Fundamental Considerations related to Chromatic Adaptation

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Abstract

There remain some important issues related to the von Kries model which have not been clarified up until now. This article discusses fundamental mathematical characteristics related to one of the issues of the model. The important issue which have not been clarified and will be discussed in this paper is what is the conditions in the wavelength range for the best von Kries model fitting? Because of the multiplication between illuminant spectrums and object spectral reflectances, the conditions in the wavelength range give various resultant tristimulus values though the von Kries model is a model with only three parameters. In the removal of colored illuminants, the model fitting has uncertainty. As the result of this paper, it is shown that the simplest von Kries model provides the best chromatic adaptation for sets of illuminant spectrum and object spectral reflectance which generate the centroid tristimulus values in the tristimulus region corresponding to the uncertainty.

Introduction

Chromatic adaptation is among the most important phenomena of the human visual system. In 1902, von Kries postulated a model for the chromatic adaptation phenomena of the human visual system.

However, there remain some important issues related to the von Kries model which have not been clarified up until now. This article discusses fundamental mathematical characteristics related to one of the issues of the model which is the basis for all chromatic adaptation model. Improvements of psychophysical evaluation scores using such as non-linear model is another view point starting from the von Kries model, and will not be discussed in this article.

The important issue which has not been clarified and will be discussed in this paper is what is the conditions in the wavelength range for the best von Kries model fitting? Related to the issue, there have been indications in Ref. 1) by us using the results of Ref. 2), and in Ref. 3) by Dr. M. H. Brill who have published many historically important articles related to chromatic adaptation.

Because of the multiplication between illuminant spectrums and object spectral reflectances, the conditions in the wavelength range give various resultant tristimulus values (non unique in the same way as subtractive color mixture) though the von Kries model is a model with only three parameters. In the removal of colored illuminants, the model fitting has uncertainty.

As the result of this paper, it has been shown that the simplest von Kries model (the simple illuminant normalization) provides the best chromatic adaptation for sets of illuminant spectrum and object spectral reflectance which generate the centroid tristimulus values in the tristimulus region corresponding to the uncertainty. Although, the simplest model is called as a wrong model, the model is the best for sets of illuminant spectrum and object spectral reflectance satisfying the centroid from mathematical point of view.

Theoretical model

Notation

$$\langle s(\lambda)\rangle_\tau = \int I(\lambda)s(\lambda)\tau(\lambda)\,d\lambda = \int I^*(\lambda)\tau(\lambda)\,d\lambda, \tag{1.a}$$

$$\langle \rho(\lambda)\rangle_\tau = \int I(\lambda)\rho(\lambda)\tau(\lambda)\,d\lambda, \tag{1.b}$$

$$\begin{aligned}\langle s(\lambda)\rho(\lambda)\rangle_\tau &= \int I^*(\lambda)\rho(\lambda)\tau(\tau)\,d\lambda,\\ &= \int I(\lambda)s(\lambda)\rho(\lambda)\tau(\lambda)\,d\lambda,\end{aligned} \tag{1.c}$$

where

$\tau(\lambda) := \bar{x}(\lambda)$ *or* $\bar{y}(\lambda)$ *or* $\bar{z}(\lambda)$,

$I(\lambda)$: Illuminant spectral distribution 1,

$I^*(\lambda)$ $\left(= I(\lambda)s(\lambda)\right)$: Illuminant spectral distribution 2,

$s(\lambda)$ $\left(0 \le s(\lambda)\right)$: Illuminant operator,

$\rho(\lambda)$ $\left(0 \le \rho(\lambda) \le 1\right)$: spectral reflectance.

The notational Eq.(1) indicates that for the calculation scheme of tristimulus values $\int I^*(\lambda)\rho(\lambda)\tau(\lambda)d\lambda$ $\left(= \int I(\lambda)s(\lambda)\rho(\lambda)\tau(\lambda)d\lambda\right)$, $s(\lambda)$ and $\rho(\lambda)$ corresponds to $\rho_1(\lambda)$ and $\rho_2(\lambda)$ in the discussions related to subtractive color mixture (References 2, 4 through 8)), respectively. Illuminant operator converts from $I(\lambda)$ to $I^*(\lambda)$. $I(\lambda)$ implies an illuminant before adaptation and $I^*(\lambda)$ implies an illuminant after adaptation.

In this paper, the notation $\rho(\lambda)$ is employed for all color types, and the following discussions are consistent not depending on color types. The following color types are defined in $\rho(\lambda)$ for Eq.(5).

Ideal color $\rho_i^{(I)}(\lambda)$: $\rho_i^{(I)}(\lambda)$ $\left(i\in\{1,2,\cdots,n\}=I_n\right)$ are assumed to either perfectly block or perfectly pass (take only 0 or 1 values), and the restriction of the four types of ideal color can be removed in our discussions.

General color $\rho_i^{(G)}(\lambda)$: $\rho_i^{(G)}(\lambda)$ takes any value in $[0,1]$,

Realistic color $\rho_i^{(R)}(\lambda)$: correlations between $\rho_i^{(R)}(\lambda)$ related to λ are posed on general color.

Tristimulus values

J denotes tristimulus values of X or Y or Z corresponding to $\bar{x}(\lambda)$ or $\bar{y}(\lambda)$ or $\bar{z}(\lambda)$, respectively. Define tristimulus values as follows:

$$J_{I^*}=\langle s(\lambda)\rangle_\tau, \tag{2.a}$$

$$J_{obj}=\langle \rho(\lambda)\rangle_\tau. \tag{2.b}$$

Eq.(2.a) corresponds to the tristimulus values of the illuminant $I^*(\lambda)$ derived from $I(\lambda)$ operated by $s(\lambda)$. Eq.(2.b) corresponds to the tristimulus values of an object of $\rho(\lambda)$ under illuminant of $I(\lambda)$. J_I denotes tristimulus values of the illuminant $I(\lambda)$. Equations (2.a) and (2.b) are the defining constraint of the whole problem, and J_{I^*} and J_{obj} are inputs that define the constraints.

Normalization

The following normalizations are performed so Y stimulus value corresponds to 100.0.

$$J_{I^*,norm}=100.0\langle s(\lambda)\rangle_\tau/\langle s(\lambda)=1\rangle_{\bar{y}}, \tag{3.a}$$

$$J_{obj,norm}=100.0\langle \rho(\lambda)\rangle_\tau/\langle \rho(\lambda)=1\rangle_{\bar{y}}. \tag{3.b}$$

For simplicity of the conventional definition of J_{norm} devolves to $J_{I^*}=\langle s(\lambda)\rangle_\tau$ and $J_{obj}=\langle \rho(\lambda)\rangle_\tau$.

The following theorems are provided.

[Theorem 1]

Posit a random ensemble of pairs $\{s(\lambda),\rho(\lambda)\}$ such that $\langle s(\lambda)\rangle_\tau=J_{I^*}$ and $\langle \rho(\lambda)\rangle_\tau=J_{obj}=J_I/2$. Then the centroid of $\langle s(\lambda)\rho(\lambda)\rangle_\tau$ is as follows:

$$E\left[\langle s(\lambda)\rho(\lambda)\rangle_\tau\right]=J_{I^*}J_{obj}/J_I, \tag{4}$$

where

E : expectation which calculates the centroid.

Proof

In APPENDIX.

Theorem 2 describes the boundary conditions for the centroid formula.

[Theorem 2]

For $J_{obj}=0$, the centroid of $\langle s(\lambda)\rho(\lambda)\rangle_\tau$ is 0.0, and for $J_{obj}=J_I$, the centroid of $\langle s(\lambda)\rho(\lambda)\rangle_\tau$ is J_{I^*}, and for both cases, the centroid is described in the form of $J_{I^*}J_{obj}/J_I$.

Proof

In APPENDIX.

Combining Theorem 1 ($J_{obj}=J_I/2$ (middle)) with the boundary conditions of $J_{obj}=0$ (minimum) and $J_{obj}=J_I$ (maximum) in Theorem 2, an approximation formula of the centroid has been constructed.

In Refs. 4) through 8), the optimal color theory by Schrodinger has been extended into subtractive color mixture. The extension can be applied in the following theorem 3.

[Theorem 3]

The following inequalities are consistent.

$$\langle s(\lambda)\rho_R(\lambda)\rangle_{\tau,\max}\le\langle s(\lambda)\rho_G(\lambda)\rangle_{\tau,\max}\le\langle s(\lambda)\rho_I(\lambda)\rangle_{\tau,\max} \tag{5.a}$$

$$\langle s(\lambda)\rho_I(\lambda)\rangle_{\tau,\min}\le\langle s(\lambda)\rho_G(\lambda)\rangle_{\tau,\min}\le\langle s(\lambda)\rho_R(\lambda)\rangle_{\tau,\min} \tag{5.b}$$

Proof

Can be proven as an extension of Refs. 4) through 8).

Analysis of Von Kries model using the theorems

Theorems 1 and 2 are applied to analysis of the von Kries model. The following equation in Theorems 1 and 2 is transformed as follows:

$$E\left[\langle s(\lambda)\rho(\lambda)\rangle_\tau\right]=E\left[\sum I(\lambda)s(\lambda)\rho(\lambda)\tau(\lambda)\right]\cong J_{I^*}J_{obj}/J_I. \tag{6}$$

Eq.(6) is transformed as follows:

$$E\left[\sum I^*(\lambda)\rho(\lambda)\tau(\lambda)\right]\cong J_{obj}J_{I^*}/J_I. \tag{7}$$

The right side of J_I, J_{I^*} in Eq.(7) are moved to the left side of Eq.(7), and Eq.(8) is derived.

$$J_I \cdot E\left[\sum I^*(\lambda)\rho(\lambda)\tau(\lambda)\right] / J_{I^*} \cong J_{obj}\ . \qquad (8)$$

Assume that $I_c^*(\lambda)$ and $\rho_c(\lambda)$ represent a set of illuminant spectrum and object spectral reflectance satisfying the centroid coordinate condition. Then Eq.(8) is represented using the notations as follows:

$$J_I \cdot \left\{\sum I_c^*(\lambda)\rho_c(\lambda)\tau(\lambda)\right\} / J_{I^*} \cong J_{obj}\ . \qquad (9)$$

Eq.(9) is just the ratio model using the ratio between two illuminant coordinates which is the most primitive model in the von Kries type models. Eq.(9) derives the solution of the chromatic adaptation for the illuminant spectrum $I_c^*(\lambda)$ and the object spectral reflectance $\rho_c(\lambda)$ corresponding to the centroid coordinate.

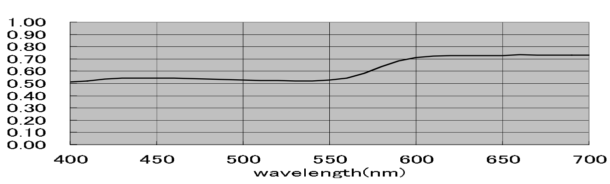
Numerical illustrations and considerations

Figures 1 No.(1) through No.(8) show spectral reflectances employed in this numerical illustrations. These are JIS standard color patches (Munsell color patches) indexed by H , V , C values. In this illustrations, the chromatic adaptation predicts from under A illuminant to $D65$ illuminant.

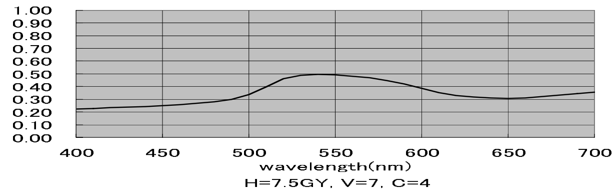
Table 1 shows the theoretical centroid in the right side of Eq.(4), and $\langle s(\lambda)\rho(\lambda)\rangle_\tau$ for each case. These are plotted in Figures 2(a)(b).

No.(1) through No.(4) are cases that the resultant stimulus values (adapted stimulus values) are on the centroid (Eq.(9) is consistent), and the simplest von Kries model is the best fitting for the prediction. On the other hand, No.(5) through No.(8) are cases that the resultant stimulus values are not on the centroid, and Eq.(9) is not consistent.

From the inequalities of Eq.(5), this is explained that as approaching to ideal color (No.(5) through No.(8)), the existable min-max bound is enlarged, and the probabilities not on the centroid is increased.



(1)



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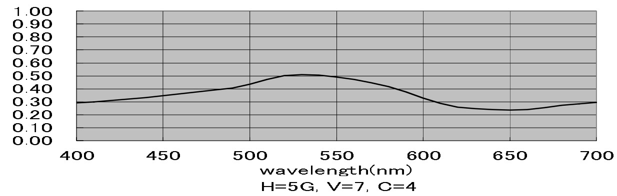

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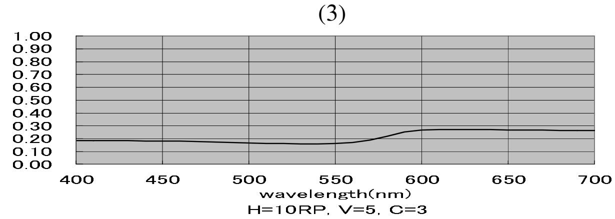

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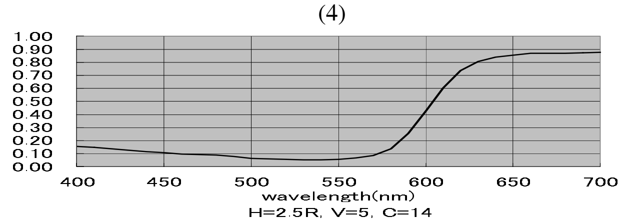

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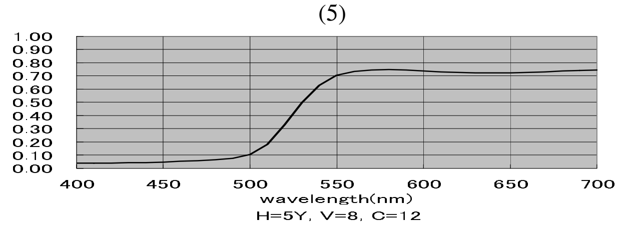

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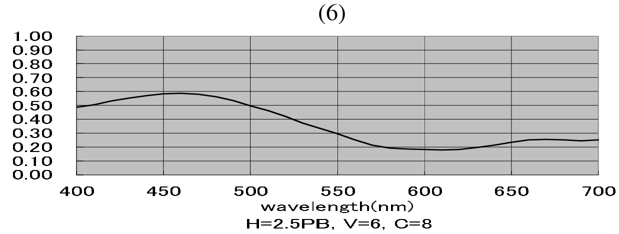

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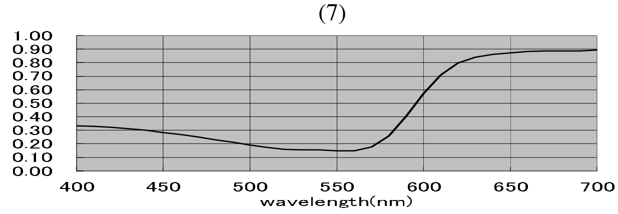
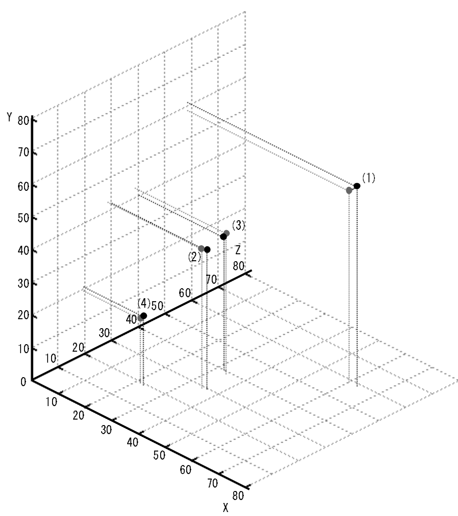


(8)

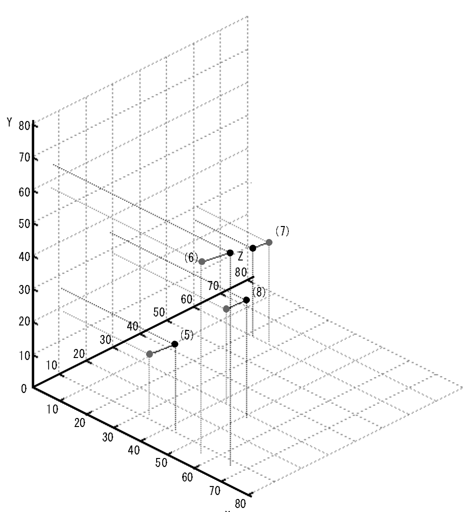
Figure 1 Spectral reflectances.

Table 1

No.	HVC			Centroid			$\langle S(\lambda)\rho(\lambda)\rangle$		
	H	V	C	X	Y	Z	X	Y	Z
(1)	2.5R	8	3	63.49	61.26	58.32	60.31	58.58	58.41
(2)	7.5GY	7	4	36.00	42.29	29.54	35.42	42.90	28.44
(3)	5G	7	4	32.14	40.58	39.92	33.96	42.92	38.83
(4)	10RP	5	3	22.77	20.89	19.34	21.21	19.48	19.48
(5)	2.5R	5	14	42.35	25.93	10.80	32.05	18.55	11.31
(6)	5Y	8	12	65.68	64.60	7.89	56.29	58.31	6.57
(7)	2.5PB	6	8	21.44	26.38	60.36	26.75	30.06	61.02
(8)	7.5RP	6	12	51.12	35.85	28.44	42.40	28.95	29.53



(a)



(b)

Figure 2 Theoretical centroid coordinate (black circle) and adapted coordinate $\langle s(\lambda)\rho(\lambda)\rangle_{\tau}$ (gray circle).

Although, the main results of this paper are the centroid formula and its relation to the simplest von Kries model, considerations are provided in relation to psychophysical experiments.

In the existing psychophysical experiments, there has been no theoretical background for the selection of spectral reflectances in the experiments. Selection of spectral reflectances derives the best fitting model from the theoretical point of view. Considering the model structure of the von Kries model, also psychophysical experiments will be improved from theoretical point of view.

After the discussions above, differences that remain between the human visual system and chromatic adaptation models should be investigated.

Conclusions

There remain some important issues related to the von Kries model which have not been clarified up until now. An important issue discussed in this paper is what is the conditions in the wavelength range for good von Kries model fitting? In this paper, our theorems derived for subtractive color mixture problems have been applied to the important issue related to the von Kries model. The theorems applied are for the centroid calculation and for the minimum and the maximum bounds calculation.

As the result of this paper, it has been shown that Eq.(9) which is the simplest model in the von Kries type models derives the solution of the chromatic adaptation for the combination of illuminant spectrum and object spectral reflectance corresponding to the centroid coordinate.

Numerical illustrations were provided for concrete explanations in which not only the centroid but also the minimum and the maximum bounds have been employed. Further theoretical analysis between the centroid and figures of spectral reflectances will be performed in our future presentations.

The mathematical structure of the von Kries model revealed in this paper will serve to analyze the essence of the set of various von Kries type models. Also psychophysical experiments will be improved from theoretical point of view.

References

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7)N. Matsushiro, Theoretical Analysis of Subtractive Color Mixture Characteristics IV, Color Research and Application, Vol.30, No.6, pp.427-437, 2005.

8)N. Matsushiro and N. Ohta, Theorem and Formula of Subtractive Color Mixture, AIC05, May, 2005.

APPENDIX

Through the following sequence, the main Theorems 1 and 2 are proven.

[PROPERTY 1]

Define $\rho_c(\lambda)=1-\rho(\lambda)$. So $\langle s(\lambda)\rho(\lambda)\rangle_\tau = J$ and $-\langle s(\lambda)\rho(\lambda)\rangle_\tau = -J$ are on inside out relation, and J_{I^*} is a fixed offset value that $\langle s(\lambda)\rho(\lambda)\rangle_\tau = J$ and $\langle s(\lambda)\rho_c(\lambda)\rangle_\tau = J_{I^*}-J$ are on inside out relation, and take the same length of the min-max range of J . Therefore, there exists one to one correspondence between $\langle s(\lambda)\rho(\lambda)\rangle_\tau = J$ and $\langle s(\lambda)\rho_c(\lambda)\rangle_\tau = J_{I^*}-J$ in the min-max range.

[PROPERTY 2]

Indicate a pair of $s(\lambda)$ and $\rho(\lambda)$ as $(s(\lambda),\rho(\lambda))$. Because of the metamerism, there exist plural number of $(s(\lambda),\rho(\lambda))$ satisfying $\langle s(\lambda)\rho(\lambda)\rangle_\tau = J$ for a given J value. For one of $(s(\lambda),\rho(\lambda))$ satisfying $\langle s(\lambda)\rho(\lambda)\rangle_\tau = J$, there exists the corresponding $(s(\lambda),\rho_c(\lambda))$ satisfying $\langle s(\lambda)\rho_c(\lambda)\rangle_\tau = J_{I^*}-\langle s(\lambda)\rho(\lambda)\rangle_\tau = J_{I^*}-J$ in the metamerism of a given J value.

[PROPERTY 3]

For $J = J_I/2$, the relation of $\{\rho(\lambda)\}=\{\rho_c(\lambda)\}$ is consistent, where { } indicates a set.

[LEMMA 1]

Assume that $\{s(\lambda),\rho(\lambda)\}$ indicates a set of $(s(\lambda),\rho(\lambda))$. Posit a random ensemble of pairs $\{s(\lambda),\rho(\lambda)\}$. The centroid of $\{\langle s(\lambda)\rho(\lambda)\rangle_\tau + \langle s(\lambda)\rho_c(\lambda)\rangle_\tau\}/2$ is $J_{I^*}/2$.

[Proof]

Let the ensemble represent equally probable states, each state is characterized by a particular pair of $s(\lambda)$ and $\rho(\lambda)$. Divide all the allowed $\rho(\lambda)$ realizations into pairs of $\rho(\lambda)$ and $\rho_c(\lambda)$ by choosing a $\rho_1(\lambda)$, computing its $\rho_{1c}(\lambda)$, choosing another $\rho_2(\lambda)$ that is not equal to either $\rho_1(\lambda)$ or $\rho_{1c}(\lambda)$, and continuing in this way until exhausting all the realizations of $\rho(\lambda)$. From PROPERTY 2, for a value of J satisfying $\langle s(\lambda)\rho(\lambda)\rangle_\tau = J$, there exist the number of pairs corresponding to the number of metamers of J (intra-pairs). For variations of J in the min-max range, inter-pairs exist in the symmetrical relation described in PROPERTY 1. For the inter-pairs, the exhausting process can be finished with complete pairs.

Now consider a pair of realizations $(s(\lambda),\rho(\lambda))$ and $(s(\lambda),\rho_c(\lambda))$ both with the same realization $\rho(\lambda)$. The following equation is valid for $\langle s(\lambda)\rho_c(\lambda)\rangle_\tau$.

$$\langle s(\lambda)\rho_c(\lambda)\rangle_\tau = \langle \rho(\lambda)\rangle_\tau - \langle s(\lambda)\rho(\lambda)\rangle_\tau . \qquad \text{(A.1)}$$

Compute the centroid of $\{\langle s(\lambda)\rho(\lambda)\rangle_\tau + \langle s(\lambda)\rho_c(\lambda)\rangle_\tau\}/2$ by first averaging $\langle s(\lambda)\rho(\lambda)\rangle_\tau$ and $\langle s(\lambda)\rho_c(\lambda)\rangle_\tau$ for the pair as follows:

$$\begin{aligned}&\{\langle s(\lambda)\rho(\lambda)\rangle_\tau + \langle s(\lambda)\rho_c(\lambda)\rangle_\tau\}/2\\ &=\{\langle s(\lambda)\rho(\lambda)\rangle_\tau + \langle s(\lambda)\rangle_\tau - \langle s(\lambda)\rho(\lambda)\rangle_\tau\}/2 \qquad \text{(A.2)}\\ &= J_{I^*}/2 .\end{aligned}$$

And then the average of the intra-pairs for a given value of J is derived as $J_{I^*}/2$. The averaging all the intra-pair averages gives the inter-pair average $(=J_{I^*}/2)$ for all possible J which corresponds to the centroid.

$$E\left[\{\langle s(\lambda)\rho(\lambda)\rangle_\tau + \langle s(\lambda)\rho_c(\lambda)\rangle_\tau\}/2\right] = J_{I^*}/2 . \qquad \text{(A.3)}$$

where

E : expectation which calculates the centroid.

[LEMMA 2]

For $J_{obj} = J_I/2$,

$$\left\{E\left[\langle s(\lambda)\rho(\lambda)\rangle_\tau\right] + E\left[\langle s(\lambda)(1-\rho(\lambda))\rangle_\tau\right]\right\}/2 = E\left[\langle s(\lambda)\rho(\lambda)\rangle_\tau\right]. \qquad \text{(A.4)}$$

Proof

$\{\rho(\lambda)\}=\{1-\rho(\lambda)\}$ in PROPERTY 3 under the condition of the lemma derives the following equation.

$$E\left[\left\langle s(\lambda)\rho(\lambda)\right\rangle_\tau\right] = E\left[\left\langle s(\lambda)\left(1-\rho(\lambda)\right)\right\rangle_\tau\right], \qquad \text{(A.5)}$$

and the lemma is proven.

[Theorem 1]

Posit a random ensemble of pairs of $\{s(\lambda), \rho(\lambda)\}$ such that $\left\langle \rho(\lambda)\right\rangle_\tau = J_{obj} = J_I / 2$ and $\left\langle s(\lambda)\right\rangle_\tau = J_{I^*}$. Then the centroid of $\left\langle s(\lambda)\rho(\lambda)\right\rangle_\tau$ is as follows:

$$E\left[\left\langle s(\lambda)\rho(\lambda)\right\rangle_\tau\right] = J_{I^*} J_{obj} / J_I . \qquad \text{(A.6)}$$

[Proof]

Using the LEMMAs 1 and 2, and the relation of $J_{obj} = J_I / 2$, the centroid is derived as follows:

$$E\left[\left\langle s(\lambda)\rho(\lambda)\right\rangle_\tau\right] = J_{I^*} / 2 = J_{obj} J_{I^*} / J_I . \qquad \text{(A.7)}$$

Theorem 2 describes the boundary conditions for the approximation in Theorem 3.

[Theorem 2]

For $J_{obj} = 0.0$, the centroid of $\left\langle s(\lambda)\rho(\lambda)\right\rangle_\tau$ is 0.0, and for $J_{obj} = J_I$, the centroid of $\left\langle s(\lambda)\rho(\lambda)\right\rangle_\tau$ is J_{I^*}, and for both cases, the centroid is described in the form of $J_{I^*} J_{obj} / J_I$.

Proof

In the case of $J_{obj} = 0.0$, $\rho(\lambda)$ is always 0.0 and the centroid of $\left\langle s(\lambda)\rho(\lambda)\right\rangle_\tau$ becomes 0.0. In the case of $J_{obj} = J_I$, $\rho(\lambda)$ is always 1.0 and the controid of $\left\langle s(\lambda)\rho(\lambda)\right\rangle_\tau$ becomes J_{I^*}. The both centroids satisfy the equation of $J_{I^*} J_{obj} / J_I$.

Biography

Nobuhito Matsushiro received his PhD degree (Information engineering) from University of Electro. Communications, Tokyo, Japan, in 1996 and PhD degree (Color science) from Chiba University in 2006. He is a PhD (Neuro and Brain Science) candidate at School of Medicine, Chiba University, Chiba, Japan. He works for Oki Electric. Co. Ltd., Printing Company Oki Data.