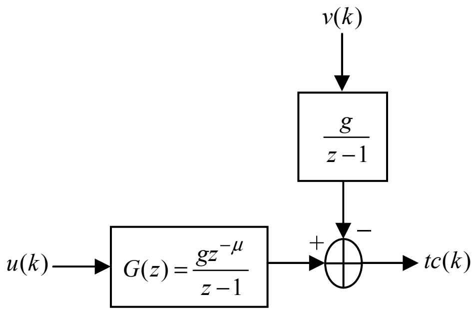

Figure 1: Block diagram of the open loop TC model

To control the TC system shown in Figure 1, different approaches have been taken in the past. For example, a TC control methodology proposed by Y. R. Wang and L.K. Mestha [2] involves making TC measurements on a regular basis and comparing them with the TC set points to generate the error signal. The error signal is then passed through a proportional integral controller with an anti-windup compensator and Smith Predictor. The drawback of this approach is in its sensitivity to noise and the need for a Smith Predictor. A modified version of this algorithm incorporating a Kalman filter for noise reduction is given in [2]. In this paper, we propose a method based on model predictive control. The proposed approach is robust to noise, system disturbance, , input constraints and eliminates the need for a Smith Predictor and anti-windup compensators. The open loop system described by Equation 2 has a delay of μ cycles. This delay is split into μ states. Derivations of the state equations are shown in Reference [1]

General Model Predictive Control

Model predictive control (MPC) [3] is the most useful control methodology for the control of systems with a constrained dynamic. In model predictive control, the control action at time k is determined by solving a finite horizon open loop-optimal control problem. The first sample of the control action is applied and the process is repeated for the next time instant $k+1$. The block diagram of a MPC is shown in Figure 2.

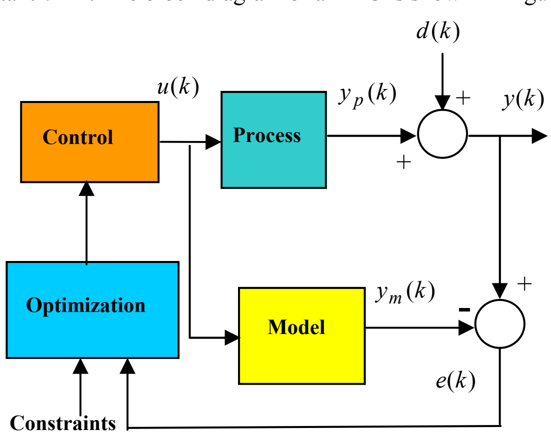

Figure 2: Model Predictive Control

In an MPC, the set points are computed based on a constrained optimization of a steady-state process model. Then a model-based predictive algorithm ensures that the process outputs track the desired set points over a prescribed horizon. MPC uses the process model in two ways:

1. It uses the model to predict the effect of the past control on P future output samples.
2. It uses the model to compute the optimal M input control samples.

Once the controller is obtained, the first sample of the control signal is realized and the process is repeated for the next time instant. This is shown in Figure 4. Model predictive control algorithms can be defined in terms of the system time response such as step response or state equations.

Model Predictive Controller – A MIMO Design

The results obtained for SISO-MPC [3] can be extended to the MIMO systems. Consider a MIMO system with N inputs and N outputs as shown in Figure (3).

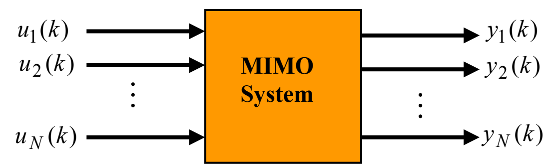

Figure 3: A MIMO system

Let the system be described by the following step response equations,

$$\begin{aligned} \hat{y}_{1k} &= A_{11}\Delta u_{1k} + A_{12}\Delta u_{2k} + \cdots + A_{1N}\Delta u_{Nk} + y^P_{1k} + d_{1k} \\ &\vdots \\ \hat{y}_{Nk} &= A_{N1}\Delta u_{1k} + A_{N2}\Delta u_{2k} + \cdots + A_{NN}\Delta u_{Nk} + y^P_{2N} + d_{Nk} \end{aligned} \tag{3}$$

The above equations can be written in a more compact form as

$$\hat{y}_k = A\Delta u_k + y^P_k + d_k \tag{4}$$

where $\hat{y}_k$, y^P_k, d_k are $PN \times 1$ column vectors, Δu_k is $MN \times 1$ and A is an $NP \times NM$ matrix defined as:

$$\hat{y}_k = \begin{bmatrix} \hat{y}_{1k} \\ \hat{y}_{2k} \\ \vdots \\ \hat{y}_{Nk} \end{bmatrix}, \quad y^P_k = \begin{bmatrix} y^P_{1k} \\ y^P_{2k} \\ \vdots \\ y^P_{Nk} \end{bmatrix}, \quad d_k = \begin{bmatrix} d_{1k} \\ d_{2k} \\ \vdots \\ d_{Nk} \end{bmatrix},$$

$$\Delta u_k = \begin{bmatrix} \Delta u_{1k} \\ \Delta u_{2k} \\ \\ \Delta u_{Nk} \end{bmatrix} \text{ and } A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \tag{5}$$

$\hat{y}_{jk}$ is the step response of the j^{th} output at time k, y^P_{jk} is the j^{th} future output at time k, d_{jk} is the disturbance at the j^{th} output at time k, and $\Delta u_{jk} = u_j(k) - u_j(k-1)$. Each sub-matrix A_{ij} has the following structure.

$$A_{ij} = \begin{bmatrix} a_{ij}(1) & 0 & 0 & \cdots & 0 \\ a_{ij}(2) & a_{ij}(1) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a_{ij}(M) & a_{ij}(M-1) & a_{ij}(M-2) & \cdots & a_{ij}(1) \\ \vdots & \vdots & \vdots & & \vdots \\ a_{ij}(P) & a_{ij}(P-1) & a_{ij}(P-2) & \cdots & a_{ij}(P-M+1) \end{bmatrix} \tag{6}$$

Each vector Δu_{ik} has the following structure.

$$\Delta u_{ik} = \begin{bmatrix} \Delta u_i(k) \\ \Delta u_i(k+1) \\ \vdots \\ \Delta u_i(k+M-1) \end{bmatrix} \tag{7}$$

The goal is to make sure that $\hat{y}_k$ tracks the desired trajectory or y^{target}. Therefore, we minimize the norm of prediction error with respect to the future control moves. That is,

$$\min_{\Delta u(k)} J = \|e(k)\| = \left\| A\Delta u_k + y_k^P + d_k - y^{target} \right\| \tag{8}$$

Subject to the following constraints

$$|\Delta u(k)| \le M_1 \quad \text{and} \quad u_{\min} \le u(k) \le u_{\max} \tag{9}$$

There is no closed form solution to this optimization problem. Numerical optimization algorithms that can handle constraints can be used. Let the solution to this optimization problem be $\Delta^* u_k$, then the control law Δu_k is computed as

$$\Delta u_k = [b \quad b \quad \cdots \quad b]\, \Delta^* u_k = [b \quad b \quad \cdots \quad b] \begin{bmatrix} \Delta u_{1k} \\ \Delta u_{2k} \\ \vdots \\ \Delta u_{Nk} \end{bmatrix} \tag{10}$$

where b is a $1 \times M$ row vector given by: $b = [1 \;\; 0 \cdots 0]$

State Space MIMO Model Predictive Control.

Consider an open loop control system given by the state space equations

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + \tilde{B}d(k) + w(k) \\ z(k) &= Cx(k) \\ y(k) &= z(k) + v(k) \end{aligned} \tag{11}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the input vector, $d(k) \in R^m$ is the disturbance vector, $w(k) \in R^n$ is the zero mean white process noise vector, $y(k) \in R^r$ is the measurement vector, $z(k) \in R^r$ is the noise free output vector, $v(k) \in R^r$ is the zero mean white measurement noise vector, $A \in R^{n \times n}$, $B, \tilde{B} \in R^{n \times m}$, and $C \in R^{r \times n}$. It is assumed that the system given in (3) is controllable. Define the difference state vector $\Delta x(k) = x(k) - x(k-1)$ and the output vector $z(k)$ as new states, then

$$\Delta x(k+1) = x(k+1) - x(k) = A\Delta x(k) + B\Delta u(k) + \tilde{B}\Delta d(k) + \Delta w(k)$$

and $z(k+1) = CA\Delta x(k) + CB\Delta u(k) + C\tilde{B}\Delta d(k) + C\Delta w(k) + z(k)$

The above equations in an augmented vector form are:

$$\begin{bmatrix} \Delta x(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix} \begin{bmatrix} \Delta x(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ CB_1 \end{bmatrix} \Delta u(k) + \begin{bmatrix} B_2 \\ CB_2 \end{bmatrix} \Delta d(k) + \begin{bmatrix} I \\ C \end{bmatrix} \Delta w(k)$$

$$z(k) = [0 \quad I] \begin{bmatrix} \Delta x(k) \\ z(k) \end{bmatrix}$$

$$y(k) = z(k) + v(k)$$

which can be written in a more compact form as

$$\begin{aligned} \bar{x}(k+1) &= \bar{A}\bar{x}(k) + \bar{B}_1 \Delta u(k) + \bar{B}_2 \Delta d(k) + \bar{B}_3 \Delta w(k) \\ z(k) &= \bar{C}\bar{x}(k) \\ y(k) &= z(k) + v(k) \end{aligned} \tag{12}$$

where

$$\bar{x}(k) = \begin{bmatrix} \Delta x(k) \\ z(k) \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A & 0 \\ CA & I \end{bmatrix}, \quad \bar{B}_1 = \begin{bmatrix} B_1 \\ CB_1 \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} B_2 \\ CB_2 \end{bmatrix},$$

$\bar{C} = [0 \quad I]$, and $\bar{B}_3 = \begin{bmatrix} I \\ C \end{bmatrix}$

The state vector $\bar{x}(k)$ is estimated using a state observer given by

$$\begin{aligned} \hat{x}(k+1 \mid k) = {} & \bar{A}\hat{x}(k \mid k-1) + \bar{B}_1 \Delta u(k) + \bar{B}_2 \Delta d(k) + \\ & K(y(k) - \bar{C}\hat{x}(k \mid k-1)) \end{aligned} \tag{13}$$

The estimator (or observer) of equation (13) provides the one-step prediction of the extended state vector $\bar{x}(k)$. For a simple state observer design reader is referred to Reference 1. The gain K for the observer equation 13 is obtained using Kalman filtering algorithm or observer design algorithm. Define

$$\Delta U(k) = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+M-1) \end{bmatrix} \quad \text{and} \quad Y(k) = \begin{bmatrix} \hat{y}(k) \\ \hat{y}(k+1) \\ \vdots \\ \hat{y}(k+P-1) \end{bmatrix} \tag{14}$$

Where M and P are the control horizon and prediction horizon. The predicted output over the prediction horizon is given by:

$$Y(k+1 \mid k)=\begin{bmatrix} \overline{C}\ \overline{A} \\ \overline{C}\ \overline{A}^2 \\ \vdots \\ \overline{C}\ \overline{A}^P \end{bmatrix}\hat{x}(k \mid k-1)+$$

$$\begin{bmatrix} \overline{C}\ \overline{B}_1 & 0 & \cdots & 0 \\ \overline{C}\ \overline{A}\ \overline{B}_1 & \overline{C}\ \overline{B}_1 & & 0 \\ \vdots & \vdots & & 0 \\ \overline{C}\ \overline{A}^{P-1}\overline{B}_1 & \overline{CA}^{P-2}\ \overline{B}_1 & \cdots & \overline{C}\ \overline{A}^{P-M}\overline{B}_1 \end{bmatrix}\Delta U(k) \qquad (15)$$

$$+\begin{bmatrix} \overline{C}\ \overline{B}_2 \\ \overline{C}\ \overline{A}\overline{B}_2 \\ \vdots \\ \overline{C}\ \overline{A}^{P-1}\ \overline{B}_2 \end{bmatrix}\Delta d(k)+\begin{bmatrix} \overline{C}\ K \\ \overline{C}\ \overline{A}K \\ \vdots \\ \overline{C}\ \overline{A}^{P-1}K \end{bmatrix}e(k)$$

where $e(k)=y(k)-\overline{C}\hat{x}(k \mid k-1)$

Equation (15) can be written in a more compact form as

$$Y(k+1 \mid k)=S_x\hat{x}(k \mid k-1)+S_u\Delta U(k)+S_d\Delta d(k)+S_e e(k) \quad (16)$$

The error over the prediction horizon is the difference between the prediction and the future set points

$$E(k+1)=Y(k+1 \mid k)-y^{target}(k) \qquad (17)$$

The cost function to be optimized is

$$J=E^T(k+1)WE(k+1)+\Delta U^T(k)R\Delta U(k) \qquad (18)$$

where W and R are the positive definite weight matrices. The cost function in (18) is optimized subject to the constraints on the input, input increment and the output. The constraints are given as:

$$\left|\Delta u(k)\right| \le M_1 \ \text{ and } \ u_{\min} \le u(k) \le u_{\max} \qquad (19)$$

The MPC framework leads to minimization of cost function in (18) under the constraints of equation (19). The above optimization was carried out using MATLAB optimization toolbox [4]. The function file **quadprog** was used for the constrained optimization.

Simulation Results

A comparison of the state feedback control and MIMO-MPC is shown in Figure 3. The initial TC is 3 and the final TC is set to be four. The response of state feedback (SF) and MIMO-MPC are compared with ideal response by computing figure of merit F. It is defined as area under the TC response normalized with respect to the area under the ideal TC response. It is given by:

$$F=100\frac{\sum_{0}^{K_{\max}} tc(k)}{K_{\max}t_c^{target}} \qquad (20)$$

The comparison of figures of merit F for the two methods for different noise levels are shown in Table 1.

Percentage of noise a	Figure of merit (F)	
	State Feedback	MIMO-MPC
0	96.90	97.22
1	95.47	96.83
3	94.42	95.97
5	93.79	94.41

Table 1. Comparison of figures of merit for SF and MIMO-MPC

Conclusion

In this paper, we proposed a state estimator/state feedback solution to TC control. Simulation results have shown that the new methodology works better than the conventional PI controller with or without a Smith Predictor.

References

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Biography

Sohail A. Dianat *received the B.S. degree from Arya-Mehr University, Tehran, Iran in 1973, and the M.S. and D.Sc. degrees from the George Washington University in 1976 and 1981, respectively, all in electrical engineering. Professor Dianat has been with the EE Department at RIT since 1981. He is the author of numerous publications in the areas of signal/image processing. He is a fellow member of SPIE and a senior member of IEEE.*

Lalit. K. Mestha*, a Principal Scientist at Xerox, received his PhD from the University of Bath, England in 1985 and his BE in 1982, from the University of Mysore, India, all in EE. He has led & worked on sensing and control of several large scale engineering systems since 1987. He holds 68 US Patents and has a total of 198 publications including journal articles, conference papers, patents & patent filings. Prior to joining Xerox, Mestha was at the SSC Laboratory in Dallas. He is a Senior Member of IEEE and teaches at RIT as an Adjunct Professor in his spare time.*

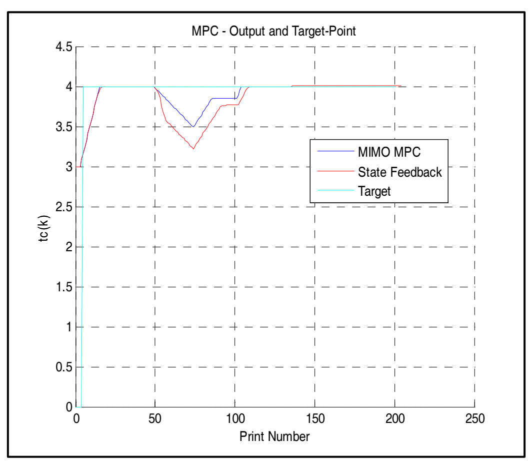

Figure 4. TC control loop response