# Estimation of developed toner mass in two-component electrophotographic system by large-scale Discrete Element Method

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#### **Abstract**

The developing behavior of toner particles in a twocomponent development system was simulated to find out the correlation between experimental and simulated results by using large-scale Discrete Element Method (DEM). The effects of developing bias frequency or circumferential speed of photoreceptor on the developed toner mass per area (DMA) was discussed both in experimental and simulation works. The toners were adhered on the outside of latent images under the low frequency, and its mass decreased with increasing the frequency. DMA decreased with increasing circumferential speed of photoreceptor, because the toners had less chances to contact the latent image. The image was also damaged by the magnetic brush under the high speed due to an increase of the shear force. The similar tendency was observed under all conditions, and it was found that the simulation results correlate with experimental ones very well. Therefore, this simulation method is useful for the analysis of the developing behavior in electrophotographic system and the optimization or designing of this system will be possible.

# Introduction

A two-component electrophotographic system is one of the printing machines, and widely used for printer, copy machine and facsimile. Demand and requests for printing technology (e.g. printing speed, high quality, low cost, high stability, energy saving, ecological and so on) are increasing with rapid development of information technologies in past dozen years. To satisfy these requests, the grasp of the phenomenon found in the system is very important and it is absolutely imperative to control the developing behavior of toners. However, it is difficult to observe it in the experimental work because the diameter of toner particle becomes smaller and smaller and the inside of the development system becomes complicated due to the downsizing or colorization of electrophotographic system.

Discrete Element Method (DEM) [1] is one of the most famous and reliable simulation methods for the analysis of solid particle behavior, and there are many researches on powder processing [2,3]. This method is applicable for the analysis of developing behavior in the two-component development system, and some studies on the modeling of magnetic brush have been already reported [4-6]. However, it has been hard to simulate both carrier and toner particles in DEM because of its calculation load. The particle size ratio between them is about 10, and the toner concentration is around 5.0 wt%. So that, the huge number of

toner particles is needed in the simulation of two-component development system. The authors had achieved speeding up of DEM calculation by optimizing the particle detection [7-9]. This high-speed DEM algorithm has high potential to simulate the developing behavior in detail [10].

In this paper, the developing behavior in the two-component development system was simulate to find out the correlation between experimental results and simulated ones by using large-scale DEM. The effect of developing bias frequency or circumferential speed of photoreceptor on the developed toner mass was discussed both in experimental and simulation works.

## **Simulation**

The three-dimensional developing behavior in the two-component development system was simulated by using the large-scale Discrete Element Method (DEM). DEM is one of the most popular and reliable simulation methods for the numerical analysis of solid particle behavior. This simulation method consists of the idea of determining the kinematic force to each finite-sized particle. All forces acting on each particle are modeled and calculated at every discrete-time step. The trajectories of particles are updated by Newton's law of motion, as the following equations.

$$\dot{\mathbf{v}} = \frac{\sum_{m_{g}} \mathbf{F}}{m_{g}} \tag{1}$$

$$\dot{\mathbf{o}} = \frac{\sum \mathbf{M}}{I} \tag{2}$$

Where,  $\mathbf{v}$  is the particle velocity,  $\mathbf{F}$  is the summed force acting on a particle,  $m_{\rm g}$  means the mass of a particle,  $\boldsymbol{\omega}$  is the angular velocity,  $\mathbf{M}$  and I denote the moment of force and the moment of inertia. The contact force,  $\mathbf{F}_{\rm CT}$ , magnetic force,  $\mathbf{F}_{\rm M}$ , Coulomb force,  $\mathbf{F}_{\rm C}$ , electrostatic force,  $\mathbf{F}_{\rm E}$ , van der Waals force,  $\mathbf{F}_{\rm v}$ , and gravitational force,  $\mathbf{F}_{\rm G}$ , act on carrier or toner particles in the two-component development system, and all forces are summed.

$$\mathbf{F}_{\text{carrier}} = \mathbf{F}_{\text{CT}} + \mathbf{F}_{\text{M}} + \mathbf{F}_{\text{C}} + \mathbf{F}_{\text{v}} + \mathbf{F}_{\text{G}}$$
(3)

$$\mathbf{F}_{\text{toner}} = \mathbf{F}_{\text{CT}} + \mathbf{F}_{\text{C}} + \mathbf{F}_{\text{E}} + \mathbf{F}_{\text{v}} + \mathbf{F}_{\text{G}}$$
 (4)

#### Contact force

A contact model between two particles is given by the Voigt model, which consists of a spring-dashpot and a slider for the friction in the tangential component. The contact forces,  $F_{\rm n}$ , compressive and  $F_{\rm t}$ , shear, can be calculated by the following equations.

$$F_n = K\Delta u_n + \eta_n \frac{\Delta u_n}{\Delta t} \tag{5}$$

$$F_{t} = \min \left\{ \mu_{f} |F_{n}|, K_{t} \left( \Delta u_{t} + r\varphi \right) + \eta_{t} \left( \frac{\Delta u_{t} + r\varphi}{\Delta t} \right) \right\}$$
 (6)

Where, K and  $\eta$  mean the spring and the damping coefficients. u and  $\varphi$  are a relative displacement of the gravitational center between two particles and a relative angular displacement. r is a particle radius and  $\mu_f$  is the frictional coefficient. The subscript n and n denote the normal and tangential components.

## Magnetic force

Carrier particles in the magnetic field caused by the magnet roll form the magnetic brush on the sleeve. The magnetic force is calculated from Eq. (7) [11].

$$\mathbf{F}_{\mathbf{M},i} = (\mathbf{m}_i \cdot \nabla) \mathbf{B}_i \tag{7}$$

$$\mathbf{B}_{i} = \mathbf{B}_{i,filed} + \sum_{j=1(\neq i)}^{n} \mathbf{B}_{i,j}$$
 (8)

$$\mathbf{B}_{i,j} = \frac{\mu_0}{4\pi} \left[ \frac{3(\mathbf{m}_j \cdot \mathbf{R}_{ji})}{|\mathbf{R}_{ji}|^5} \mathbf{R}_{ji} - \frac{\mathbf{m}_j}{|\mathbf{R}_{ji}|^3} \right]$$
(9)

$$\mathbf{m}_{i} = \frac{4\pi}{\mu_{0}} \frac{\mu_{r} - 1}{\mu_{r} + 2} r_{i}^{3} \mathbf{B}_{i} \tag{10}$$

Where,  $\mathbf{m}_i$  and  $\mathbf{B}_i$  mean the magnetic dipole moment and the magnetic flux density of the *i*-th particle, respectively.  $\mathbf{B}_{i,j}$  is the magnetic flux density caused by the magnetized *j*-th particle.  $\mu_0$  is the magnetic permeability of vacuum (=12.57×10<sup>-7</sup> A/m²) and  $\mathbf{R}_{ji}$  is the position vector from the *j*-th particle to the *i*-th one. The magnetic dipole moment is defined by Eq. (10). Although  $\mathbf{B}_i$  should be used the summed value of the magnetic flux density from magnet roll and the ones from magnetized particles (Eq. (8)),  $\mathbf{B}_i$  was assumed to be the value of the magnetic flux density from magnet roll due to its convergent calculation load.  $\mu_r$  means the relative magnetic permeability.

#### Coulomb force

Coulomb force,  $\mathbf{F}_{\mathbf{C}}$ , acts on toner/toner or toner/carrier particles, because the toner has triboelectric charge, which is caused by agitating in a developer tank.  $\mathbf{F}_{\mathbf{C}}$  is calculated from the following equation.

$$\mathbf{F}_{\mathrm{C},ij} = \frac{1}{4\pi\varepsilon} \frac{q_i q_j}{\left|\mathbf{R}_{ii}\right|^2} \frac{\mathbf{R}_{ji}}{\left|\mathbf{R}_{ji}\right|} \tag{11}$$

Where,  $\varepsilon$  is the permittivity of air (=8.86×10<sup>-12</sup> F/m) and q denotes charge of toner. The carrier particle has the opposite charge of toner on the surface.

## Electrostatic force

The toner particles, which were delivered to the development area, fly to latent images by the electrostatic force,  $\mathbf{F_E}$ . A toner having q in charge receives  $\mathbf{F_E}$  in the electric field,  $\mathbf{E}$ .

$$\mathbf{F}_{\mathrm{E}} = q\mathbf{E} \tag{12}$$

$$\mathbf{E} = -\nabla \phi \tag{13}$$

 $\phi$  is the electric potential distribution and given by Eq. (14).

$$\nabla^2 \phi = -\frac{\rho}{\varepsilon} \tag{14}$$

Where,  $\rho$  denotes the charge density of the development area. It was assumed that the magnetic brush and the charge of toners don't affect the electric potential (i.e.,  $\rho = 0$ ) because it costs huge calculation load at every time step.

### van der Waals force

The van der Waals force,  $\mathbf{F}_{\mathbf{v}}$ , between *i*-th and *j*-th particles of  $d_i$  and  $d_i$  in diameter is given by the following equation.

$$\mathbf{F}_{v,i} = -\frac{\sqrt{A_i A_j}}{12h^2} \frac{d_i d_j}{d_i + d_j} \frac{\mathbf{R}_{ji}}{|\mathbf{R}_{ji}|}$$
(15)

Where, A demotes the Hamaker constant and h is a surface distance between the particles. When particles are in contact, h is determined to be 0.4 nm by considering the Born repulsion.

## Simulation conditions

5000 carrier particles having 50 or 70  $\mu m$  in diameter and mono-sized toner particles with 10 µm were used in the simulation of the two-component development system. The densities of carrier and toner particles were 3600 and 1100 kg/m<sup>3</sup>, respectively. It was assumed that all toner particles had the same charge (-25 μC/g) at the center of the particle, and the charge wasn't distributed on the toner particle surface and the charge transfer wasn't considered during the simulation. The number of toners was 50000 (1.31wt%). The developing bias frequency or the circumferential speed of the photoreceptor were changed from 0 to 8.0 kHz or  $0.50 \sim 1.44 \text{ m/s}$  to investigate the effect on the developed toner mass. The dot latent images were formed on the photoreceptor in the analysis of the effect of bias frequency, and the line latent image was formed when the developed mass was compared to experimental results. The electric potentials of photoreceptor or latent image were set at -700V or -50V, respectively. The calculations were executed in a workstation, which had AMD Opteron<sup>TM</sup> Processor 1.8GHz and 2.0 Gbyte of RAM.

## **Results and Discussions**

Figure 1 shows the snapshots of developing behavior of toner particles in the two-component development system. In this figure, carrier or toner particles were colored gray or pink. It was observed that the toners were developed to the latent images on the photoreceptor as time progresses, and this simulation is possible for the evaluation of developing characteristics.

Figure 2 shows the snapshots of developed toner on the photoreceptor under the different developing bias frequency (0.5  $\sim$  4.0 kHz). The toners were scattered under the low bias frequency, on the other hand, the clear dot images are developed under the high frequency. Figure 3 shows the snapshots of dot images in the simulation work. Lots of toner particles were adhered on the outside of latent images. The similar tendency was seen in the simulated results. Figure 4 shows the relation between the adhered toner mass on the outside of latent images and the developing bias frequency in the simulation work. The toner mass decreased with an increase of bias frequency, and it becomes constant at more than 2.0 kHz.

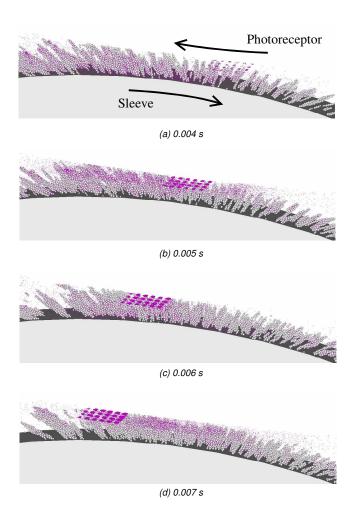


Figure 1. Snapshots of developing behavior of toner particles.

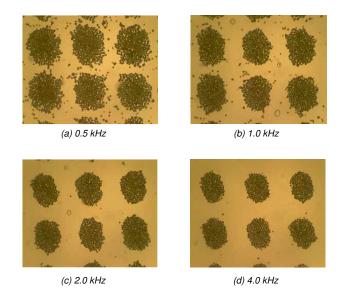


Figure 2. Dot images on photoconductor in the experimental work.

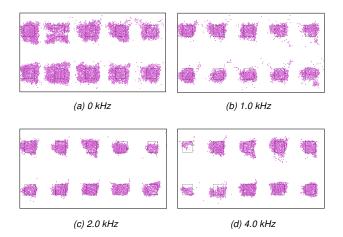


Figure 3. Dot images on photoconductor in the simulation work.

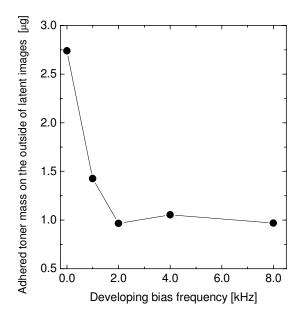


Figure 4. Relation between the adhered toner mass on the outside of latent images and developing bias frequency.

Figure 5 shows the relation between the developed mass per area (DMA) and the circumferential speed of photoreceptor for the experimental and simulated results. It was found that DMA decreased with increasing photoreceptor speed. The toners had less chances to contact the latent image, because the image passed though the development nip quickly under the high speed. The scavenging effect also seems to be another reason for decreasing DMA. Figure 6 shows the snapshots of developed toner on latent image under the different photoreceptor speed. The developed images were damaged by the magnetic brushes and the scratch of image becomes clearly with increasing the speed. Figure 7 shows the relation between the mean shear force acting on the photoreceptor and the circumferential speed. The mean shear force

under the condition of high speed is larger than that of low speed because the relative speed between the photoreceptor and the magnetic brush increases, and it leads serious scavenging effect on the developed image.

Similar results were observed both in experimental and simulation works under different conditions. Although the particle size, toner charge, its concentration and electric field were simplified in this analysis due to the calculation load, it would be concluded that this simulation results correlates with experimental ones. The further progresses can be expected in the future, and this simulation method is useful for the analysis of the developing behavior in the electrophotographic system and the optimization or designing of this kind of system by using the simulation will be possible.

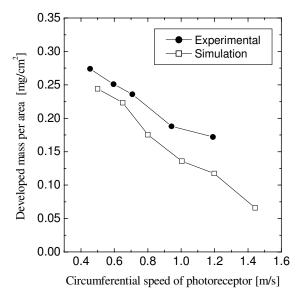


Figure 5. Relation between developed mass per area and circumferential speed of photoreceptor.

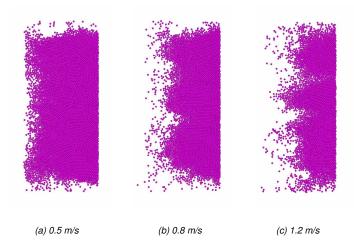


Figure 6. Snapshots of developed image under different photoreceptor speed.

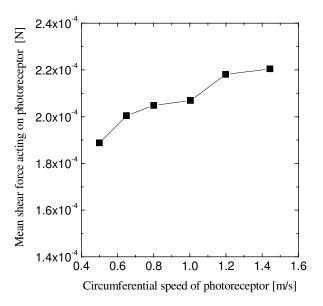


Figure 7. Relation between mean shear force acting on photoreceptor and circumferential speed.

#### **Conclusions**

The developing behavior in the two-component development system was simulated by using large-scale Discrete Element Method (DEM) to find out the correlation between the experimental and simulated results. The followings are summaries of this work.

- This large-scale DEM algorism made the analysis of the developing behavior in the two-component development system possible.
- ii) The toners were scattered under the low developing bias frequency, on the other hand, the clear dot images are developed under the high frequency. The adhered toner mass on the outside of latent images decreased with an increase bias frequency, and it becomes constant at more than 2.0 kHz.
- iii) DMA decreased with increasing circumferential speed of the photoreceptor, because the toners had less chances to contact the latent image. The developed image was damaged by the magnetic brush under the high speed condition because it receives large shear force from the brushes.
- iv) The similar tendencies were observed both in experimental and simulation works, and this simulation results correlates with experimental ones, although there were some assumptions. This large-scale DEM made the evaluation of developing efficiency possible. Therefore, this simulation method is useful for the analysis of the developing behavior in electrophotographic system and the optimization or designing of this system will be possible in the future.

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# **Author Biography**

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