Stability Analysis of a Drop Generation from a Nozzle in an Electric Field

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Abstract

Stability of a conducting drop hanging from a nozzle in an electric field was examined theoretically. With this static model, stability of electrostatic inkjet process was estimated. The basic equations are the augmented Young-Laplace equation for drop shape and the Laplace equation for electric field. These coupled equations were solved by the Finite Element Method. By the initial condition of its shape, a drop could be deformed into different shapes, such as "conical shape," "nipple" or "dog bone" with the increment of non-dimensional electric field. The concentration of electric field around the corner of a nozzle was found to be the cause of these multiple shapes.

Introduction

Since the first inkjet printer, "Mingograph" appeared in market from Seimens Co., Ltd. [1][2], the inkjet technology has progressed tremendously in quality and print speed. Although the electrostatic inkjet technology has not applied to commercial printers, it is attractive in the industrial application. It can make various forms of jet, such as individual drop (drop on demand), micro spray or spindle that could be utilized to make fibers. Additionally, it could jet highly viscous liquid [3] and make such a super fine drop as less than 1 femto litter. Fundamental studies are indispensable to apply this technology to industrial usage. In this point of view, stability of an electrified drop hanging from a nozzle was examined theoretically. Although the jetting is totally a dynamic process, it was found that the electro-hydrostatic approach was useful to estimate how the stable jet was achieved. Also the converged solution of the static problem can provide the initial condition of the dynamic problem in the future.

The equilibrium shape of an inviscid and conductive drop hanging from a nozzle is governed by the augmented Young-Laplace equation that describes the balance of the forces from surface tension, gravity, hydrostatic and electrostatic pressures on the interface [4][5]. Without corona discharge, the electrostatic pressure is governed by a linear differential equation, the Laplace equation. The drop shape is unknown a priori, varied with the increment of the electrostatic pressure so that these equations should be coupled and both the drop shape and the electric field should be solved simultaneously. Basaran et al. [4][5] and Harris et al. [6][7] studied the similar problems with the same equations, although their interest was limited to parallel-plates geometry. Tsukada et al. [8] examined nozzle-plate geometry, which was relevant to the jetting process, both theoretically and experimentally. Tsukada et al., however, did not study the effect of geometrical parameters, such as the nozzle length or the gap between the nozzle and plate, on the shapes and stability of electrified drops. Providing the effect of the parameters is the goal of this paper.

Theoretical Model

Calculation Domain

Similar geometry that Harris et al. [6] used is examined. Fig. 1 shows an axisymmetric, conducting drop hanging from a nozzle of length H_2. The nozzle is at potential u_0 and the bottom plate, a distance H_1 from it, is grounded. L is length between center of the axis and asymptotic boundary of calculation domain. The horizontal plane $z = 0$ is located at the tip of the nozzle. The z-axis is parallel to gravity.

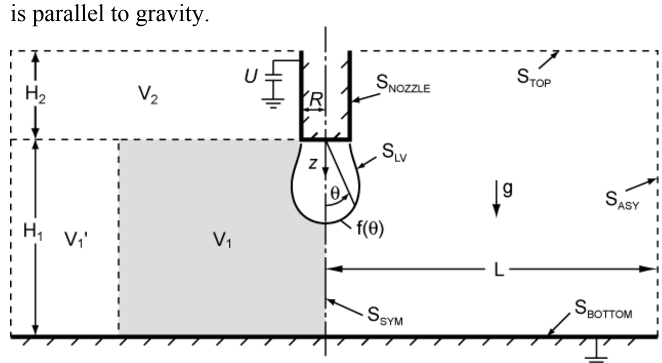

***Fig.** 1* *Axsymmmetric drop hanging from a nozzle in the presence of an electric field*

Two coordinate systems are used with the origins of both systems located in the plane $z = 0$ along the axis of symmetry. A spherical coordinate system (r, θ, ϕ) is applied to the domain V_1, a gray zone of Fig. 1, where r is the radial coordinate and θ and ϕ are the meridional and azimuthal angles, respectively. A cylindrical coordinate system (x, ϕ, z), where x is the projection of r onto the plane $z = 0$, is applied to all other domains. Because an unbounded domain is impracticable, the length L must be finite. To reduce the influence of the length L, cylindrical coordinate domain V_1' is added to the domain V_1.

Governing Equations and Boundary Conditions

The inviscid drop shape and the electrostatic field are governed by the augmented Young-Laplace and Laplace equations as follows;

$$\underline{\nabla}_s \cdot \underline{n}_{LV} = K + Gz + NeE_n^2 \quad on\ S_{LV}\ , \tag{1}$$

$$\nabla^2 U = 0 \quad in \quad V_1 \,\&\, V_2\ . \tag{2}$$

Two dimensionless numbers are defined; electrical Bond number as $Ne \equiv \varepsilon R/2\sigma R$ and gravitational Bond number as $G \equiv gR^2\Delta\rho/\sigma$. A drop is so small that the gravitational Bond number is negligible. Equations (1) and (2) have already been dimensionless. Lengths are measured in unit of R and potential U is in unit of u_0. Here, σ is the surface tension of a drop and $\Delta\rho$ is the density difference between the drop and the ambient fluid. $\underline{n}_{LV}$ is the unit normal vector of the drop surface. E_n ($\equiv R\,\widetilde{E}_n/u_0$, $\widetilde{E}_n$ is dimensional) denotes the normal component of the electric field. Reference pressure K ($\equiv R\Delta p_0/\sigma$) is the pressure difference Δp_0 between the drop and the ambient fluid in the horizontal plane $z = 0$. The reference pressure K is set by constraining the drop volume to be a fixed amount V_0:

$$V = V_0 \, . \tag{3}$$

The governing equations (1) and (2) are solved subject to the boundary conditions:

$$\begin{aligned} & f_\theta \equiv df/d\theta = 0 \quad at \quad \theta = 0 \\ & f = 1 \quad at \quad \theta = \pi/2 \\ & U = 1 \quad on \quad S_{LV} \ and \ S_{NOZZLE} \\ & U = 0 \quad on \quad S_{BOTTOM} \\ & \underline{n}\cdot\underline{\nabla}U = 0 \quad on \quad S_{SYM}, S_{ASY} \ and \ S_{TOP} \end{aligned} \tag{4}$$

Equilibrium Drop Shape in the Absence of External Forces

When gravitational and electrical forces are negligibly small compared to surface tension forces, equilibrium drop shapes are segments of spheres as shown in Fig. 2. The drop shape is expressed in terms of the single parameter D (= d/R), the ratio to the radius R of the signed distance from the center of the sphere to the tip of the nozzle. The shape function is

$$f(\theta) = D\cos\theta + \sqrt{1 + D^2 - (D\sin\theta)^2} \tag{5}$$

When $D = 0$ the drop is a hemisphere; as $D > -1$, the drop vanishes. The radius of nozzle is fixed to unit length.

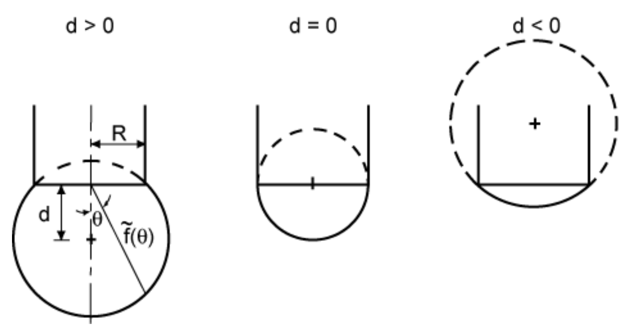

Fig. 2 Equilibrium drop shapes when Ne = 0 and G = 0.

Numerical Analysis

The domain is tessellated into a set of quadrilateral elements, as shown in Fig. 3. In the domain V_1 the elements are bordered by the fixed spines in r-direction and by the curves in θ-direction, which move proportionally to the free surface along the spines [9]. Because the domain is axisymmetric, only the domain of positive x-direction is calculated.

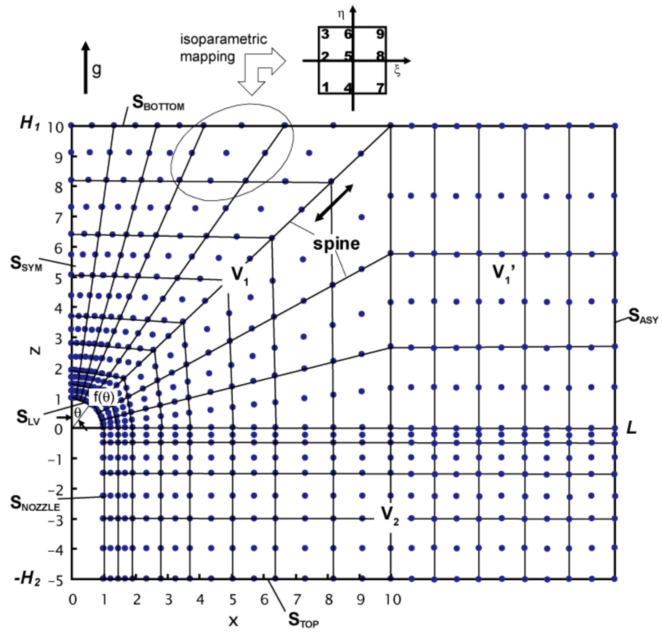

Fig. 3 An example of physical domain of calculation divided into finite elements. The figure above shows isoparametric mapping from/to physical domain to/from calculation domain.

The equations (1) and (2) are coupled and solved by the Galerkin Finite Element Method (GFEM) simultaneously. Mutiplying equations (1) and (2) by weighting functions which are identical to the bi-quadratic basis functions and integrating them by parts, weak forms of the equations (1) and (2) are obtained. Without the boundary conditions, they are

$$\begin{aligned} R_s^{YL} = \int_{S_{LV}} & [\underline{\nabla}_s \cdot (\varphi^s \underline{n}_{LV}) - (K + NeH^2E_n^2) \\ & \varphi^s \underline{e}_r \cdot \underline{n}_{LV}] dS_{LV} \quad s = 1,\cdots,S \end{aligned} \tag{6}$$

$$R_i^L = \int_{V'} \underline{\nabla}\varphi^i \cdot \underline{\nabla}U dV'_{LV} = 0 \quad i = 1,\cdots,I \, . \tag{7}$$

Here, S and I denote numbers of nodes.

Electric potential and drop shape are isoparametrically mapped onto the calculation domain [10] and expressed by the bi-quadratic basis functions as follows;

$$U(\theta,r) \ or \ U(z,x) = \sum_{i=1}^{I} \alpha_i \varphi^i(\xi,\eta) \tag{8}$$

$$f(\theta) = \sum_{s=1}^{S} \beta_s \varphi^s(\xi, \eta = 0) \, . \tag{9}$$

So far, $N = S + I$ residuals and unknowns (β, α) are defined. The volume constraint (4) is rewritten as the $N + 1^{st}$ residual,

$$R_{N+1} \equiv R^{VC} = V - V_0 = 0 \, . \tag{10}$$

To search for the turning points (TP), another residual is defined which specifies an adaptive choice of parameter P,

$$R_{N+2} = P - P_0 - \Delta P = 0 \, , \tag{11}$$

where P_0 is the value of the parameter at a known solution $\underline{\omega}^*$ on a family of solutions and the parameter step size ΔP is a specified increment to a new solution of $\underline{R}(\underline{\omega})$ on the same family. The method of choosing the parameter P from among $\underline{\omega}$ is described by Abott [11]. At the start of calculation the electric Bond number Ne is chosen as the parameter P. Finally the $N + 2$ residuals and unknown vector are defined respectively as $\underline{R}(\underline{\omega}) \equiv (R^{VC}, R_{N+2}, \underline{R}^{YL}, \underline{R}^{L})$, $\underline{\omega} \equiv (K, Ne, \underline{\beta}, \underline{\alpha})$. The nonlinear set of $N + 2$ algebraic equations $\underline{R}(\underline{\omega}) = 0$ is solved simultaneously by Newton's method. With initial guess, $\underline{\omega}^{(0)}$, $k + 1^{st}$ ($k = 1,2,\ldots$) solution can be solved as follows;

$$\underline{\underline{J}}(\underline{\omega}^{(k)})(\underline{\omega}^{(k+1)} - \underline{\omega}^{(k)}) = -\underline{R}(\underline{\omega}^{(k)}) \tag{12}$$

Here $\underline{\underline{J}}(\underline{\omega})$ is a Jacobian matrix. By applying boundary conditions (4), the equation (12) is solved iteratively until the L_2-norm of residuals $\underline{R}(\underline{\omega}^{(k)})$ were less than a prescribed tolerance Δe. Quality of initial guess is critical for Newton method. With the following initial guess, all the calculations were converged quadratically within 8 iterations.

$$f^{(0)}(\theta) = D\cos\theta + \sqrt{1 + D^2 - (D\sin\theta)^2}$$
$$U^{(0)}(\theta, r) = \begin{cases} 0, & everywhere\ in\ V'\ except\ on\ S_{LV}\ and\ S_{NOZZLE} \\ 1, & on\ S_{LV}\ and\ S_{NOZZLE} \end{cases} \tag{13}$$
$$K^{(0)} = 2, \quad Ne^{(0)} = 0$$

Results

Stability analysis of equilibrium drops

Fig. 4 shows the effect of a distance H_1 from the bottom plate on aspect ratio of initially hemispherical drops as the parameter P increases. The aspect ratio, a/b is defined as $a/b \equiv f(\theta = 0)/R(=1)$. It is reasonable that higher electric potential is required to obtain the same aspect ratio when H_1 is widened. The drop shapes are stable up to turning points in effective potential and beyond the turning points, drops become unstable. By applying the scheme that Abott proposed [11], the family of calculation could be continued into unstable parameter space.

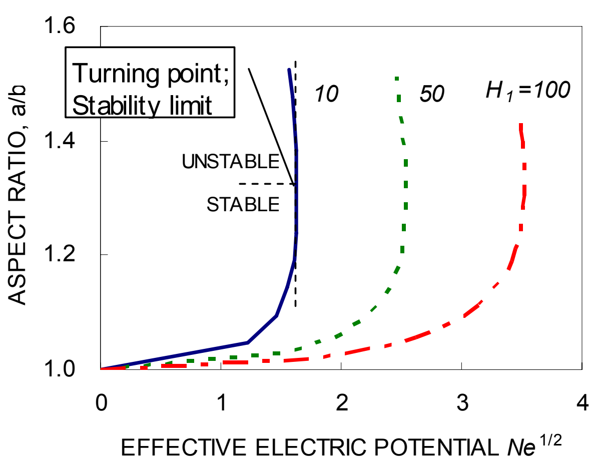

Fig. 4 *The effect of a distance H_1 from the bottom plate on the aspect ratio. G = 0, H_2 = 100, L = 100, D = 0.*

Drop shapes

Figs. 5 to 7 show the effect of the drop shape parameter D on the evolution of drop shapes with effective electrical potential. All other geometrical parameters are fixed. In Fig. 5, initially hemispherical drop ($Ne^{1/2} = 0$, solid curve) deforms as the potential rises until turning point, $Ne^{1/2}$ = 1.74 (broken curve). Although effective potential decreases over the turning point, the drop deforms furthermore toward conical shape. Assumed that radius of nozzle is 0.1 mm and the drop is tap water, the permittivity of which is 7.17×10^{-10} s^4A^2/m^3kg and the surface tension σ = 73 mN/m, the dimensional electrical potential at nozzle $\tilde{u}$ is equal to 248V at the turning point.

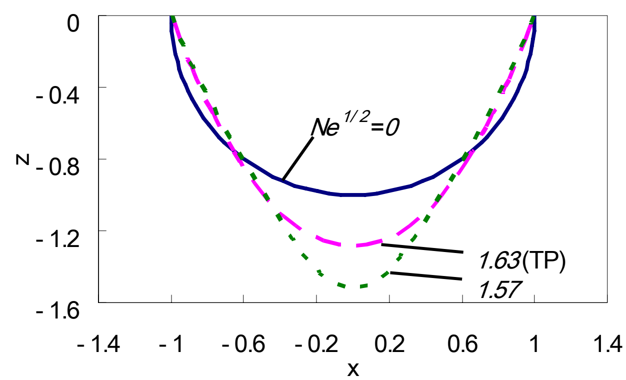

Fig. 5 *Evolution of drop shapes with effective potential when G = 0, H_1 = 10, H_2 = 100, L = 100, D = 0.*

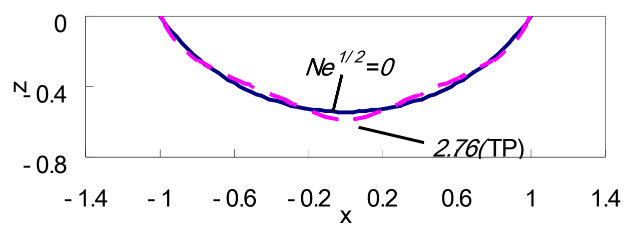

Fig. 6 *Evolution of drop shapes with effective potential when G = 0, H_1 = 10, H_2 = 100, L = 100, D = -0.65.*

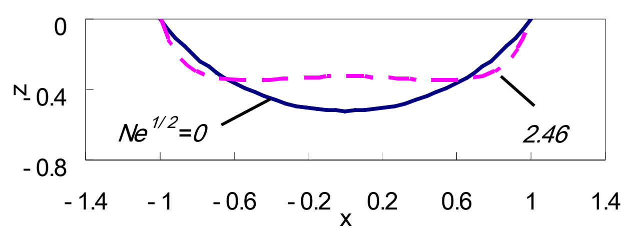

Fig. 7 *Evolution of drop shapes with effective potential when G = 0, H_1 = 10, H_2 = 100, L = 100, D = -0.7.*

In Figs. 6 and 7, drops form anomalous shapes, called nipple and dog-bone, respectively [6]. Such dog-bone shapes have been observed in experiment [12]. More than two peaks of drop shape are thought to be the sites of jetting drops. Cloupeau and Prune-Foch summarized main functioning modes of electrostatic spraying of liquids [13]. The cases of Figs. 6 and 7 resemble onset of multijet mode in which several jetting sites are established around the end of the capillary [13].

The equilibrium drop shape is determined by the balance of forces from surface tension, hydrostatic and electrostatic pressure. Among all the forces on a drop the origin of such anomalous shapes as nipple and dog-bone is distribution of electrostatic force on the drop surface. Fig. 8 shows the electrostatic field surrounding the drop in Fig. 7 at $Ne^{1/2}$ = 2.46, at the case which deforms to dog-bone shape. Strong electrostatic field near the corner of a nozzle elongates the drop toward positive *x*-direction to be dog-bone shape. Nipple is in-between cone and dog-bone, appeared in only small range of effective electrostatic potential.

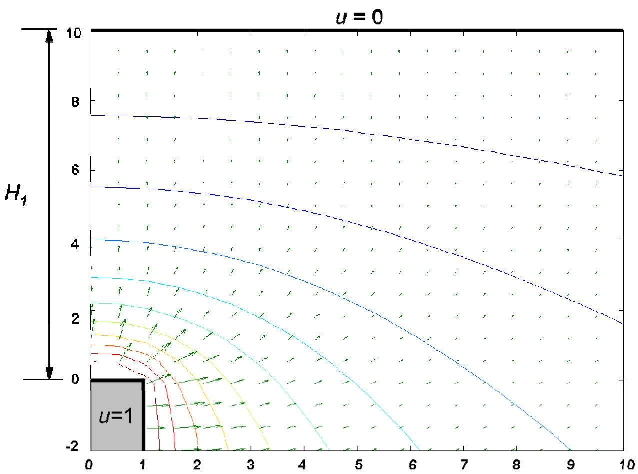

Fig. 8 *Electrostatic field surrounding a nozzle when G = 0, H_1 = 10, H_2 = 100, L = 100, D = -0.7, $Ne^{1/2}$ = 2.46. *Limited area is shown.*

Concluding remarks

Stability of a drop hanging from a nozzle in an electric field was examined theoretically.

1) Higher electric potential is required to obtain the same aspect ratio when the distance H_1 is widened. At small distance H_1, turning point of stability appears.
2) As drop shape parameter *D* decreases from 0 to -1, drop shapes transit from conical shapes to nipple-like shapes to dog-bone shapes. Concentration of electric field around the corner of a nozzle is the cause of these multiple shapes.

At narrow gap between nozzle and plate, drop can be jetted at relatively low electrical potential. Concentration of electric field around the corner of a nozzle, however, could form multiple sites of jetting. The nozzle shape and appropriate set-ups of nozzle and parameters, such as electric potential should be considered for stable jetting. Although the foregoing results are equilibrium profiles provided that the drop volume is constant, they can suggest the way to achieve stable electrostatic inkjet process. Also they can be the guidance to the studies of dynamic phenomena.

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Author Biography

"Kazu" Tada joined Fuji Xerox in 1990 following completion of a M.S. degree in Instrumentation Engineering from Keio University. Since then he has been developing method of coating photoreceptors. From 1997 to 1990 he has studied coating science and technology in University of Minnesota and received another M.S. degree. Currently he enjoys double statuses of his career as of an engineer in Fuji Xerox and a Ph.D candidate in Waseda University.