A Voronoi Based Framework for Multilevel AM Screen Design

Stefaan Lippens, Dirk Broddin, Wilfried Philips; TELIN/Ghent University-IBBT, Ghent, Belgium; Punch Graphix, Lier, Belgium

Abstract

Halftoning by screening is a popular technique because of its computational simplicity. The screen matrix is very determining for the quality of the generated halftone patterns. In this paper we will introduce a novel screen design framework for the generation of multilevel AM halftoning screens. A first important novelty is the support for multilevel screens. The screen design is targeted towards multilevel electrophotographic print technologies, where the intermediate output levels are not as reproducible as the outer levels. In this case it is important to control the so called hardness of the screen, which comes down to the fraction of pixels with intermediate levels in the halftone patterns. In the proposed algorithm we can control this hardness directly. Another novelty is that the framework isn't limited to the classical orthonormal AM halftoning grids. The geometrical foundation of the algorithm is based on the concept of Voronoi tessellation. This approach makes it possible to handle for example slanted AM grids or grids with different ruling in the main directions.

Introduction

Classical halftoning is the process of transforming a continuous tone image into a binary image to make it printable on a bi-level printing device. The traditional way of halftoning is AM halftoning, also called autotypical halftoning, and is still used in many applications today. In AM halftoning the binary pattern is built of clusters positioned on a regular orthonormal grid. The size of the clusters is modulated to obtain the desired gray value.

Screening is a very popular halftoning algorithm, especially for AM halftoning. The important advantage of this algorithm is the low complexity, both in terms of required number of operations per pixel and the lack of dependencies between the processing of individual pixels. In screening, each pixel of the input image is compared with a spatially varying threshold, implemented as a *threshold matrix*, also called screen, mask or dither array (Fig. 1). The values in the threshold matrix determine how the the clusters of black and white output pixels grow as a function of the input gray value. The threshold matrix also determines the characteristics of the resulting patterns, such as the ruling and angle of the AM grid. In practice a screen has to be designed for a desired ruling and angle combination. This is especially true in AM halftoning for color printing, where the halftones of four or more separations have to be combined, and Moiré effects have to be minimized.

In the previous paragraphs we assumed only two output levels of the printing device, but nowadays there exist also multilevel print technologies, such as electrophotography. In these technologies intermediate output density levels can be achieved. These intermediate levels however prove to be less reproducible as a consequence of the physical details of the electrophotographic process [2, 5, 1]. Too much use of the intermediate density levels results in low print quality with high variability and noise in the densities and edge and shadowing artifacts. On the other hand, the intermediate density levels are needed to obtain an acceptable amount of reproducible gray levels for a given ruling r (in lpi) and print resolution d (in dpi). As a rule of thumb, the amount of reproducible gray levels for a print device with M output levels is upper bound by

$$(M-1)\frac{d^2}{r^2}+1 \tag{1}$$

For example, for a 150 lpi screen on a 1200 dpi device, at least 16 density levels are needed to obtain 1000 reproducible output gray levels.

Because of the limited reliability of the intermediate levels mentioned before, multilevel printing still requires (multilevel) halftoning. In the multilevel halftone design, there is a trade-off between the stability of gray scale reproduction offered by a *hard* screen (low use of intermediate levels) and the added value of using intermediate levels. This added value is not limited to the number of reproducible output levels. The usage of multilevel output pixels is also valuable for the countering of internal Moiré effects in the screen for example. In this context, it is important that the fraction intermediate density levels can be controlled during screen design.

In this paper we develop a framework for multilevel AM screen design. A first novelty of the framework is the Voronoi based spot function, which enables non traditional AM grid geometries. A second, possibly more important aspect is the direct control over the amount of intermediate output levels or the hardness of the screen.

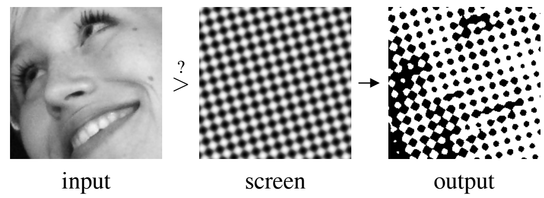


Figure 1. The screening algorithm (for a bi-level output device): the input image is compared pixel-wise with a space varying threshold (an AM halftoning screen here) resulting in a binary output image. The individual binary dots or pixels in the output image are clustered in spots.

General screen design concepts

For a given desired ruling and angle of the AM screen a certain (super) cell size or tile size is chosen in which the screen will be defined. This tile is periodically repeated so that the whole input image can be covered. For bi-level halftoning, the screen is usually based on a spot function, which defines the order in which pixels should be turned on [4]. Translation of the spot function to the threshold matrix is rather straightforward.

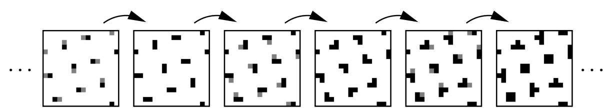


Figure 2. *Incremental generation of multilevel AM halftone patterns, obeying the stacking constraint. In each step, the value of some pixels in the pattern are increased.*

Multilevel screening for M output levels is similar to bi-level screening. Instead of one threshold matrix, there are $M-1$ threshold matrices, one for each transition between successive output levels. The design of this set of threshold matrices is less straightforward than in the bi-level case because there are several thresholds to choose per pixel. Especially the hardness of the screen, which is not relevant in bi-level screen design, should be taken into consideration here.

As an introduction to the multilevel screen design approach we employed, we consider a threshold matrix (or a set of threshold matrices) and analyse how the clusters in the halftone pattern grow as a function of the input gray value. An important property of the threshold operation and the generalised multilevel threshold operation is the monotonic behaviour: given two input gray values g_1 and g_2 with $g_1 < g_2$, the (multilevel) thresholded output values will obey $h_1 \leq h_2$. In the context of screening, this property is usually called the 'stacking constraint' for the set of patterns that can be generated by the screen [6]. This property can also be used in the other direction. Given a set of multilevel halftone patterns $h_i[x,y]$ that obey the stacking constraint, it is possible to construct a set of threshold matrices that can generate the given patterns. Based on this idea, the screen design problem can be translated to the easier problem of generating a set of halftone patterns obeying the stacking constraint.

A set of halftoning patterns satisfying the stacking constraint can easily be generated in an incremental way. Say that we start from a completely white[1] pattern $h_0[x,y] = 0$. In each step of the iterative algorithm we only increase the value of some pixels, making them darker. For example, by increasing the value of some pixels from $h_0[x,y]$, we obtain a new pattern $h_1[x,y]$, which complies, by construction, to the stacking constraint: $h_0[x,y] \leq h_1[x,y]$. In the same way, the patterns $h_2[x,y]$, $h_3[x,y]$ and $h_i[x,y]$ in general can be generated. Fig. 2 illustrates this incremental generation of multilevel halftone patterns.

Screen design for AM halftoning

In order to obtain a proper AM halftoning screen, the generated patterns should have AM characteristics. This implies that the pixels should be clustered around the given regular fixed grid positions. There are actually two grids in play here: the positions of the black clusters in the highlights, called the headpoints, and the positions of the white clusters in the shadows, called the antiheadpoints (Fig. 3). Because of the symmetry between the headpoint and antiheadpoint geometry we will use two variations of the incremental pattern generation from Fig. 2, shortly called 'trips' hereafter. One trip will cover the spot growth of the headpoint spots, evolving from completely white to the 50% coverage point[2] where white and black spots meet. The other trip will cover the antiheadpoint spot growth from completely black to the 50% coverage point. In this last trip, the pixel values start at $M-1$ (black) and in each iteration some pixel values are decreased instead of increased as in the headpoint spot grow trip. Fig. 4 shows these two trips. Patterns within the same trip obey the stacking constraint by design of course. Special care should be taken however that the patterns also follow the stacking constraint between both trips. One possibility to ensure this is for example setting a precomputed pattern for the 50% coverage point and using this as the upper bound, respectively lower bound for the patterns in the headpoint spot trip, respectively antiheadpoint spot trip.

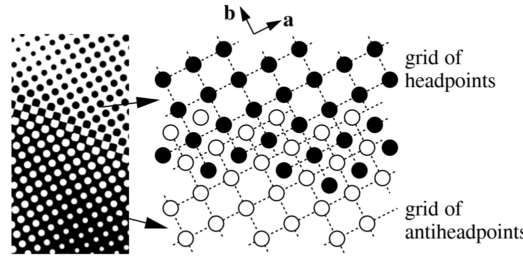


Figure 3. *Anatomy of an AM halftoning pattern: black spots positioned on headpoints and white spots on antiheadpoints. The headpoint grid can be defined by the (orthonormal) vectors* $\mathbf{a}$ *and* $\mathbf{b}$*.*

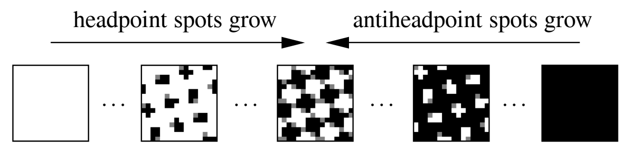


Figure 4. *The headpoint spot grow trip and antiheadpoint spot grow trip meet at the 50% coverage point.*

In each iteration of the incremental pattern generation algorithm, each spot in the screen tile grows one unit. Our contribution is an algorithm for selecting the order in which spots are incremented or decremented. We will limit the discussion to the headpoint spots, for the sake of simplicity. Several requirements are important in the selection of the pixel to increase. The pixel should for example be in the neighbourhood of the headpoint. Pixels that have already reached their maximum value (defined by the upper bound of the 50% coverage pattern for example) cannot be incremented. These simple conditions just limit the pool of candidate pixels, but do not favor one of the candidate pixels above the other. For optimal quality the clustering of the spots, and more specifically, how the spots grow is important. The concept of the spot function, as used in binary screen design, can be reused for this end. A given spot function can be used as a cost function to select the best pixel from the candidate pixels.

Voronoi based spot function

The spot function expresses the priority for each pixel to increase to ensure an AM screen. For pixels close to the headpoint, at the center of the spot, the spot function value should be high and it should decrease further away from the headpoint. The spot function can also be seamlessly extended towards the antihead-

[1] We use the hard-copy convention for gray values here, where lower values mean lighter and higher values mean darker. Value 0 corresponds to complete white, uncovered paper, value $M-1$ means maximum density black.

[2] This is the theoretical 50% coverage point, ignoring any dot gain.

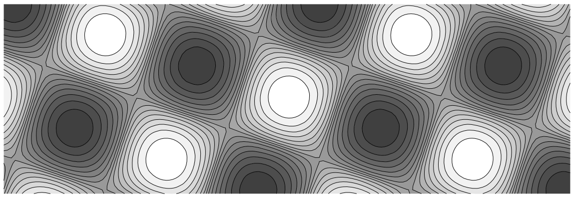
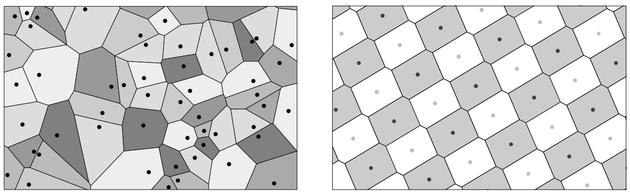


Figure 5. *Contour plot of the spot function defined by (2).*



(a) General. (b) Slanted AM halftoning grids.

Figure 6. *Examples of Voronoi tessellations for (a) a general set of points p_i and (b) the headpoints and antiheadpoints of a slanted AM halftoning grid.*

points. As selection criterion for the antiheadpoint pixels we need then to look for the pixels with the lowest spot function values however. This way, the spot function should be the the lowest around the antiheadpoint and increase further away. A well known spot function (given as example in [4]) can be constructed in the (a,b) coordinate grid defined by the headpoint grid vectors **a** and **b** from Fig. 3:

$$\cos(2\pi a)+\cos(2\pi b) \tag{2}$$

This function is maximal at integer values of a and b (which correspond with the headpoints) and minimal for $a=\frac{2k+1}{2}$ and $b=\frac{2l+1}{2}$ with k and l integer (the antiheadpoints). When used in screen design, it generates the euclidean spot shape as can be observed from the spot function contours in Fig. 5. The cosine based spot function (2) is very straightforward but lacks flexibility for alternative grid geometries and more advanced spot growth control.

A more flexible spot function can be obtained based on the Voronoi principle. Given a set of points p_i, a Voronoi tessellation associates with each point p_i a region of points that are closer to this point p_i than any other point p_j from the set. Fig. 6(a) shows a Voronoi tessellation of a set of randomly distributed points p_i. When applied to the regular distribution of the headpoints and antiheadpoints of an AM halftoning grid, a tessellation like Fig. 6(b) is obtained. The grid in this last example is non orthogonal and the cells around the points adapt to this geometry. Also note that the cosine based spot function (2) is limited to square and parallelogram shaped geometries and can not be adapted to the configuration of Fig. 6(b).

Within these Voronoi concepts it is also possible to define a spot function. Based on the desired properties stated before (maximal at headpoints and minimal at antiheadpoints), we define the spot function for every point (x,y) as follows:

$$S(x,y)=\frac{d^{(A)}(x,y)}{d^{(A)}(x,y)+d^{(H)}(x,y)} \tag{3}$$

with $d^{(H)}(x,y)$ the distance from (x,y) to its closest headpoint and $d^{(A)}(x,y)$ the distance to the closest antiheadpoint for (x,y)

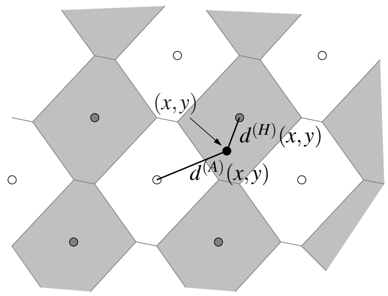


Figure 7. *Illustration of the elements in the Voronoi based spot function (3).*

as illustrated in Fig. 7. This function is maximal with value 1 where $d^{(H)}(x,y)=0$, which is the locus of the headpoints, and it is minimal with value 0 where $d^{(A)}(x,y)=0$, which is the locus of the antiheadpoints. Also note the symmetry between the headpoints and antiheadpoints in this spot function: when $d^{(H)}(x,y)$ and $d^{(A)}(x,y)$ are swapped in (3), the expression can be rewritten as $1-S(x,y)$.

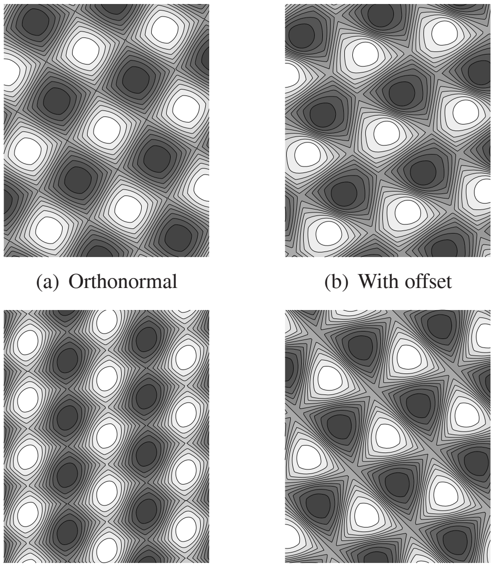
Fig. 8 shows some example Voronoi spot functions for different headpoint and antiheadpoint configurations. Fig. 8(a) shows the traditional case with orthonormal headpoint and antiheadpoint grids. This spot function is very similar to the cosine based spot function from Fig. 5. In Fig. 8(b) the same grids are used but there is an additional offset between the antiheadpoint grid and headpoint grid. The spot growth is not symmetrical in this case. This decoupling of the headpoint and antiheadpoint grid can for example be used to optimize the rosette structures in shadows and highlights separately, such as in [3]. In Fig. 8(c) the grids are not orthogonal but a bit slanted and the ruling in the two main directions are not equal in addition, like in Fig. 6(b). Note that this spot function can not be obtained from Fig. 8(a) with a proper affine transformation. Fig. 8(d) is a last illustration of the flexibility of the Voronoi based spot function. In this example, the angle between the main directions and the offset between the antiheadpoint grid and headpoint grid are wisely chosen to obtain a hexagonal structure.

Multilevel compensation

In the context of multilevel screen design, only using a spot function is not sufficient to fully exploit the multilevel aspect. As noted before, in each iteration the screen design algorithm has to select a pixel for increasing (or decreasing) its value. When only the spot function value is used as cost function of pixel selection, the same pixel will be picked in successive iterations, as long as that pixel has not reached its upper (or lower) bound. Consequently, each spot of the generated pattern will contain at most one pixel at an intermediate density level, while the other pixels will be at 0 or at $M-1$.

To encourage multilevel content in the screen we add a negative feedback term to the cost function as multilevel compensation. Say we are at a certain iteration i of the algorithm and the previously generated pattern is $h_{i-1}[x,y]$. This pattern is then normalized and subtracted with a weight s from the discretized spot function $Z[x,y]$ to get the cost function for iteration i:

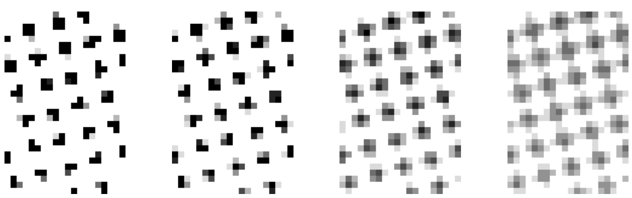
$$C_i[x,y]=Z[x,y]-s\frac{h_{i-1}[x,y]}{M-1} \tag{4}$$



(a) Orthonormal (b) With offset

(c) Slanted (d) Hexagonal

Figure 8. *Examples of Voronoi based functions.*



(a) $s = 0.0$ (b) $s = 0.1$ (c) $s = 0.5$ (d) $s = 1.0$

Figure 9. *Examples of halftone patterns generated for different values of the multilevel compensation factor s for 3 bit multilevel output (8 levels).*

In case of the headpoint spot trip, we select for each headpoint spot the pixel from the spot cell with the highest $C_i[x,y]$ value and increase it one unit.[3] Because of the negative feedback term $-sh_{i-1}[x,y]$ in (4), the pixels that already have a high value $h_{i-1}[x,y]$ are less likely to be selected than pixels with lower $h_{i-1}[x,y]$ values. This way picking the same pixel in successive iterations is discouraged. The higher the multilevel compensation factor s, the more the intermediate density levels are encouraged as illustrated in Fig. 9. Fig. 10 also shows the relation between the fraction of the intermediate density levels, the s-factor and the bit depth of the multilevel output. It shows how the s-factor can be used to control the hardness of generated screen (the lower the fraction of intermediate levels, the harder the screen). The practical use of generating screens with a desired level of contrast based on the proposed algorithm is addressed in a separate NIP24 contribution by F. Deschuytere et al. (Image Quality and Productivity of the New Xeikon Digital Presses using "true 1200 dpi" Multilevel Print Head Technology).

Conclusions

In this paper we proposed a novel screen design algorithm for multilevel AM halftoning. By using a Voronoi based spot function, the algorithm is not limited to the traditional orthonormal AM grids. It is capable of tackling a broader range of grids, including grids with different rulings in the main directions, slanted grids, hexagonal grids and even configurations with an additional offset between the headpoint and antiheadpoint grid. This makes the algorithm an interesting tool in the minimization of Moiré artifacts. The algorithm is also capable of directly controlling the hardness of the generated multilevel screen with a multilevel compensation factor.

[3] The antiheadpoint spot trip is much the same, except that the pixels with the *lowest* $C_i[x,y]$ values are selected for *decreasing*.

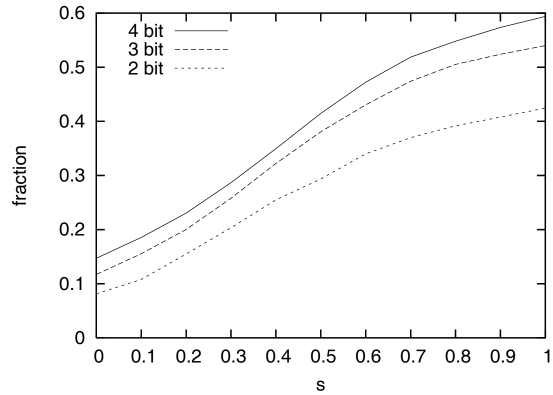


Figure 10. *The average fraction of intermediate levels in the halftone patterns in function of the multilevel compensation factor s for different multilevel bit depths.*

References

[1] Dirk Broddin and Frank Deschuytere. US patent 5,799,137: Clustered dot and line multilevel halftoning for electrophotographic color printing, August 1998.

[2] Katrien Daels and Paul Delabastita. US patent 5,903,713: Moiré free multilevel halftoning of color images, May 1999.

[3] Paul Delabastita. US patent 5,828,463: Printing plate for a halftone image having toner dependent rosette structures by phase modulation, October 1998.

[4] Adobe Systems Incorporated. *PostScript Language Reference Third Edition*. Addison-Wesley Publishing Company, 1999.

[5] Dhiraj Kacker, Tom Camis, and Jan P. Allebach. Electrophotographic process embedded in direct binary search. *IEEE Transactions on Image Processing*, 11(3):243–257, March 2002.

[6] Daniel L. Lau and Gonzalo R. Arce. *Modern Digital Halftoning*. Marcel Dekker, 2001. ISBN 0-8247-0456-8.

Acknowledgment

This research is funded by the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT-Vlaanderen).

Author Biography

Stefaan Lippens received the B.S.E. degree from Ghent University, Belgium in 2000 and the M.S.E.E. degree from Ghent University in 2003. He works as a researcher at the research group for Image processing and Interpretation (IPI) at the department for Telecommunications and Information Processing (TELIN) of Ghent University. He is active in the area of print modelling and digital halftoning techniques, ranging from AM halftoning to error diffusion based halftoning techniques.