# Spectral Modeling of an n-Ink Printer via Thin Plate Spline Interpolation

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## Abstract

*Reducing metamerism is one of the main goals of spectral printing. There are three main steps in spectral printing to address:*

1. *What is the printer gamut in spectral space (Printer Forward Model)*
2. *Given an input spectrum, what is the closest point on the printer gamut to the input reflectance (Spectral Gamut Mapping)*
3. *What ink density combination produces the given point on the gamut (Ink Separation)*

*In this paper we address the first issue for printers with a large number of inks. The performance of the model is evaluated based on an 8- ink printer and is compared to the most commonly used technique. We show that the new model performs as accurately as the existing models but does not require as many as constraints to set up the model.*


## Introduction

For color printers with a low number of inks (4 to 6), metamerism (colors that change under different illuminants) can be a problem. As well, such printers have limited color gamuts. In order to minimize metamerism, we are seeing printers with as many as 12 inks being introduced into the market. However, in order to take advantage of the larger number of inks in reducing metamerism, an input spectral reflectance needs to be matched accurately in the spectral space. Finding the best ink combination to match an input reflectance in the spectral space is referred as spectral printing. For the printing model to perform well, a printer forward model is needed to predict reflectance of any ink combination accurately. One printer model that works well when the number of inks is small is the Yule-Nielsen Spectral Cellular Neugebauer (YNSCN). However, a problem with the Yule-Nielsen Neugebauer model is that its computational complexity grows exponentially with the number of inks. Another difficulty with using this model is that it requires very specific uniform sampling points for the training data set, which makes it hard to setup and use the model.

In this paper we present a printer model based on Thin Plate Spline (TPS) interpolation. This model has the advantage that the number of training points and the computational requirements grow much more slowly than in the case of the YNSCN model. In addition, TPS does not require training data to be sampled on an evenly spaced grid. Accuracy comparison between YNSCN and TPS model is presented for an 8-ink printer.

## Data Collection

An 8-ink printer with the following inks was used to study the performance of the calibration model for printer output: Cyan, Magenta, Yellow, Light Cyan, Light Magenta, Black, Gray, and Light Gray. The Yule-Nielson Cellular Neugebauer Model (YNCN) was implemented as a reference forward printing model for comparison to the performance of the proposed model. [6],[7],[8].

For training 6048 patches were used, and 939 for testing. The patches are printed on glossy media with an ink limit of either 1.5 or 2 drops of ink (depending on the ink type).

A GretagMacbeth Spectralino was used to measure the spectrum reflectance of the printed patches with a10nm sampling from 380nm to 730nm. The Spectralino has an accuracy of around 0.30 $\Delta E_{94}$ between two different sets of measurements under D65 illumination. The printer has an average 0.75 $\Delta E_{94}$ page-to-page variation (including instrument variation).

## Thin Plate Spline Interpolation

As is typical of interpolation methods, thin-plate spline (TPS) interpolation constructs a function that matches a given set of data values $y_i$, corresponding to a given set of data vectors $\mathbf{X_i} = [X_{i,1}, X_{i,2}, \ldots X_{i,D}]$ in the sense that $y_i = f(\mathbf{X_i})$ [1] Xiong et al. extended the TPS model to N-Dimensions and applied it to illumination estimation successfully [1].

For the spectral printing process, in this paper TPS is used to find a continuous function that maps between the set of inks and each of the output dimensions. For instance if the output spectral reflectance of an 8-ink printer is measured from 380nm to 730nm with a 10 nm sampling, TPS is used to create 36 separate functions mapping from the 8 input dimensions to each reflectance wavelength 380nm, 390nm, …. to 730nm individually

## Spectral Printer Modeling Directly

In this paper, TPS is used to model the printer output using two different approaches. The first approach finds a continuous function between an input ink combination and a specific wavelength band of the resulting printed spectrum. For this paper, there are 36 separate continuous functions representing the spectral output of the printer at a 10nm sampling from 380nm to 730nm. Since the light incident on coated media follows Beer's law, many researches recommend transforming the reflectance of ink combinations through a logarithmic function in order to reduce complexity of the output space [2],[3],[4] and [5]. Figure 1 compares the underlying dimensionality of the printer spectral output when the gamut dimension was analyzed directly and after logarithmic transformation. Residual error in constructing the space using Principal Component analysis was used to evaluate the dimensionality of the gamut before and after transformation. Figure

1 shows that, as might be expected, the printer gamut has lower intrinsic dimensionality (around 3) than the 8 inks used to create the gamut. The figure also shows that the printer gamut in the logarithmic space has still lower dimensionality. Knowing this, the first printer model is based on finding 36 continuous functions between input ink combinations and logarithmic transformation of the printer gamut in spectral space.

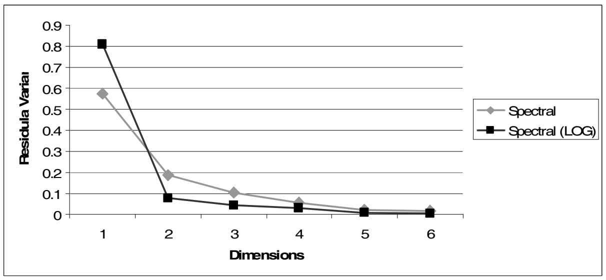


***Figure 1:The residual variance of PCA on spectral reflectances from an 8-ink printer (Spectral) and logarithmic transformation of the spectral reflectance of 8-ink printer ( Spectral (LOG) )***

## Spectral Printer Model using Isomap

The second printer model is based on understanding the manifold of the printer output space and transforming the printer spectral gamut space to a simpler space before trying to apply any interpolation technique. We found that the logarithm helped reduce the dimensionality, but we would like to understand if there is a better transformation to be done in order to lower dimensionality of the printer gamut and thus be able to improve speed and accuracy of the printer forward model. Isomap [9] is used to map the data into a lower dimensional space while preserving the geodesic distances between the original data points.

### *Isomap and Multidimensional Scaling*

Isomap [9] is a nonlinear generalization of classical Multidimensional Scaling (MDS) [10]. MDS maps the input data to a lower dimensional space, subject to the constraint that pairwise distances between data points are preserved as much as possible. The main idea of Isomap is to perform MDS, not on the input space distances, but on the geodesic distances between points on the data manifold. The geodesic distances represent the shortest paths along the curved surface of the manifold. This can be approximated by a sequence of short steps between neighboring sample points. Isomap then applies MDS to the geodesic, rather than straight line, distances to find a low-dimensional mapping that preserves these pairwise distances.

### *Applying Isomap to the Printer Model*

Figure 2 shows the dimensionality of the spectral gamut as found by Isomap. The figure shows that using Isomap we can reduce output gamut of the 8-ink printer by one more dimension from 4 to 3 dimensions. The figure also shows that after Isomap transformation, running through a second transformation (based on logarithmic function) does not reduce complexity of the gamut.

The 2nd printer forward model is based on using TPS interpolation to:

1. Create continuous functions between input ink combinations and Isomap transformed gamut space

2. Create continuous functions between input Isomap transformed gamut space and printer gamut space in spectral space

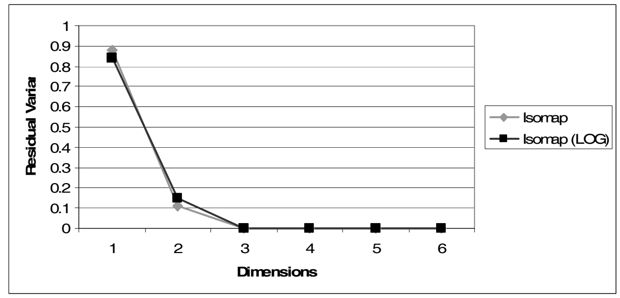


***Figure 2: The residual variance of Isomap on spectral reflectances from an 8-ink printer (Isomap) and logarithmic transformation of the Isomap mapped spectral reflectance of an 8-ink printer ( Isomap( LOG) )***

### *Result*

To evaluate performance of the two models discussed in this paper, an 8-ink Yule-Nielson Cellular Neugebauer [6] model was implemented and trained using the same training data as for TPS.

A search was done to find the best Yule-Nielson value and it was set at 4.1. Table 1 shows both the spectral prediction accuracy of the model and the color variation under D65 illumination.

**Table 1: Yule-Nielson Neugebauer Model Performance with Yn parameter optimized at 4.1. RMS represents error in spectral space measured as root mean squared. $\Delta E_{94}$ is calculated based on D65 illumination**

| | RMS | | $\Delta E_{94}$ | |
|---|---|---|---|---|
| | Mean | Max | Mean | Max |
| YNCN | 0.0047 | 0.0284 | 1.48 | 5.12 |

The next figure shows the performance of TPS using the first characterization method (characterizing spectral and logarithmic transformation of the spectral gamut directly):

**Table 2: Performance of printer model using TPS in predicting spectral gamut directly. RMS represents error in spectral space measured as root mean squared. $\Delta E_{94}$ is calculated based on D65 illumination**

| | RMS | | $\Delta E_{94}$ | |
|---|---|---|---|---|
| | Mean | Max | Mean | Max |
| Spectral | 0.0097 | 0.1027 | 3.1 | 47 |
| Spectral (LOG) | 0.0086 | 0.09 | 2.1 | 23 |

Table 3 shows the accuracy of TPS in predicting the spectral gamut of the printer using intermediate Isomap embedding. The results are based on using a set of 4 TPS interpolations to map between input ink combinations (8 dimensions) and the 36-dimensional spectral data embedded by Isomap into 4 dimensions followed by a second set of 36 TPS interpolations mapping between the embedded 4-dimensional space and 36-dimensional spectral reflectances.

**Table 3: Performance of printer model using TPS in predicting spectral gamut through Isomap embedding. RMS represents error in spectral space measured in root mean squared. $\Delta E_{94}$ is calculated based on D65 illumination**

| | RMS | | DeltaE | |
|---|---|---|---|---|
| | Mean | Max | Mean | Max |
| Isomap | 0.005 | 0.0629 | 1.99 | 6.71 |
| Isomap (LOG) | 0.0049 | 0.068 | 2.1 | 7.2 |

The next figure compares the performance of all the different printer-characterization models discussed in this paper. It shows that TPS performs as accurately as the best known printer model when the printer gamut space dimensionality is reduced using Isomap.

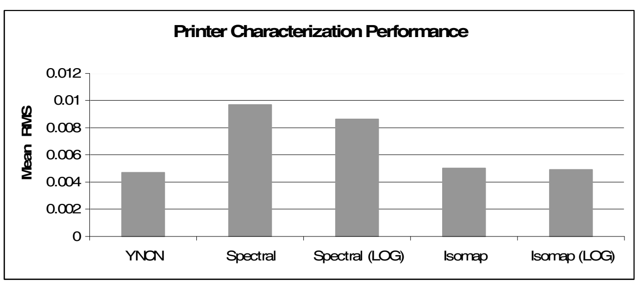


***Figure 3:*** *Summary performance of different proposed models compared to the reference model (YNCN). RMS represents error in spectral space measured as root mean squared*

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## Conclusion

We showed that a printer forward model based on Thin Plate Spline interpolation can perform as accurately as Yule-Nielsen Cellular Neugebauer Model that was used as the reference printer model, but with fewer constraints. We also show that by lowering the dimensionality of the space in which the predictions are run from 36 to 4, we can improve the accuracy of the interpolation. Two methods for lowering the dimensionality of the printer spectral gamut were compared. One approach is based on logarithmic transformation of the spectral reflectances of the printer gamut and the second is based on Isomap.

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## Author Biography

*Behnam Bastani joined Hewlett-Packard Company in 2004 where his research focus has been on designing auto-calibration models for stand-alone high-end ink-jet printers with focus in Retail Printing market. He has been involved in Retail Photo market analysis and collaborated on protecting HP Intellectual Property in this market. He is also a PhD candidate at Simon Fraser University where his research focus is in Spectral Printing and automatic Ink Separation. He has 18 publications and 8 patents in the field of color science and computer vision.*