# **Evaluation the Printing Fidelity of Glossy Ink-jet Paper Based on Measure Theory**

Xiao-Nao Liu, ; Zhejiang University; Hangzhou, China

Zhong Liu, Yong-Hua Zang, Yin-Li Liu; Tianjin University of Science and Technology, Tianjin Key Laboratory of Pulp & Paper; Tianjin, China

Jie Fan; Zhejiang University; Hangzhou, China

#### **Abstract**

Printing fidelity of ink-jet paper is crucial to obtain high quality printing. A new parameter  $S_{PMD}$  is proposed to evaluate the printing fidelity of glossy ink-jet paper.  $S_{PMD}$  consists of four quality factors, which are obtained by PMD (pixel microdifference) model calculated from mass distribution and measure theory. In the model, the extent of spreading and penetration, and color accuracy of printed pixel dot could be quantitatively determined. The results showed that the parameter,  $S_{PMD}$ , was reliable and consistent with human visual perception. It is appropriate for the printing fidelity evaluation of glossy ink-jet paper on professional ink-jet printers.

# **Background**

Ink-jet printing is the fastest growing area in the printing industry, especially for home and office markets<sup>[1]</sup>. This tendency is accompanied by a growing demand for super-grade ink-jet paper. Ink-jet paper must hold the ink near the surface to prevent bleeding-through and control the vertical penetration to minimize wicking and feathering<sup>[2]</sup>. Printing fidelity of ink-jet paper is crucial to obtain high printing quality. Less wicking or bleeding-through leads to good fidelity.

In order to evaluate of the printing quality, image analysis has been widely used. Kowalczyk and co-worker<sup>[2]</sup> used an image analysis system for the evaluation of ink-jet print quality, which consisted of a microscope, charged coupled device camera, video capture board and image analysis software. The test evaluated the intensity of the strike-through and recorded optical densities of the color. Printed quality could also be analyzed by ImageXpert[3] software, including printed dot area, mean gray scale, dot axis ratio and dot roundness. Fleming et al. [4] introduced the concept that the size of an ideal dot covering smallest area should be circular dot. They defined a nominal dot area and an "ideal" dot area. The dot fidelity of ink-jet printing could be measured by their nearness to circularity. However, traditional methods only reflected the geometrical shape of printed dot. The extents of spreading and penetration, and color accuracy of pixel dot have not been considered in their methods.

The reproducing pixel dots with desired shape and size are crucial to maintaining image uniformity and sharpness. Since quality of a printed product is, in the end, always judged by an

observer, the consumer of the printed product, the goal of this research is to find a new method which simulates human visual perception to measure printing fidelity of glossy ink-jet paper. A new parameter  $S_{PMD}$ , which based on measure theory<sup>[5]</sup> and triangular fuzzy numbers<sup>[6]</sup>, is proposed to evaluate the printing fidelity of glossy ink-jet paper..

# **Experimental**

## Samples

Three commercial available glossy ink-jet papers from different manufacturers were used for ink-jet printing, which were denoted as 1#, 2# and 3#, respectively (Table I). According to ISO standards, the paper properties were tested under constant temperature and humidity.

Table I Paper properties of the samples

	Coating	Gloss	Roughness	Absorption
	type	(%)	(µm)	(°)
1#	Micro-porous	72.2	0.47	34
2#	Micro-porous	70	0.37	31.8
3#	Resinous	74.4	1.37	45.4

#### **Printing**

The printing pattern consisted of a array of 36×36 dots on a 180×180 grid of pixels<sup>[4]</sup>, which was created with Adobe Photoshop™ in CMYK mode and printed for each of the four process colors (cyan, magenta, yellow and black) on Epson Stylus Photo 2100<sup>[7]</sup> ink-jet printer at a resolution of 720dpi. In order to "force" the printers to print four pure colors, the printing trials included calibration of the printer for each paper. ICC (International Color Consortium) profiles were generated for all combinations of papers and printer by professional proofing software—EFI ColorProof™ XF3.1<sup>[8]</sup>.

## Image capture system

Printed pixel dot images were captured after a drying time by HIROX 3D video microscope system HIROX KH-7700<sup>[9]</sup>. This system consists of digital camera, light source, integrated PC with LCD monitor and the software. For the pixel dot measurement, the magnification was 700.

#### Calculation

#### PMD model

A measure is a way of ascribing a numerical 'size' to Borel subsets on  $\mathbb{R}^n$ , such that if a set is decomposed into a finite or countable number of pieces in a reasonable way, then the size of the whole is the sum of the sizes of the pieces. For each Borel set on  $\mathbb{R}^n$ , we call  $\mu$  a measure if  $\mu$  assigns a non-negative number, possibly  $\infty$ , to each subset of  $\mathbb{R}^n$  such that<sup>[5]</sup>:

$$(a) \mu(\phi) = 0 ; (1)$$

(b) 
$$\mu(A) \le \mu(B)$$
 if  $A \subset B$ ; (2)

(c) if A<sub>1</sub>, A<sub>2</sub>,... is a countable (or finite) sequence of sets then

$$\mu\left(\bigcup_{i=1}^{\infty} A_i\right) \le \sum_{i=1}^{\infty} \mu(A_i) \tag{3}$$

with equality in (3), i.e.

$$\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i) \tag{4}$$

if the  $A_i$  are disjoint Borel sets. We call  $\mu(A)$  the measure of the set A, and think of  $\mu(A)$  as the size of A measured in some way.

A measure on a bounded Borel subset of  $R^n$  for which  $0 < \mu(R^n) < \infty$  will be called a mass distribution, and we treat  $\mu(A)$  as the mass of the set A. For example, let a be a point in  $R^n$  and define  $\mu(A)$  to be 1 if A contains a, and 0 otherwise. Then  $\mu$  is a mass distribution, thought of as a point mass concentrated at a. For a function f(x) on  $R^n$  (n=2), if  $\int_A |f| d\sigma$  is exist, then  $R^n$ 

$$\mu_f(A) = \int_A |f| d\sigma \tag{5}$$

where A is a bounded subset of  $R^2$ . Thus, we regard  $\mu_f$  as a mass distribution, in a sense, f(x) is 'density'.

Usually a norm is denoted by  $\|\cdot\|^{[10]}$ . A normed space is a pair  $(\chi, \|\cdot\|)$ , where  $\chi$  is a vector space and  $\|\cdot\|$  is a norm on  $\chi$ . A metric on  $\chi$  is defined by  $d(x, y) = \|x - y\|$ . A Banach space is a normed space that is complete with respect to the metric defined by the norm.

In this study, for a continuous function f of boundary or subsection on  $\mathbb{R}^2$ , we define

$$\|\mu_f\| = \sup_{A \subset \mathbb{R}^2} \mu_f(A) \tag{6}$$

We define two continuous functions f, g on  $\mathbb{R}^2$ . Then, the distance between function f and function g is d(f,g), which can be characterized with mass distribution difference. The formula is as follow:

$$d(f,g) = \|\mu_{f-g}\|.$$

$$(R^2, d(f,g)) \text{ is a Banach space.}$$

$$(7)$$

For a printed drop, it initially spreads with the region beneath the porous sublayer becoming immediately saturated. At longer time, the region of saturation extends beyond the edge of the drop, and eventually suction overpowers spreading and causes the drop to retract<sup>[11]</sup>. This behavior is shown in Figure 1.



Figure 1. Spreading and penetration behavior of a drop. Drop initially spread, but eventually retract under the influence of suction.

Thus, we regard a printed pixel dot image as a function f(x) and define f(x) to be gray value if printed ink on paper surface, and 0 otherwise.

$$f(x) = \begin{cases} kB_L & inking\\ 0 & blank \end{cases}$$
 (8)

where  $B_L$  is brightness level of RGB,  $B_L \in [0,255]$ ; k is the scale factor to make f(x) close to 1. Because the maximum of  $B_L$  is 255, thus, we choose k = 1/255.

For ink-jet printers, the theoretical shape of the smallest dot is circular<sup>[4]</sup>. Thus, we set the ink absorbed on paper surface as  $\mu(A)$ , then a corresponding function of the ideal pixel image is g(x). We define g(x) to be 1 if  $x \in D$ , and 0 otherwise.

$$g(x) = \begin{cases} 1 & x \in D \\ 0 & x \notin D \end{cases} \tag{9}$$

where D is a circle centered on (0,0), which satisfied

$$\mu_f(R^2) = \mu_g(R^2). \tag{10}$$

For the functions f(x) and g(x), corresponding to printed pixel image and ideal pixel image, f and g are the bounded subsection continuous functions on  $R^2$ . Then we propose a new metric PMD to denote the difference between a printed pixel dot image and a reconstructed ideal pixel dot image for evaluating the printing fidelity of a pixel dot image,

$$PMD = d(f, g) = \|\mu_{f-g}\|.$$
 (11)

The computational flowchart of PMD model is illustrated in Figure 2.

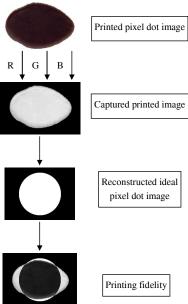


Figure 2. Flowchart of calculation when using PMD model as image printing fidelity metric

# Calculation of $S_{PMD}$

Since much of human reasoning is based on imprecise, vague and subjective values, most of decision-making processing, in reality, requires handling and evaluation of fuzzy numbers. Zadeh's<sup>[6]</sup> fuzzy logic has given analysts a tool to represent the

human behavior more precisely, especially where relatively few data exist, and where the expert knowledge about the system is vague and linguistic<sup>[12]</sup>. An easy way to define a fuzzy number is to determine its lowest value, its highest value, and the most likely value. We use complementary triangular fuzzy numbers<sup>[13-15]</sup> for the numerical evaluation of quality level of fidelity by introducing a set of factors for a color specialist. The degree of color pixel fidelity can vary due to numerous factors. The printing standard colors are black, cyan, magenta and yellow. Thus, we choose four major factors affecting a color specialist's decision that are Black pixel, Cyan pixel, Magenta pixel and Yellow pixel.

The fuzzy set  $a=(a_l,a_m,a_u)$ , where  $a_l \leq a_m \leq a_u$  and  $a_l,a_m,a_u \in R$ , is called a triangular fuzzy number, if the membership function of a is given by

$$\mu_{a} = \begin{cases} \frac{x - a_{l}}{a_{m} - a_{l}} & a_{l} \leq x \leq a_{m} \\ \frac{x - a_{u}}{a_{m} - a_{u}} & a_{m} \leq x \leq a_{u} \\ 0 & otherwise \end{cases}$$

$$(12)$$

where  $a_l$  and  $a_u$  represent the lower bound and upper bound respectively,  $a_m$  the most likely value, which represent the fuzzy degree. When  $a_l = a_m = a_u$ , then a is a real number.

Criteria standardization is normally done on 0 to 1 scale, or 0-10 or 0-100 etc. [15]. These criteria at the lowest level with different suitability classes is standardized using maximum Eigen vectors on 0 to 1 scale. The fuzzy number 0.5 represents equal importance and 0.9 expresses extreme importance on linguistic variables. The fuzzy number means very strong, essential, and moderately important which correspond to 0.8, 0.7 and 0.6 respectively.

The data collection forms for the 15 color experts (eight female and seven male) were analyzed using triangular fuzzy numbers. The printing fidelity value could be expressed by PMD<sub>i</sub> (i=K, C, M, Y). According to the triangular fuzzy number complementary judgment matrix<sup>[16, 17]</sup>, weight vectors of quality factors were obtained, as shown in Table II.

Table II Weight vectors of quality factors

Quality factors	Expression	Weight vector	
Black pixel	$PMD_{K}$	0.2701	
Cyan pixel	$PMD_C$	0.2421	
Magenta pixel	$PMD_{M}$	0.2414	
Yellow pixel	$PMD_Y$	0.2464	

Thus, a new parameter  $S_{PMD}$  for evaluating printing fidelity of glossy ink-jet paper is defined based on weight vectors. The formula is as follow:

$$S_{PMD} = w_1 \cdot PMD_k + w_2 \cdot PMD_C + w_3 \cdot PMD_M + w_4 \cdot PMD_Y$$
  
= 0.2701 \cdot PMD\_K + 0.2421 \cdot PMD\_C + 0.2414 \cdot PMD\_M + 0.2464 \cdot PMD\_Y

The  $S_{PMD}$  value can be used as an overall indicator of printing fidelity of glossy ink-jet paper.

#### Results and discussion

#### Results for PMD value

The printing fidelity values of printed pixel dots were summarized in Table III. The closer the PMD value is to 0, the closer the printed pixel dot is to ideal that showing less XYspreading, evenness topography and exact printing color. For all the printed pixel dots, the results for PMD<sub>Y</sub> and PMD<sub>M</sub> were larger than PMD<sub>K</sub> and PMD<sub>C</sub>. Black and cyan pixel dots were smaller, rounder and with less color variation than other color pixel dots. The PMD values of colored pixel dots of 2# paper were better than other samples. Figure 3 is the images of standard four color pixel dots. From Figure 4 to Figure 6, it could be seen that the more roundness and evenness colored pixel dots by far were obtained from 2# paper. The pixel dots of sample 2# were close to the standard pixel dots. The black pixel dot was rendered greater accuracy by 1# paper. It was coincident with visual perception as shown in Figure 7. The  $S_{PMD}$  values of 1# and 2# paper was similar that showing high printing fidelity. The 1# and 2# sample used in this experiment was micro-porous coating type, while 3# sample was resinous coating type. The micro-porous paper has small and inert pigment particles that create numerous minute cavities in which the ink is deposited so the print pixel dots could be handled immediately with less ink spreading. Therefore, it has a high printing fidelity than resinous coating type under the printing conditions used in this work.

Table III PMD values of printed pixel dots

	$PMD_K$	PMD <sub>C</sub>	$PMD_M$	$PMD_Y$	$S_{PMD}$
1#	0.35	0.39	0.44	0.5	0.419
2#	0.43	0.36	0.44	0.47	0.425
3#	0.44	0.41	0.43	0.49	0.443



Figure 3. Standard four color pixel dots images



Figure 4. Yellow color pixel dot images of paper samples

Figure 5. Cyan color pixel dot images of paper samples



Figure 6. Magenta color pixel dot images of paper samples

Figure 7. Black color pixel dot images of paper samples

## Results for S<sub>PMD</sub> values and Roundness values

This can be compared with the fidelity values estimated using roundness equation  $^{[4, 18]}$ :

$$Roundness = p^2 / 4\pi A \tag{14}$$

where p is the perimeter of printed color pixel dot and A is the area of printed color pixel dot. The closer the roundness is to the unity, the better the printing fidelity.

Table IV listed the printing fidelity data obtained experimentally and using roundness. For all the samples, the pixel dot fidelity estimated by PMD model was closer to visual perception. The relative deviation of S<sub>PMD</sub> value was smaller than that of roundness value. It implied that PMD value has less variation than roundness in fidelity measurement. Roundness value reflected the geometrical shape of pixel dot. However, not only the geometrical shape but also the extent of spreading and penetration, and printing color evenness of pixel dot could be calculated quantitatively by PMD model.

Table IV Printing fidelity data of printed pixel dots

	$S_{PMD}$	Relative Deviation	Roundness	Relative Deviation
1#	0.419	0.078	1.581	0.119
2#	0.425	0.048	1.722	0.131
3#	0.443	0.097	1.551	0.113

#### Conclusion

This research introduced a new parameter  $S_{PMD}$ , based on the measure theory and triangular fuzzy number, to measure the X-Y spreading, Z-penetration and color rendering accuracy of printed pixel dot for evaluating the printing fidelity of glossy ink-jet paper. The calculated results by PMD model were stable, precise and consistent with visual perception. The parameter  $S_{PMD}$  is appropriate for evaluating printing fidelity of glossy ink-jet paper. It is useful to improve paper coating structure and quality.

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# **Author Biography**

Xiao-Nao Liu received her BS in printing engineering (2003) and her PhD in pulp and paper making engineering from Tianjin University of Science and Technology (2008). She joined the Fan group (Zhejiang University, China) as a postdoctoral fellow. Her current research centers on the patterning nanostructured materials and online analysis of printing quality.