Control Analysis of a Hybrid Two-Component Development Process

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Abstract

In this paper, we will illustrate the utility of a control oriented model for a hybrid two component development process. The model used for control extracts relevant characteristics from an experimentally validated numerical model based on toner age distribution and development probability. Equilibrium analysis of the model shows that developability loss associated with low area coverage, low humidity operating conditions can be attributed to the fact that the development process is not operating around an equilibrium point. Velocity field analysis indicates the process has limited controllability under these operating conditions. Based on these observations, a constrained optimal control problem is formulated to develop a control strategy to maximize the operating time while maintaining acceptable developability. The numerical solution shows that for a given operating condition, with bounded toner concentration and development voltage, the optimal dispense strategy is a concatenation of a zero-dispense profile and a controlled dispense rate associated with operating the development voltage at its maximum value.

Introduction

The development process is one of the six key steps in xerography [1]. In this paper, we consider a hybrid two component development process [2] as illustrated in Figure 1. The development process can be modeled in various ways depending on the objectives. First principle models have been developed to provide phenomenological understanding and predict system behavior [3, 4]. Although comprehensive and accurate, these models tend to be complex and computationally difficult to be integrated into system analysis and to be used for synthesizing control strategies. Control oriented models need to capture the essential process behaviors (system dynamics) by relating the relationship from a set of input variables to a set of output variables through the evolution of a set of state variables that characterizes the internal dynamics of the process. Various system analysis techniques and controller synthesis methods can then be applied to the resulting model to characterize system behavior and develop appropriate control strategies. In our previous work [5], a control-oriented model was constructed based on a comprehensive phenomenological and probabilistic model [4]. In this work, we will use a common xerographic development issue as a case study to illustrate the benefit and utility of a control oriented model in system analysis and control synthesis.

It has been observed that toner particles tend to lose developability during low area coverage printing. This phenomenon has also been called the "toner aging" effect [4]. It is one of the main causes for print-to-print inconsistency in tone reproduction since it causes variation in the developed toner mass under the same operating parameters. One obvious method to maintain consistent developed mass per unit area (DMA) is to appropriately adjust the development voltage actuator, denoted by V_{dev} [6]. As developability decreases, the corresponding development voltage can be increased to compensate for the loss in the effective electrostatic potential. However, as illustrated in Figure 2, this strategy fails to maintain DMA when V_{dev} saturates then the process fails and certain "maintenance" measures are required to restore developability, which reduces printing productivity and increases the operation cost. To help maintain development performance, we consider using toner dispense as an additional actuator variable. The idea is to improve developability by adding 'fresh' toner to the system. The scenario outlined above raises the following questions:

1. Is it possible to avoid saturating V_{dev} under low area coverage conditions? This question can be answered by performing *equilibrium* analysis on the control oriented model.
2. Can the process be driven to another desired operating point using dispense and development voltage? This question addresses the *controllability* [7] of the development system and can be explored using vector field analysis.
3. Given the working range of the actuators and bounds from other practical factors, what is the proper control strategy that will maximize the time before the system cannot maintain acceptable DMA? We refer to this as *time to failure*, denoted by $T_{failure}$, and develop a control strategy that maximizes $T_{failure}$ by formulating and solving a nonlinear constrained optimization problem.

There are obviously many other questions and issues that one can formulate and try to address. In this work, we will address the above three issues using the control-oriented model and the appropriate analysis and synthesis techniques.

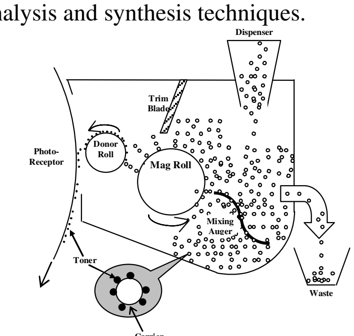

***Figure 1** Schematic of a hybrid two-component development process*

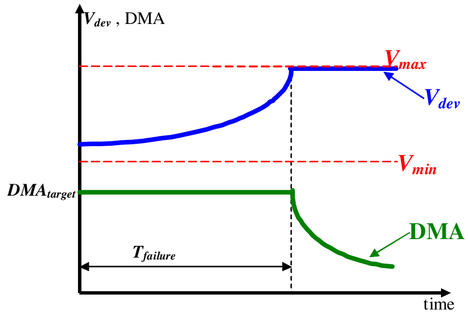

Figure 2 *Development voltage saturation in low area coverage jobs*

The remainder of this paper is organized as follows. The control-oriented model will be briefly summarized in the next section. Equilibrium and velocity field analyses will be given in section 3. In section 4, the optimization problem is formulated and an optimal control strategy is numerically solved to reduce maintenance related down time. Finally, conclusions are given in the last section.

Control Oriented Model

In this section, we briefly summarize the hybrid, two component development process model used in the control analysis. For a detailed derivation, the reader is referred to [5]. The control oriented model is a cascade structure of a nonlinear static mapping and toner aging dynamics modeled using three state variables as shown in Figure 3. The term $D_t(t)$ is the dispense rate and $V_{dev}(t)$ is the development voltage, these two are the control inputs to the process (i.e. actuators). The term $C_t(t)+R_w(t)$ is the effect of throughput and waste, which can be regarded as the disturbance input to this process. The output of the process is the DMA on the photoreceptor. To achieve constant DMA, active adjustment of the development voltage is usually required. This will partially compensate the toner aging effect, which is represented by γ_L in this model. The states are toner mass $M_t(t)$, sump state $\gamma_s(t)$ and donor state $\gamma_d(t)$, where the sump and donor states characterize the "goodness" of the toner at different points in the system such that smaller values for these states correspond to reduced developability. The sump state and the donor state are scaled to take on values between 0 and 1, where 1 represents fresh toner. With the assumption that the carrier mass $M_c(t)$ is constant in the sump, the toner mass is proportional to toner concentration (referred to as "TC"), which is assumed to be measurable in practice. As shown in [5], the evolution of the states can be represented by three first-order ordinary equations

$$\dot{M}_t(t) = D_t(t) - C_t(t) - R_w(t)\,, \qquad (1)$$

$$\dot{\gamma}_s(t) = \frac{D_t(t)}{M_t(t)}(1+p_a)(1-\gamma_s(t)) - \beta\cdot\gamma_s(t)\,, \qquad (2)$$

$$\dot{\gamma}_d(t) = \frac{D_t(t)p_a}{M_t(t)}(1-\gamma_d(t)) + \lambda\frac{C_t}{M_d}(\gamma_s(t)-\gamma_d(t)) - \beta\gamma_d(t)\,, \qquad (3)$$

where p_a and β are parameters related to developer material properties, and M_d is the estimated toner mass on the donor roll. The output DMA is given by

$$DMA = f_1(\gamma_L, V_{dev})\,, \qquad (4)$$

where f_1 is the static nonlinear mapping between the development voltage and DMA, and

$$\gamma_L = f_2(tribo(M_t, RH), \gamma_d)\,, \qquad (5)$$

where f_2 is a function that relates TC, relative humidity (referred to as "RH") and donor state to developability.

As discussed in [5], the operating conditions assumed for the model include low area coverage (i.e. low throughput) and low RH, which drive the toner aging dynamics that lead to developability loss.

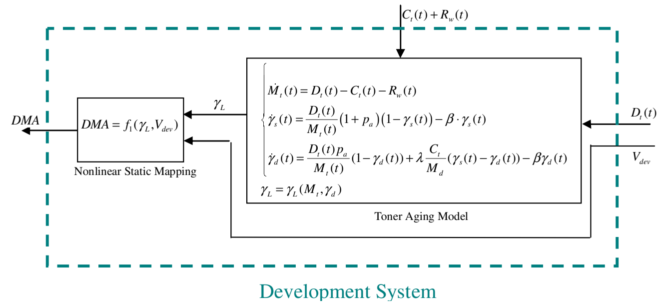

Figure 3 *Block diagram of the control oriented model*

Equilibrium and Velocity Field Analysis

Avoiding developability loss is equivalent to finding an operating condition where the toner developability will not change for a specific throughput. This is equivalent to demonstrating the existence of an equilibrium state within the acceptable operating range of the development process. By definition, when a system is at equilibrium, the system states will not change with respect to time, i.e. the time derivative of the states are zero. Ideally, the system operating conditions would correspond to an equilibrium state so that the system could operate 'indefinitely' without changes in system properties. However, if the system is not operating at equilibrium, the process cannot stay at the current operating point and the system dynamics (toner aging dynamics) will drive the system away from the current operating point. In this case, control may be useful for improving system performance. To understand the impact of control, we first need to analyze the controllability of the process, which, roughly speaking, quantifies the influence of the actuators on the process states. In this section, we first discuss the equilibrium properties of the development model, and then we use a velocity field analysis to investigate controllability of the process.

Equilibrium Analysis

Since the toner mass $M_t(t)$ and the sump state $\gamma_s(t)$ dynamics do not depend on the donor state $\gamma_d(t)$ (see Eq. (2)), we can limit the analysis to just the toner mass $M_t(t)$ and the sump state $\gamma_s(t)$. As will be shown in subsequent discussion, the behaviors of these two states are adequate to explain the loss of developability phenomenon.

Assuming constant relative humidity (RH), Figure 4 shows the required development voltage to achieve a specific DMA with different values of $\gamma_s(t)$ and $M_t(t)$. The horizontal axis of Figure 4 has been normalized by a nominal TC level. As shown in Figure 4, $V_{dev}(t)$ is a monotone decreasing function of both TC and $\gamma_s(t)$. To achieve a specific DMA, the required development voltage will increase as sump state drops or TC drops, where the first case corresponds to the toner aging effect, and the second case corresponds to the tribo-charging effect.

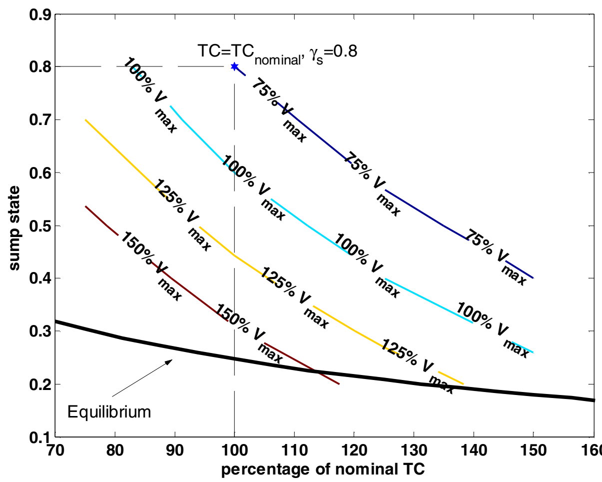

Figure 4 *Isolines of normalized development voltage for a constant DMA*

Here it is assumed that the nominal operating condition is at nominal TC and γ_s=0.8 (the "star" point in Figure 4), which corresponds to a development voltage of 75% of the maximum development voltage, V_{max}. If at this time, a low area coverage (e.g. < 2% area coverage) job starts, the control oriented model can be used to find the resulting equilibrium. Assuming constant throughput and zero waste, the equilibrium can be found by setting the time derivatives of TC and γ_s to be zero in Eqs. (1)-(2), resulting in the following equation for equilibrium (see Figure 4)

$$\gamma_s = \frac{1}{1+K_e \cdot TC} \tag{6}$$

where $K_e=(M_c\beta)/[(1+p_a)C_t]$. Figure 4 also shows that the nominal operating point is not on the equilibrium, indicating the process will not stay at the nominal operating condition. However, operating at an equilibrium point is a necessary condition for a process to be "controllable" [7]. Roughly speaking, violation of the necessary condition means the system governed by Eqs. (1)-(2) cannot move in any arbitrary directions on the TC - γ_s plane, which is also called the phase plane. To understand the limitation of the limited controllability of the system dynamics in Eqs. (1)-(2), we would like to know the 'controllable direction' on the phase plane, which is the subject of the next subsection.

Velocity Field Analysis

The equations for the development process given in Eqs. (1)-(2) govern both the speed and direction for the process states i.e. the equations characterize the "velocity" of the states. To understand the influence of dispense as an actuator, we consider the impact of dispense on the velocity of the states. Figure 5 shows three velocity vectors corresponding to zero dispense rate, dispense rate equal to throughout (2%), and the maximum dispense rate, respectively. The maximum dispense rate is set at ten times the throughput. From Fig. 5, it is clear that the dispense input can only move the system state in a limited set of directions. This matches the analysis that the system is not "completely controllable." With dispense constrained from zero to its upper limit, the state trajectory can only move in the right-lower direction for points above the equilibrium. For points below the equilibrium, the state trajectory can only move in the upper-left direction. For points above the equilibrium, isolines of V_{dev} are in the possible directions, and can only move in the lower-right direction. This observation suggests that it is possible to drive the states along the isolines of constant development voltage with an appropriate designed dispense strategy.

For the process model and operating conditions under consideration, equilibrium and velocity analyses have indicated that the loss of developability is unavoidable and that using development voltage and dispense as control actuators to avoid developability loss has limited impact. Both conclusions match empirical observations. Given the inherent system limitations revealed by these analyses, the question of what is the maximum achievable performance still remains and will be addressed in the next section.

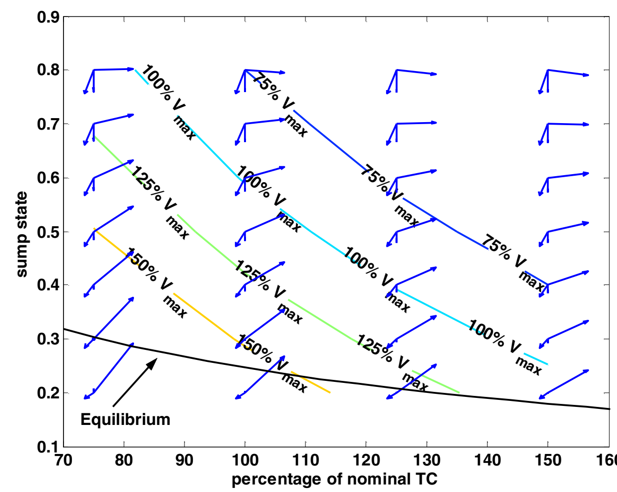

Figure 5 *Velocity field of the development process*

Time to Failure Maximization

Using the control oriented model, this section formulates and solves an optimization problem to find a dispense strategy that maximizes time to failure.

Problem Formulation

Using the model, i.e., Equations (1)–(5), the time to failure maximization problem can be mathematically formulated as follows: Let the allowable maximums of TC and development voltage be given by TC^u and $V_{dev}{}^u$, respectively, and let the DMA target be given by DMA$_{target}$. For given TC(*0*) and $\gamma_s(0)$, find a dispense rate $D_t(t) \ge 0$ for $0 \le t \le T_f$ that maximizes

$$J = T_f \tag{7}$$

subject to

$$\dot{M}_t(t) = D_t(t) - C_t(t) - R_w(t), \tag{8}$$

$$\dot{\gamma}_s(t) = \frac{D_t(t)}{M_t(t)}(1+p_a)(1-\gamma_s(t)) - \beta\cdot\gamma_s(t), \tag{9}$$

$$f_1(\gamma_L, V_{dev}(t)) = DMA_{target} \qquad 0 \le t \le T_f, \tag{10}$$

$$\gamma_L = f_2(tribo(M_t, RH), \gamma_s), \tag{11}$$

$$V_{dev}(T_f) = V^u_{dev}, \tag{12}$$

$$V_{dev}(t) \le V^u_{dev} \qquad 0 \le t \le T_f, \tag{13}$$

$$M_t(t) \le M_c \cdot TC^u \qquad 0 \le t \le T_f \; . \qquad (12)$$

It turns out that this problem is a constrained nonlinear optimal control problem, analytical solution of which is very difficult to find except a few special cases [8]. Hence, we solve this problem using a numerical optimization technique [9].

Numerical Solution

Using MATLAB optimization toolbox, we solved the "time to failure maximization" problem formulated above for the case where $TC(0)=TC_{nominal}$, $\gamma_s(0)=0.8$, $V_{dev}{}^u = V_{max}$, and TC^u= 125%$TC_{nominal}$. The solution to this problem, i.e., the dispense profile that maximizes time to failure, is given in Figure 6. The corresponding time to failure is T_f = 4870 seconds.

Simulating the model with the optimal dispense rate gives state trajectory in the phase plane (See Figure 7). These figures indicate that the optimal dispense rate to achieve the longest operating time is a concatenation of zero dispense with a controlled dispense rate associated with operating the development voltage at its maximum value.

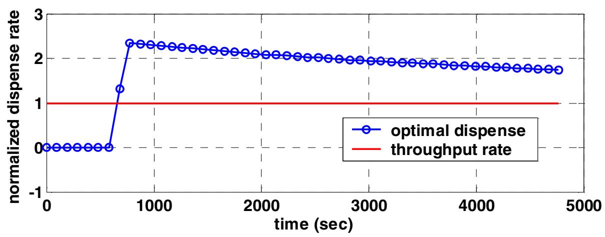

Figure 6 *Normalized optimal dispense profile*

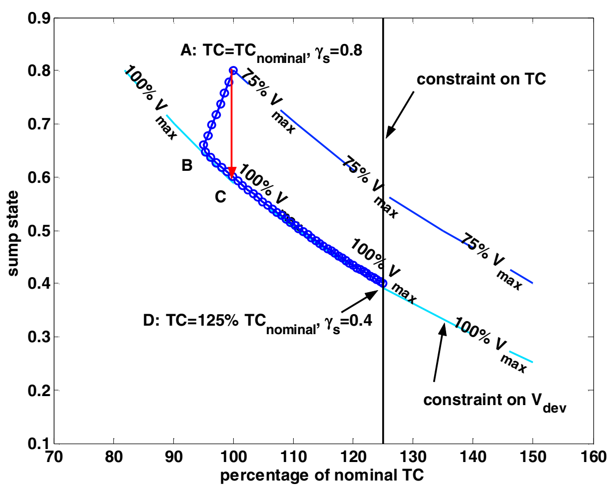

Figure 7 *State trajectory to achieve longest "service period" in phase plane*

Next we compare the performance with optimal dispense strategy with that of conventional dispense strategy. In this work, we considered dispense as an additional actuator input to maintain developability. However, in practice, dispense is typically used to maintain TC at a desired level. We refer to this as constant TC strategy. Figure 6 compares the performances of two cases in state space. The optimal path is given by (A→B→C→D) in blue circle and the path with constant TC strategy is given by (A→C) in red line. The time until failure for the first case is 4870 seconds while it is only 1200 seconds for the latter case. Therefore, the optimal dispense strategy, a solution from the optimal control problem, achieves significant performance improvement compared to the conventional strategy.

Conclusions

A case study with a hybrid two-component development process has been presented that illustrates the benefit and utility of a control oriented model in system analysis and control synthesis. For the process model and operating conditions under consideration, equilibrium and velocity analyses have indicated that the loss of developability is unavoidable and that using development voltage and dispense as control actuators to avoid developability loss has limited impact. Given the inherent system limitations revealed by these analyses, optimal dispense strategy has been sought by numerical optimization technique. Simulation results verifies that the solution yields improved performance compared to conventional dispense strategy.

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References

[1] L. B. Shein, Electrophotography and development physics (Springer-Verlag, NY, 1988)

[2] R. Clark and D. Craig, "Xerox Nuvera Technology for Image Quality," in Proc. NIP 21, pp. 671-674, (2005).

[3] H. Iimura, H. Kurosu and T. Yamaguchi, "The Effect of Surface Treatment on Toner Adhesion Force," in Proc. NIP 15, pp. 535-538 (1999).

[4] P. Ramesh, "Quantification of Toner Aging in Two Component Development Systems," in Proc. NIP 21, pp. 544-547, (2005).

[5] F. Liu, G. T.-C. Chiu, E. S. Hamby, Y. Eun and P. Ramesh, "Control Oriented Modeling of a Hybrid Two-Component Development Process for Xerography," in Proc. ICIS2006, pp. 87-90, (2006).

[6] E. Hamby and E. Gross, "A Control-oriented Survey of Xerographic Systems: Basic Concepts to New Frontiers," in Proc. the American Control Conference, pp. 2615-2629, (2004).

[7] H. J. Sussmann, "Lie brackets and local controllability: A sufficient condition for scalar-input systems, " in SIAM Journal on Control and Optimization, Vol. 21, No. 5, pp. 686-713, Sep. 1983.

[8] A. E. Bryson and Y.-C. Ho, "Applied Optimal Control", Blaisdell Publishing Company, 1969

[9] J. Liang, Y. Chen, M. Meng and R. Fullmer, "Solving Tough Optimal Control Problems by Network Enabled Optimization Server (NEOS)," in Proc. IEEE Intelligent Automation Conference, (2003).

Author Biography

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