Optimal Time-Sequential Sampling of the Color Reproduction Characteristic Function

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Abstract

This paper describes the design of an optimal time-sequential sampling (OTSS) sequence to sample the color reproduction characteristic (CRC) function which is a high dimensional signal. The CRC describes the printer's desired-color to output-color map. Time-sequential sampling (TSS) is an approach to sample a small number of n color samples (typically $n = 3$) at each time print belt cycle, while at the same time gives sufficient information to properly reconstruct the time-varying CRC with minimal reconstruction errors. Hence only minimal sensing resources are required to obtain the high dimensional CRC at each time step, making color consistency control possible. Optimality in the OTSS design is achieved by maximizing the time between each successive n samples. The proposed OTSS design can be implemented practically and to achieve nearly similar reconstruction performance, the OTSS sequence with $n = 1$ requires 63% less samples per unit time compared to full sampling.

Introduction

The color reproduction characteristics(CRC) function describes how a printer maps the desired colors into actual printed colors. These printed colors vary with time due to printer dynamics as well as environmental and material variations. For control of the color consistency, the time varying CRC are needed as feedback signal [1][2]. Unfortunately they are only partially available in typical xerographic printers. In current state-of-art printers, the CRC are monitored by periodically printing and measuring small number of test patches ($n \approx 3$). The same set of test color patches are typically used. This limits the ability of the sensing system to resolve both the color and temporal resolutions. To increase the color resolutions, more sensor patches and hence more hardware resources are needed. To increase the temporal resolution, the test patches need to be printed more frequently, hence consuming more toner and reducing productivity.

To alleviate this requirements for sensing hardware and other resources, the time-sequential sampling (TSS) approach, which was originally developed for video processing in the 1980's [3], was proposed in [2, 4]. In this approach, instead of printing and sensing the same test patches at each time, different test patches are printed and sensed. The time-varying CRC can be considered as a 4-dimensional temporal-generalized tonal signal with 3 generalized tonal dimensions and one temporal dimension. The CRC is time-varying due to disturbances on the xerographic process [1]. Using the test patches to monitor the CRC amounts to sampling the 4 dimensional signal with a small number of n test patches at each time step $t = kT$ where T denotes the sampling period. Notice that the 3 generalized tonal dimensions refer to the tones of three primary colors (e.g. Cyan, Magenta and Yellow tones) used in typical printing systems. Usefulness of these samples are determined by how well the original CRC can be reconstructed in time. The number of samples, n, is determined by available sensor hardware. Because the n generalized tonal samples (i.e. color samples) vary from time sample to time sample in time-sequential sampling, they can provide better information about the original 4 dimensional signal than if the colors are fixed in current setup. The time sequentially sampled information is then reconstructed by a causal low pass filter (Kalman filter) which can be used as feedback for color consistency control [4].

Our previous works [2, 4] have demonstrated that TSS can effectively monitor the entire time-varying tone reproduction curve(TRC) even if only $n = 1$ test patch is used at each time. TRC gives the mapping of a single desired tone to a single printed output tone. In these works it has been shown that tone consistency control performance is much better than if a small number (say $n = 3$) of fixed tones are sampled, and is nearly as good as if the entire time varying TRC is available for feedback. These results were obtained using simple lexicographic and bit-reversed sampling sequences which are somewhat ad-hoc. Indeed, neither sampling sequence completely avoids aliasing, and neither the tonal resolution nor the sampling period was optimized. By restricting the TSS sequences to be lattices, the powerful multidimensional signal processing technique of lattice theoretic framework was used in [5] to design optimal time-sequential sampling (OTSS) lattice. It is shown that the set of all feasible (i.e. avoid aliasing) TSS lattices is compact. This makes it possible to design optimization procedure that results in a TSS lattice with the largest sampling period without incurring aliasing by the optimal trade off between tonal and temporal resolutions.

This paper extends the work in [5], which only considers the 1 dimensional tone reproduction curve(TRC) by considering sequences for sampling the 4 dimensional time-varying CRC. This design requires added computational complexity. In this paper, we address this issue and related implementation issues of designing the OTSS sequence for the CRC. The designed OTSS sequence enables minimal sampling resource to reconstruct the time-varying CRC with minimal reconstruction errors.

Problem Formulation

Let $CRC(t)$ denote the time varying CRC of the printer at time $t \in \Re$ as shown in Figure 1.

$$CRC(t) : [0,A_1) \times [0,A_2) \times [0,A_3) \to \Re^3, \quad \texttt{color}_{desired} \mapsto \texttt{color}_{printed}$$

We denote the domain space by the generalized tonal space $\mathscr{S}$ i.e. $\mathscr{S} = [0,A_1) \times [0,A_2) \times [0,A_3) \subset \Re^3$ is the color space for

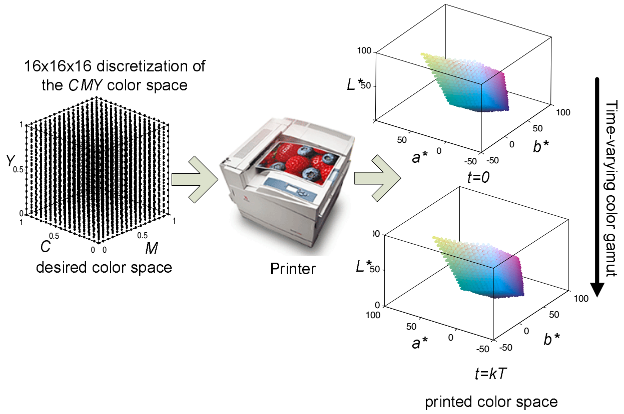

Figure 1. *The time-varying CRC, $CRC(t)$ give the mapping between the input CMY color space to the output $L^*a^*b^*$ color space. Time-sequential sampling of the $CRC(t)$ refer to specifying n sample points in the discretized CMY color space and measuring the resulting printed output in $L^*a^*b^*$ color space at each time step $t = kT$.*

the $CRC(t)$ sampling problem (e.g. each coordinate is the density of the primary CMY with A_i being the maximum density of each primary color). Sampling the $CRC(t)$ refers to specifying the n input CMY combinations to the color printing process with the resulting output-color measured in term of $L^*a^*b^*$ values. In time-sequential sampling, the n colors are sampled at discrete instances $t = kT$, given by the periodic sequence:

$$\alpha(k) = \alpha(k+p) = [\alpha_1(k), \alpha_2(k), \ldots, \alpha_n(k)] \in \mathscr{S}^n \qquad (1)$$

where the period of the sequence $B = pT$. The time-varying CRC can be considered a $N = 4$ dimensional temporal-generalized tonal function, $\omega(t, \mathbf{x}) : \Re \times \mathscr{S} \rightarrow \Re^3$ where $\mathbf{x} \in \mathscr{S} \subset \Re^3$ denotes the desired color. The generalized tonal dimension is limited to the set of $\mathscr{S}$. Let $\Omega(f, \mathbf{u})$ be its power-spectral density, where $f[Hz]$ is the temporal frequency and $\mathbf{u}$ $[cycles/generalized\ tone]$ is the generalized tonal frequency. Since the tonal domain $\mathscr{S}$ is finite, $\Omega(f, u)$ cannot be truly band-limited. In this work, following [6], we assume that $\Omega(f, \mathbf{u})$ is essentially band-limited to a closed support region, $\Theta \subset \Re^4$ in the sense that

$$\int_{\Re^4/\Theta} \Omega(f, \mathbf{u})\, df\, d\mathbf{u} \leq \varepsilon^2$$

so that all but ε^2 of the spectral energy is concentrated in a compact spectral support Θ. We assume that Θ is given by an ellipsoidal region parameterized by $(W, \mathbf{U})$ i.e.

$$\Theta(W, \mathbf{U}) = \left\{ (f, u) : \frac{f^2}{W^2} + \sum_{i=1}^{3} \frac{u_i^2}{U_i^2} \leq 1 \right\} \subset \Re^4$$

where W is the length along the temporal frequency dimension and $\mathbf{U} = [U_1, U_2, U_3]^T$ are lengths along the spatial frequency directions.

Design Example

For the optimal time-sequential sampling (OTSS) sequence design we need:

(1) The maximum required discretization of the input color space $\mathscr{S}$ given by $\mathbf{M}_{max}$.

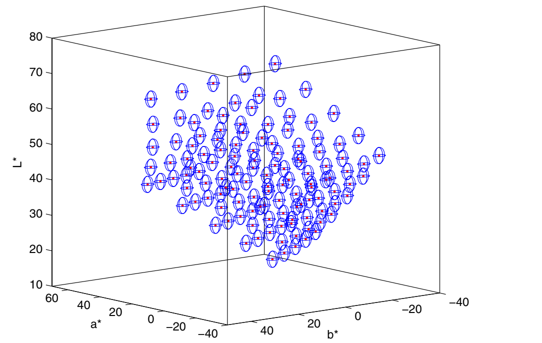

Figure 2. *Just-perceptible color difference ellipses (i.e. $\Delta E^*_{ab} \leq 2.3$) for each $L^*a^*b^*$ values corresponding to a discretization of $\mathbf{M} = [5, 5, 5]^T$ on the CMY color space*

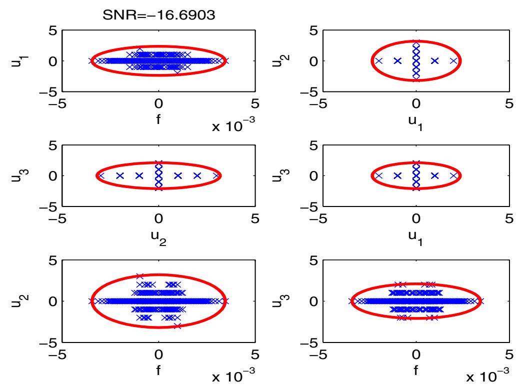

Figure 3. *Projection of the region of most significant spectral content β ($\tau_\Omega = 5$ gives a SNR=-16.7dB) and the corresponding ellipsoidal, $\Theta(W, \mathbf{U})$ region that wrap β where $W = 0.00345$ and $\mathbf{U} = [2.4, 3.1, 2.1]^T$*

(2) The ellipsoidal region $\Theta(W, \mathbf{U})$ where the spectrum of the time varying CRC, $\Omega(f, \mathbf{u})$ is concentrated.

To obtain $\mathbf{M}_{max}$, consider the effect of discretizing the printer's input color space (i.e. the CMY color space) on the printer's output $L^*a^*b^*$ color space. The dots ('.') in Figure 2 shows the $L^*a^*b^*$ values corresponding to a uniform discretization of the CMY color space. Surrounding each of these dots is an ellipsoid corresponding to region where the $\Delta E^*_{ab} \leq 2.3$. Hence, for a human viewer, two colors with different $L^*a^*b^*$ values that lay within each of these ellipsoid are not differentiable [7]. Therefore by selecting $\mathbf{M}_{max}$ such that these ellipsoid covers the entire printer's gamut, we ensure the extraction of only the most significant color information. In our case this works out to be $\mathbf{M}_{max} = [15, 15, 15]^T$. To obtain the compact spectral support $\Theta(W, \mathbf{U})$, consider the availability of a collection of time-varying CRC data. Let $\Omega_{L^*}(f_k, \mathbf{u_r}), \Omega_{a^*}(f_k, \mathbf{u_r})$ and $\Omega_{b^*}(f_k, \mathbf{u_r})$ denote the power-spectral density of the time-varying L^*, a^* and b^* components respectively. To find $\Theta(W, \mathbf{U})$, consider the maximum power spectral density at each frequency location i.e. we define $\Omega_{max}(f_k, \mathbf{u_r}) = \max[\Omega_{L^*}(f_k, \mathbf{u_r}), \Omega_{a^*}(f_k, \mathbf{u_r}), \Omega_{b^*}(f_k, \mathbf{u_r})]$. Let β denotes the region in the frequency domain where $\Omega_{max}(f_k, \mathbf{u_r})$ is above a threshold value, τ_Ω. The idea to find $\Theta(W, \mathbf{U})$ is to wrap an ellipsoidal region on β. This is done by minimizing the

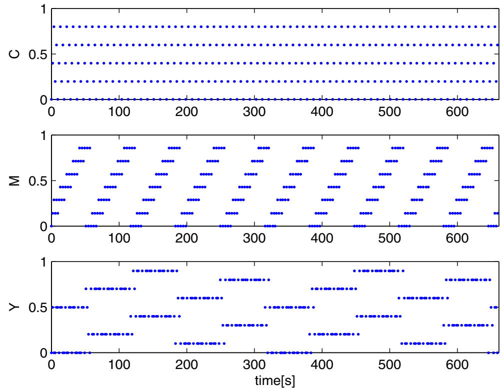

Figure 4. OTSS sequence for sampling the $CRC(t)$ with $n = 1$ for 1 period, $B = M_1 M_2 M_3 T = 661.5s$. In this case, $T = 1.89s$ and $\mathbf{M} = [5,7,10]^T$

difference in the hyper-volume between $\Theta(W, \mathbf{U})$ and the convex hull of β subject to the constraint that $\Theta(W, \mathbf{U})$ is larger than the convex hull of β. From Figure 3, we have $W = 0.00345Hz$ and $\mathbf{U} = [2.4, 3.1, 2.1]^T cycles/tone$.

Based on these results, the OTSS sequence design (as briefly summarized in the next section) for $n = 1$ sample for sampling instant (i.e. $\mathbf{A} = [1,1,1]^T$) is shown in Figure 4 where $T = 1.89s$ and $\mathbf{M} = [5,7,10]^T$. The samples per unit time for this design is 0.53. An equivalent design for full sampling (i.e. with no aliasing of Θ in the frequency domain) is arrived by ensuring that the temporal and generalized tonal sampling frequencies are within their Nyquist frequencies. This full sampling design yields a samples per unit time of 0.86. Hence the OTSS design requires 63% less samples per unit time compared to full sampling. Furthermore, if $n = 3$ color samples at each time-step is used (i.e. $\mathbf{A} = [1/3, 1/3, 1/3]^T$) the sampling period can be increased to $T = 51.08s$ which is an increase of approximately 27 times. Hence a user has the flexibility to tradeoff hardware requirements and sampling frequency as deemed appropriate. To test the performance of the OTSS design, we compare the reconstructed output-colors, $\widehat{L^*a^*b^*}(k)$ corresponding to each CMY grid coordinates (using a periodic Kalman filter [4]) with the actual output-color $L^*a^*b^*(k)$, at each time-step k. The color difference corresponding to each CMY grid coordinates is given by $\Delta E^*_{ab}(k) = \|\widehat{L^*a^*b^*} - L^*a^*b^*\|_2(k)$. The time-normalized performance index $J_L = \sum_{k=l_0}^{l_0+L} \mathscr{E}[\Delta E^*_{ab}](k)/L$ where $\mathscr{E}[\Delta E^*_{ab}](k)$ gives the mean of $\Delta E^*_{ab}(k)$ over all colors on the grid at time-step k, is taken as the measure of reconstruction performance. For $l_0 = 1000, L = 1000$, simulation study shows that the OTSS design yields $J_L = 1.89$. With a non-optimal lexicographic sequence [4] yields $J_L = 2.02$. Since the OTSS sequence design does not result in aliasing in the frequency domain, it performs as well as the case of equivalent design with full sampling. These results indicate the effectiveness of the proposed OTSS design for sampling the time-varying CRC.

Design of OTSS Sequence

An optimal time-sequential sampling (OTSS) sequence is defined as a TSS sequence that does not results in aliasing for the assumed ellipsoidal spectral support $\Theta(W, \mathbf{U})$ and maximizes

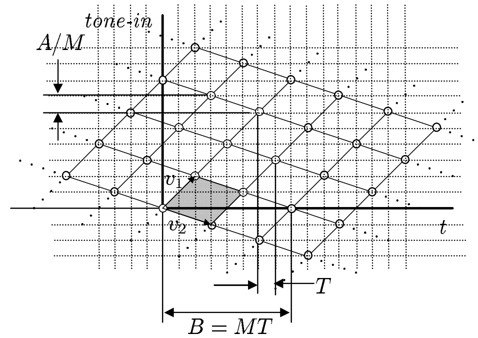

Figure 5. Point lattice $LAT(V)$, where $V = [v_1, v_2]$ on the temporal-tonal space. Also shown in dotted vertical and horizontal lines is the grid, $LAT(D)$.

the sampling period, T. Allebach [3] shows that when $\omega(t, \mathbf{x})$ is time-sequentially sampled, the resulting frequency content of the sampled signal is made up of the weighted replications of $\Omega(f, \mathbf{u})$ at location $(r/B, m/A_1, p/A_2, q/A_3)$ on the frequency domain where r, m, p, q are integers, B is the period of the sequence and A_1, A_2, A_3 give the maximum density of each CMY primaries. The idea for the OTSS design is to rearrange the replicas of $\Theta(W, \mathbf{U})$ at location $(r/B, m/A_1, p/A_2, q/A_3)$ such that they do not overlap each other and the sampling period T maximized. This design is made possible by restricting the TSS sequence to be lattices as done in [6] where the set of lattice points are generated by a nonsingular generator matrix $V \in \Re^{N \times N}$. An example of a lattice point in 2 dimensional tonal-temporal space (i.e. $N = 2$) is shown in Figure 5. This restriction allows for the use of multidimensional signal processing based on lattice theory [8]. The technique is useful here because a multidimensional signal sampled on a lattice $LAT(V)$ results in the replication of the original frequency content of the signal also on a lattice $LAT(V^*)$, where $V^* = V^{-T}$. Henceforth $LAT(V^*)$ is known as the polar lattice. The following gives the summary of the OTSS design process (for further details, see [5]):

(1) **Parameterizing the set of feasible TSS lattices**. Consider the case of time-sequential sample using $n = 1$ sample at each time instant. We also assume that all sampling points lay on a fine rectangular grid, $LAT(D)$ where $D = \text{diag}(T, A_1/M_1, A_2/M_2, A_3/M_3)$ (see Figure 5 for an example with $N = 2$). Lattices are uniquely represented by $LAT(DH)$ where H gives the lower triangular (or column reduced) Hermite Normal Form (HNF). The matrix H is defined by, (i)$h_{ii} > 0, \forall i = \{2, \ldots, 4\}$, (ii) $h_{ij} = 0$ for $j > i$ and (iii) $h_{ij} \leq 0$ and $|h_{ij}| < h_{ii}$ for $j < i$. Hence the parameters that represent lattices are given by $T, \mathbf{A} = [A_1, A_2, A_3]^T, \mathbf{M} = [M_1, M_2, M_3]^T$ and combinations of possible h_{ii} integer values that form unique H matrices. It can be proven that the parameters defining the set of all feasible TSS lattices for optimization are bounded [5] i.e.:

(i) $T \in [0, T_{max})$ where $T_{max} = \frac{1}{8WU_1U_2U_3A_1A_2A_3}$.
(ii) $\mathbf{A}$ is fixed to the desired maximum intensity range at each CMY coordinates.
(iii) $\mathbf{M} \in [\mathbf{M}_{min}, \mathbf{M}_{max}]$ where $\mathbf{M}_{min}$ is determined by the generalized tonal Nyquist frequencies and $\mathbf{M}_{max}$ is determined by the maximum sensor resolution.
(iv) $h_{11} = 1, h_{22} = M_1, h_{33} = M_2, h_{44} = M_3$ and $0 \leq h_{ij} <$

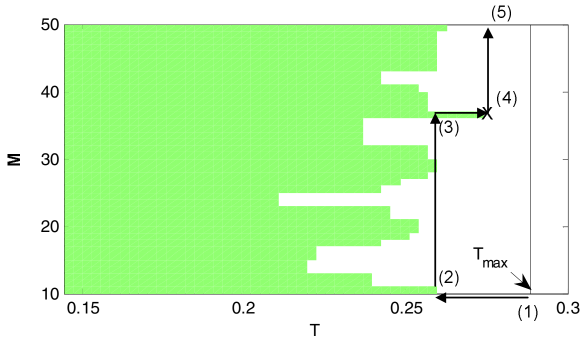

Figure 6. *Optimization search space. The optimization is as follows; start at (1) i.e. at $T = T_{max}$ and $\mathbf{M} = \mathbf{M}_{min}$; Find (2) by bisection; Reach (3) by progressively increasing the components of $\mathbf{M}$; Repeat these steps to find the OTSS lattice at (4); Go until $\mathbf{M}_{max}$ is reached at (5) (see [5] for details)*

h_{ii} for $j < i$.

(2) **Optimization algorithm**. Find a TSS lattice that maximizes the sampling period T and does not results in overlapping of the replicas of $\Theta(W,\mathbf{U})$ on its polar lattice. Figure 6 shows the optimization search space in term of the sampling period T and the possible combination of $\mathbf{M}$ in the given range. $\Gamma_{(T,\mathbf{A},\mathbf{M})}$ denotes the set of lattices $LAT(DH)$ withe these parameters setting. Our interest here is to find if there exists a polar form of $LAT(DH) \in \Gamma_{(T,\mathbf{A},\mathbf{M})}$ denoted by $LAT(V^*)$ (where $V^* = (DH)^{-T}$) which packs the ellipsoid $\Theta(W,\mathbf{U})$. The polar lattice $LAT(V^*)$ packs $\Theta(W,\mathbf{U})$ if the $\Theta(W,\mathbf{U})$ replicated on the lattice points of $LAT(V^*)$ do not overlap. For the case of an ellipsoidal body, a simple algorithm exists to check if a polar lattice do indeed packs $\Theta(W,\mathbf{U})$ [5]. In Figure 6, the shaded area denotes region where there does indeed exists such a polar lattice and the unshaded area denotes the region where there is no such polar lattice. The OTSS lattice is shown by the cross ('x') in Figure 6 and this is found by an optimization procedure proposed in [5].

The optimization process for $N = 4$ requires large computational effort (e.g. for modest $\mathbf{M} = [5,5,5]^T$, the number of lattices in the set $\Gamma_{(T,\mathbf{A},\mathbf{M})}$ is approximately $16K$). Fortunately at particular $(T,\mathbf{A},\mathbf{M})$, not all lattices belonging to the set of lattices $LAT(DH)$ with $(T,\mathbf{A},\mathbf{M})$ parameters setting, $\Gamma_{(T,\mathbf{A},\mathbf{M})}$ need to be evaluated. Firstly we note that if the polar lattice $LAT(V^*)$ packs Θ then the reflected polar lattice $LAT(\widehat{R}\,V^*)$ also packs Θ where $\widehat{R} = \Lambda^{-1}R\Lambda$, $\Lambda = \text{diag}(1/W, 1/\mathbf{U})$ and R gives the reflection matrix. Hence for those reflected polar lattices that lay in $\Gamma_{(T,\mathbf{A},\mathbf{M})}$, they can be dropped from evaluation. Moreover, since $\Gamma_{(T,\mathbf{A},{}^a\mathbf{M})} \bigcap \Gamma_{(T,\mathbf{A},{}^b\mathbf{M})} \neq \emptyset$ for ${}^a\mathbf{M} \neq {}^b\,\mathbf{M}$, it is not necessary to evaluate some lattices at ${}^b\mathbf{M}$ if lattices at ${}^a\mathbf{M}$ have all been evaluated.

The design procedure can be extended to permit $n > 1$ by designing the OTSS sequence as given previously by taking $\mathscr{S} = [0,1/n) \times [0,1/n) \times [0,1/n)$. It turns out that this simple design is optimal provided that each of the multi-dimensional signal segment have the same frequency content.

Conclusion

A design procedure to obtain the optimal time-sequential sampling (OTSS) sequence for sampling the color reproduction characteristics(CRC) function is proposed. For compact ellipsoidal spectrum, $\Theta(W,\mathbf{U})$ the OTSS design avoids aliasing in the frequency domain and maximized the sampling period T. A simple extension allows for the flexibility to time-sequentially sample $n > 1$ color samples at each time step. This allows for an increase in the sampling period, which gives a user the flexibility to trade-off hardware requirements and sampling frequency. The OTSS design achieves comparable performance and requires 63% less samples per unit time for $n = 1$ compared to full sampling. Moreover, it achieves better reconstruction performance compares to non-optimal lexicographic sequence. To further reduce the number of required samples to fully characterized the print's variation, we are currently investing a hybrid approach of time-sequentially sampling both the TRC and CRC simultaneously.

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