

Suppression of Automoiré in Multi-Level Supercell Halftone Screen Designs

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Abstract

An optimization approach is proposed for suppressing the automoiré, also known as internal moiré, in multi-level periodic screen designs. This artifact can appear when the halftone dots of a rational tangent screen do not have integer offsets in pixels. Starting with an ideal continuous space halftone pattern for a given level, a bounded, constrained convex optimization problem is solved to obtain the best discrete space halftone pattern values on the pixels of the supercell tile. The optimization minimizes a least-squares spatial frequency weighted difference between the pattern when first reconstructed to continuous space by the printing system (e.g. exposure) and the ideal pattern. The resulting halftone pattern has been found to typically contain very little energy outside of the screen's fundamental frequencies and harmonics while preserving the original ideal dot shape reasonably well. A series of such patterns so derived can be converted to multi-level threshold arrays if the pattern values are quantized and the stacking constraint is enforced during optimization.

Introduction

To obtain close approximations to desired periodic screen frequencies and angles, a supercell method can be employed whereby halftone dots are placed on crossed lines of a rational slope within a tile. This tile is designed so it can be applied periodically, producing an extension of the screen pattern. In general this approach will result in halftone dots that are not identical to one another since their placement in the tile will not necessarily be aligned to the pixel grid. Such methods can still work well when the device resolution is high compared to the screen frequency, but at the lower resolutions typical to desktop electrophotography (EP), the dot-to-dot variations can lead to noticeable artifacts, and limit the usefulness of the method.

These artifacts are referred to as “automoiré” or “internal moiré” because they represent an interference effect between the screen and the pixel grid. The effect can also be understood as consequence of aliasing caused by sampling the ideal continuous space halftone pattern. Methods to reduce automoiré typically involve using randomness to break-up the regular pattern, but the result is generally a grainier image with only partial reduction of the automoiré. A better method is that of “Well-Tempered Screening” [1] which uses feedback from the filtered pixel pattern of the previous level to modify the thresholds during the design process. The aliasing error implicit in the original discrete threshold array can be considered to be diffused across the tone scale. Much of the signal processing framework provided by this work informs the work presented here and was a source of inspiration. However, because this approach was initially proposed for bi-level screening where it is inevitable that

significant non-harmonics will be introduced at some tone levels, it is philosophically different than the method proposed here which attempts to completely optimize each level, among other differences.

Multi-level halftoning can also be used to reduce automoiré by providing a mechanism for anti-aliasing. Starting with a high resolution halftoning pattern, an anti-aliasing filter can be applied and the result sub-sampled to the device resolution. Conventional box filtering approaches for creating multi-level threshold arrays [2] can be cast into this form. However, box filters can still allow significant aliasing artifacts to pass through, whereas higher quality anti-aliasing filters will cause ringing outside of the dynamic range allowable for driving the printing system. Furthermore, the quantization of the halftone levels provides further restrictions on the quality that can be achieved.

However, the fact that continuous-level halftone patterns in principle could be completely anti-aliased is tantalizing, and is the starting point for the proposed method.

Frequency Domain Description of Rational Tangent Screens

Here periodic screening patterns where the ratio of frequency components among the fundamentals is rational are considered. The advantage of such a design is that there is necessarily a finite square tile which can be applied periodically to recreate the infinite parallelogram tiling pattern [3]. The fundamental frequencies in the x- and y-directions can then be written as integer values in units of cycles per tile. For simplicity we call them $f_1 = (A,B)$ and $f_2 = (C,D)$. In the orthogonal case $(C,D) = (B,-A)$. Any of the harmonics can be represented by integer linear combinations of these fundamentals.

In the space domain, the halftone dots can be considered to lie on the intersection of the parallel lines perpendicular to vectors (A,B) with spacing $1/\sqrt{A^2+B^2}$ and perpendicular to (C,D) with spacing $1/\sqrt{C^2+D^2}$. Note that this screen period is not the same as the length of the sides of the parallelograms thus formed, except in the orthogonal case. These sides are the dot center displacements and are given by the formulas:

$$\begin{aligned}\Delta x_1 &= B/(AD-BC) \\ \Delta y_1 &= -A/(AD-BC) \\ \Delta x_2 &= D/(AD-BC) \\ \Delta y_2 &= -C/(AD-BC)\end{aligned}\tag{1}$$

It is straightforward to derive that there are $|AD-BC|$ such dot centers in the tile, so to avoid huge tile sizes A,B,C,D need to

be kept reasonably small in practical situations, and therefore desired angles and frequencies cannot always be arbitrarily closely approximated.

In practice, the tile will contain an integer number of device pixels. For simplicity assume the resolution is square with TS pixels in both the horizontal and vertical directions. Such a discrete space pattern that is tiled periodically has an exact representation in the Discrete Fourier Transform (DFT) and the circular convolution that it implies. Extension to rectangular tiles and/or rectangular pixels is elementary. The dot displacements given in Equation 1 can be expressed in pixel units simply by multiplying by TS and in general will not be integer. Therefore the halftone dots do not all lie in the same position relative to the pixel grid, which can lead to many design issues as discussed in the next section. Instead it is often the case that the halftone cell displacements are designed in the spatial pixel domain to avoid such problems. For example, if the desired displacements are known in pixels, setting $B = \Delta x_1$, $A = -\Delta y_1$, $D = \Delta x_2$, $C = -\Delta y_2$, and $TS = |AD - BC|$ will give such a square tile.

Aliasing Formulation of Automoiré.

Consider the following approach to halftone design. Using knowledge about the behavior of the printing system, the designer of the halftone chooses a desired dot shape for each input level. In continuous space this can be considered as an ideal halftone pattern. This pattern is then converted to a discrete space halftone by sampling the ideal pattern at pixels. See Figure 1. The spot function formulation for dot growth can generally be cast into this form.

In the case where the dot centers have integer pixel displacements, each halftone dot will be identical. However in the general case described in the previous section, they will not have the same shape or even the same area. In addition, these variations will occur in a regular manner with their own, often lower, frequency, causing them to be very visible. From this point these are referred to as non-harmonic components because they are not integer multiples of the screen fundamentals; they are of course periodic in the tile period.

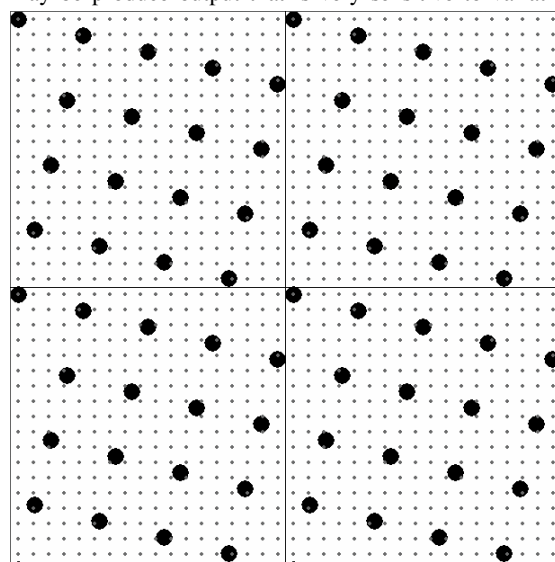
This phenomenon can be understood in signal processing terms as aliasing. Since the ideal halftone dot typically has significant harmonics above the Nyquist rate, they will be folded into the baseband and may appear as low frequency non-harmonic components. It is interesting to note that this is occurring even in the integer cell displacement case, but the aliasing is occurring on top of harmonic components, so only the dot shape is distorted but the pattern remains periodic in the fundamentals.

However, it is not mandatory to have integer dot displacement to avoid non-harmonic components. If continuous values are allowed in the discrete pattern and if the reconstruction of the pattern to continuous space is band-limited, a discrete

pattern can be created either by design or by complete anti-aliasing that will reconstruct to an image that only has components in the fundamental and harmonics below the Nyquist rate. In practice, the halftone values are quantized and bounded, so this may not be completely achievable, but the energy in the unwanted components can be reduced significantly.

A typical approach to multi-level halftoning can be considered as exactly such an attempt at anti-aliasing. Starting with a highly ($M \times N$) over-sampled ideal bi-level halftone pattern, the image is filtered with an $M \times N$ box filter (equivalent to counting the "on" pixels under the box) and sub-sampling to produce the values. It is convenient to choose $M \times N + 1$ to be equal to the number of desired levels. The strength of the non-harmonics can be significantly reduced over the bi-level case, but not eliminated. See Figure 2.

Other anti-aliasing filters are possible but practical considerations make them difficult to design. Namely, the halftone values must fit a specified dynamic range related to the maximum and minimum driving values in the printing system. Therefore, filters that are sharp in the frequency domain, like the ideal sinc filter, cannot be used because of ringing. Filters that are too smooth, like Gaussians, tend to make dots that are too fuzzy and may produce output that is very sensitive to variations in



printing system parameters.

Figure 1: The ideal halftone dot pattern at input level 243 of 255 for the (1,4),(-4,1) screen. Four tiles are shown. The lighter dots show the sampling grid for TS=18. It is apparent that the ideal halftone dots will not be sampled in the same relative positions.

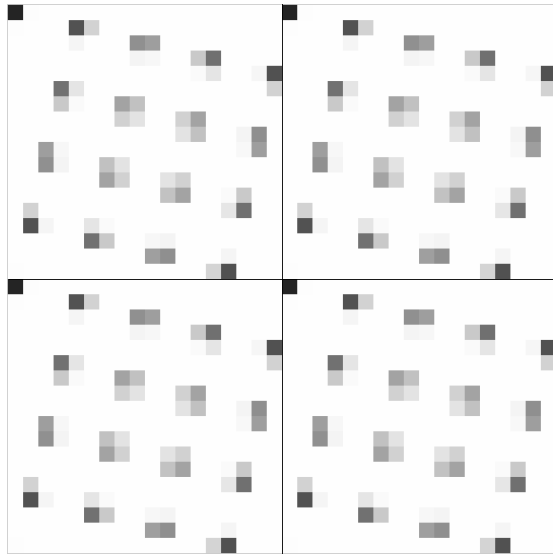


Figure 2: The discrete halftone pattern produced by applying a box filter to the ideal pattern in Figure 1 and sampling to $TS=18$. The low frequency automoiré is clearly visible.

Proposed Optimization Approach.

Instead of attempting to find optimal filters for anti-aliasing, a more direct approach to design the screen patterns is proposed here. By allowing the values of the halftone pattern on the tile to be the free variables in an optimization process, patterns are directly designed that are significantly free of non-harmonics while maintaining the ideal dot shape as closely as possible.

Let $v_k(i,j)$ be a $TS \times TS$ halftone pattern for the input level k on the pixels of the tile that will be periodically applied. When printing level k , the periodically replicated signal v will be used to drive the printing system, and, by nature of physical reality, will at some point be converted to continuous space. In electrophotography this is the optical exposure energy incident on the photosensitive drum. For this method it is required to have a reasonable model for this reconstruction process which is lumped into a transformation $H(\cdot)$, i.e. exposure energy $E(x,y) = H(v)$. In most EP contexts the reconstruction can be modeled as a nonlinear system mapping halftone values to a scanning signal followed by a convolution with the optical profile of the light source. The remainder of the printing system process $\phi(E(x,y))$ from exposure through development does not need to be modeled if the ideal halftone pattern $u_k(x,y)$ has been designed to be robust with respect to the process, as clustered dot screening of appropriately low frequencies are known to be. Now, the important point is that $\phi(\cdot)$ can be reasonably modeled by a point-wise, space-invariant nonlinear function to predict EP system output [4]. Therefore, if $E(x,y)$ could be made to be exactly periodic in $(A,B),(C,D)$ then the final predicted developed image will also be. However, if non-harmonic components are present, their effect may be amplified since the nonlinearity $\phi(\cdot)$ typically has high gain around the operating point. These non-harmonic components can be introduced by limitations (bounds and quantization) on the

halftone values and/or by reconstruction aliasing if $H(\cdot)$ is not sufficiently bandlimiting.

Given this framework, the goal is to find the discrete halftone pattern that produces an exposure as close to the ideal pattern as possible. Since it is more important to suppress non-harmonic components, especially those that are low-frequency and therefore visible, than preserve the exact dot shape, the difference between the ideal and obtained patterns is frequency weight accordingly by a transfer function Q . Combining these ideas, the goal of the optimization can be expressed:

$$\text{minimize } \|Q(H(v_k(i,j)) - u)\|^2 \quad (2)$$

$$\text{subject to: } 0 \leq v_k(i,j) \leq 1 \quad (3)$$

If $H(\cdot)$ is a linear operator the problem is convex and can be efficiently solved. An example solution is shown in Figure 3.

When designing screen patterns that are to be implemented as threshold arrays, the patterns should obey the stacking constraint. This can be incorporated into the optimization by using the bounds $v_{k1}(i,j) \leq v_k(i,j) \leq v_{k3}(i,j)$ where $k1 < k < k3$.

A more difficult practical concern to satisfy is the need for discrete levels in the halftone values. Such discrete optimization problems are much more difficult to solve. If a large enough number of levels are available (say 4-bit or more) the values $v_k(i,j)$ found in the continuous valued optimization can be quantized by rounding without introducing significant new errors. However, at 16 levels or less it is better to use an iterative procedure whereby chosen values of the continuous solution are fixed (e.g. those closest to a quantization level) and the remaining values are then re-optimized.

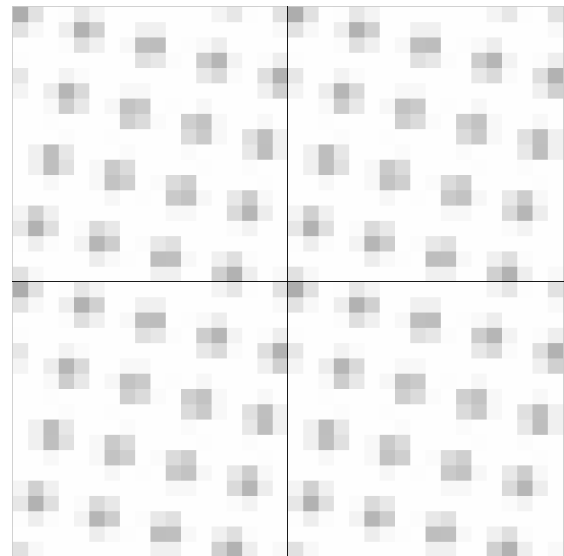


Figure 3: The discrete halftone pattern produced by the optimization approach applied to the ideal pattern in Figure 1.

Example Application

As a practical example, consider the orthogonal screen with fundamentals $(1,4), (-4,1)$ and $TS=18$ pixels. At 600 dpi this is approximately 137.44 lpi at 75.96 degrees. With $TS=17$ this is a commonly used screen since the halftone cells have integer displacement, but with $TS=18$ they do not. However, the $TS=18$ version has the nice property that when it is used for the C and M (reflected) colorants it is C-M-K moiré cancelled when K is the conventional $(3,3), (-3,3)$, $TS=18$ integer displacement screen, which at 600 dpi is approximately 141.42 lpi at 45 degrees.

When sampled at $TS=18$ pixels/tile the $(1,4), (-4,1)$ screen has a strong aliased components at $(\pm 1,0)$ $(0,\pm 1)$ which originate from the $(\pm 17,0), (0,\pm 17)$ harmonics. As shown in Figure 4 this component is present in the discrete screen patterns across the tone scale even when a box-filter is used to create a multi-level screen. When printed on a 600 dpi 4-bit LED engine this is manifested at many tone levels as a 33.33 lpi automoiré at 0 degrees, which is highly visible.

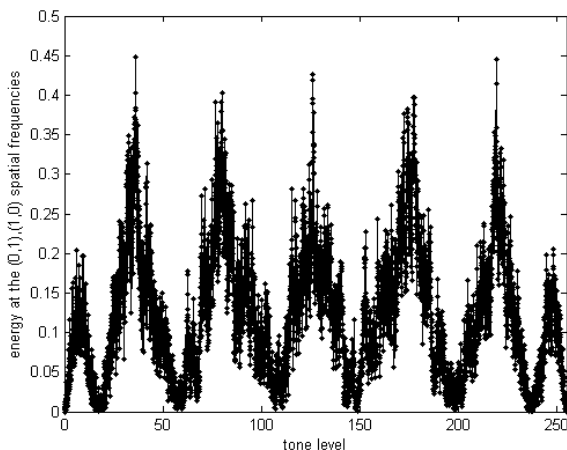


Figure 4. The energy at 33.33 lpi, 0 and 90 degrees for the 4-bit multilevel screen created using a box filter.

The optimization approach of the previous section was implemented in Matlab code and was used to create a 4-bit threshold array based also on this design. The $18 \times 18 = 324$ variables were first optimized for the middle tone level and quantized by the iterative optimization scheme. The patterns for all other levels were then determined by a recursive approach by dividing the remaining tone scale into halves and solving for the middle pattern with the appropriate constraints. The resulting patterns obey the stacking constraint and can be converted to a multi-level threshold array. As shown in Figure 5, the $(\pm 1,0), (0,\pm 1)$ components are significantly suppressed in the discrete screen patterns. More importantly, they were not visible in the printed output of the test engine.

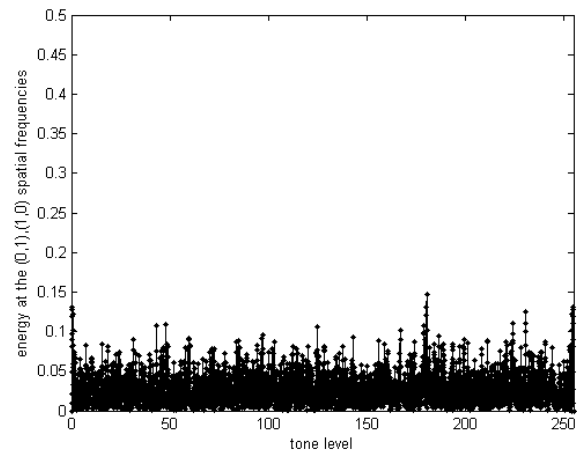


Figure 5: The energy at 33.33 lpi, 0 and 90 degrees for the 4-bit multilevel screen created by the optimization approach.

Conclusion

An optimization approach was presented for suppressing the automoiré in multi-level periodic screen patterns. A series of such patterns can be converted to multi-level threshold arrays if the pattern values are quantized and the stacking constraint is enforced during optimization. Even under the practical constraints of 4-bit halftone value quantization and the stacking constraint, the automoiré in the optimized screen was suppressed across the tone scale and was not visible in the resulting experimental printed output.

References

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Author Biography

Kenneth R. Crouse received his B.S. in Electrical Engineering from the University of Illinois Urbana-Champaign (1988) and his Ph.D. in Electrical Engineering from University of California, Berkeley (1997). He was a post-doc at U.C. Berkeley until 2000, when he joined Monotype Imaging Inc. He is currently a member of the Research Group working in areas of visual fidelity, anti-aliasing, and halftoning.