A Digital Halftoning Using Magic Square

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Abstract

Digital halftoning is essential to print or display a continuous tone with binary devices. Error Diffusion (ED) has been a major stream to produce a pictorial image on inkjet prints, but is not always acceptable for commercial printing or EP printer, because it generates the dispersed dots hard to print.

This paper proposes a novel method for generating ordered dither mask using "magic square". *Magic square provides spatially balanced threshold values in row, column, and diagonal directions by the rule of "*even-sum*". The proposed magic square satisfies the energy minimum criterion in the local area and is expandable to a large scale for sufficient gray scales. We selected the magic square candidates with Fourier spectra concentrated in the middle spatial frequency to produce a smoothed gradation suitable for* LBP *or commercial printing. The paper introduces a systematic method for expanding small mask to large mask by "*master-slave" *algorithm. The experimental results show the improved grayscale reproduction with reduced artifacts in comparison with the conventional* Bayer *dither or* ED.

Introduction

During past 40 years, so many digital halftoning methods have been developed.[1] They are divided broadly into two categories of *"independent"* or *"adaptive"* thresholding as shown in Fig. 1.

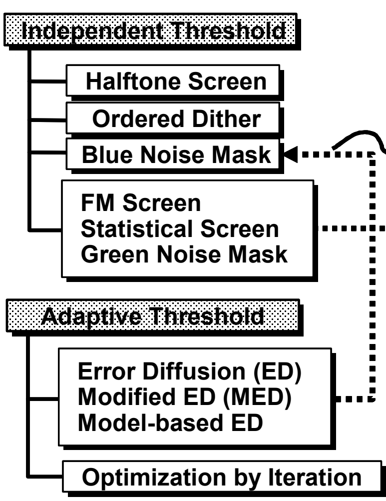

Figure 1. Categories in Digital Halftoning

Recently "*Blue Noise Mask*"[5,6] or "*statistical screen*" is widely noticed as a substitute for *ED* or classical "*halftone screen*" because of high-speed and high-resolution and "*Green Noise Mask*"[7] as a compromise plan applicable to *Electro-Photographic* (*EP*) print or conventional printing with "small-clustered but dispersed" dots.

Ordered Dither and Magic Square Masks

"*Ordered Dither*" is a most easy and fast method for bi-level halftoning. The ordering rule in threshold values was first given by Limb[2] as 2 × 2 minimum mask and extended to general size of $2^n \times 2^n$ by *Bayer*[3] as follows.

$$D_2 = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}, \quad D_N = \begin{bmatrix} 4D_{N/2}+0 & 4D_{N/2}+2 \\ 4D_{N/2}+1 & 4D_{N/2}+3 \end{bmatrix}$$
$$(\textit{Basic by Limb}) \qquad (\textit{Extended by Bayer}) \tag{1}$$

Bayer mask is known to generate the dots pattern with the highest spatial frequency but has a drawback of "textural visibility" in tonal gradation, while *ED* gives the better tonal rendition but causes unwanted "*wormy pattern*".

In this paper, we introduce a "*Magic Square* (*MS*)" in place of ordered dither mask. *MS* originates from ancient China in 2000 B.C., which was discovered as 3 × 3 figures carved on the shell of a tortoise as shown in Fig. 2. This is the minimum *MS* whose figures "1~8" are uniquely ordered to have the same sum of "15" in any row, column, and diagonal cells in 3 × 3 matrix.

4	9	2
3	5	7
8	1	6

Figure 2. World-first Magic Square in B. C. 2000 at China

There are various types of magic square. Roughly they are classified into "*perfect MS*" having equal sum of figures in all row, column, and diagonal cells, and "*imperfect MS*" not equal in diagonal. As the dimension N increases, the number of *MS* runs into astronomical figures, for example, 880 for N=4, 275,305,224 for N=5, and becomes too numerous to be counted for larger N than 6.

The threshold value $\boldsymbol{D}=[d_{mn}]$ in *MS* mask is modified to take integer value starting from zero as $0 \le d_{mn} \le N$-1.

The total sum of $[d_{mn}]$ is given by:

$$S_N = \sum_{m=1}^{N}\sum_{n=1}^{N} d_{mn} = \frac{N^2(N^2-1)}{2} \tag{2}$$

The local sum of $[d_{mn}]$ in each row m and column n equals:

$$S_{row=m} = \sum_{m=1}^{N} d_{mn} = S_{col=n} = \sum_{n=1}^{N} d_{mn} = \frac{1}{N} S_N \tag{3}$$

Figure 3 shows an example of N=4 *MS* mask in comparison with that of *Bayer*'s ordered dither. The well-known *Bayer* matrix in (a) is characterized by the large and small threshold values are alternately arranged across one to next cell in each row and column of matrix $[d_{mn}]$. Although this feature generates the dithered dots pattern with the highest spatial frequency, the figures' arrangement in threshold matrices $[d_{mn}]$ is not so well balanced in local as compared with that of MS. While, (b) and (c) are the examples of "*imperfect*" and "*perfect*" *MS* with S_N=120 and S_{row}= S_{col}=30. Among many "*perfect*" *MS*, (d) is a special type called "*super perfect*" with the local sum equal to 30 in the 2 × 2 cells of center, four corners, most left/right of 2 center rows, and most upper/lower 2 center columns. It is known 48 "*super perfect*" exist among 880 *MS* for N=4 in total. Furthermore, we found "*perfect*" *MS* (e) and (f) with the local sum equal to 30 in all of adjacent 2 × 2 cells. We call them "*pan-perfect*" *MS*, which are expected to have the well-balanced characteristics similar to "*super perfect*" *MS*.

Interlace-structured Magic Square

The major purpose of our paper is to find the *MS* masks to generate "small-clustered & moderately dispersed" dots than the "highly dispersed" *Bayer* or *ED*.

The mask size of ordered dither should be larger than 8 × 8 to reproduce a continuous tone. However it's impossible to search all possible *MS* for N=8, because of the astronomical number of combinations.

Hence, firstly we tried to extract the candidates with

1. strong *Fourier* spectra for threshold mask in "middle" spatial frequencies by applying *DCT*
2. interlaced-structure in threshold entries like as *Bayer*.

By chance, two distinctive *MS*, one designed by *I. Hoshi* (1958) and other by *M. Saito* (1991) were discovered from hundreds of known 8 × 8 *MS*.

I. Hoshi's MS

The *"perfect" MS by I. Hoshi* has an interlaced structure in Fig. 4 (b) like as *Bayer* in (a). Its mask $[d_{mn}]$ is arranged in "*stepped stones"* by *four* pair of entries with *DCT* spectra more concentrated in middle frequencies than *Bayer*.

M. Saito's MS

The threshold mask $[d_{mn}]$ *by M. Saito* in Fig. 3(c) is uniquely interlaced in "*rhombic"* structure and *DCT* spectra more concentrated in middle frequencies than *I. Hoshi.*

Among these two, *M. Saito*'s *MS* attracted our attention.

In our preliminary experiments, *Saito*'s *MS* showed the better tonal renditions for natural images than *Bayer*.

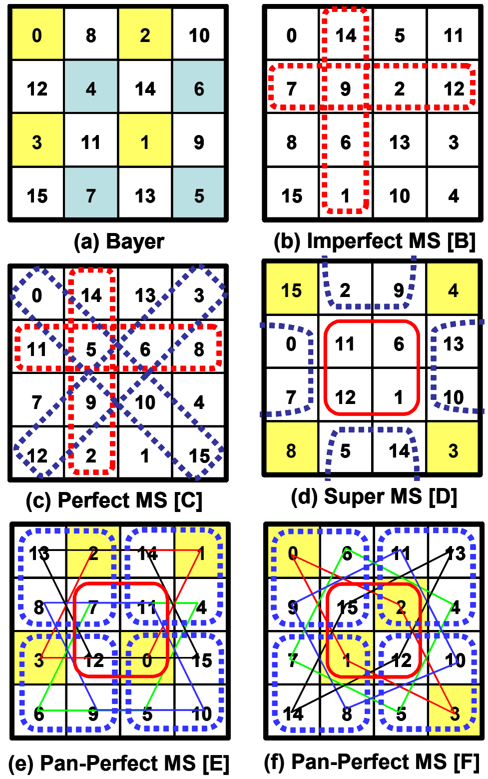

Figure 3.

Pan Magic Square and Its Extension

Our "*pan-perfect*" *MS* [E] or [F] in Fig. 3 is more interested because their threshold matrices have a marvel of order and neatness interlaced in geometry.

As the size of matrices is not enough to use, we developed a "*master-slave"* method for extending them into large size.

Master-Slave Extension Method for MS

Magic square has the basic characteristics of "*additivity*" and "*proportionality*" between the two different masks that can produce a new MS.

Figure 4 illustrates how a small size *MS* is extended to large size in the case of *M=N=4*.

The master *MS* with dimension *M* is extended to *M* × *N* by a combination of slave *MS* with dimension *N* as follows.

1. Magnify the entries by N^2 in master mask $\boldsymbol{D}_M = [m(i, j)]$: *i, j=1~M*, and repeat copies *M* times both in row and column direction resulting in *MN*×*MN* enlarged mask.
2. Add the entries in slave mask $\boldsymbol{D}_S = [s(k, l)]$: *k, l=1~N* to each corresponding *M*×*M* block of enlarged mask."

The threshold mask of extended new "*pan-perfect*" *MS* $\boldsymbol{D}_E = [e(m, n)]$: *m, n=1~MN* is simply calculated by

$$e(m,n) = M^2 m\big(Mod[i-1,M]+1, Mod[j-1,M]+1\big) + s\big(Floor[(i-1)/M]+1, Floor[(j-1)/M]+1\big) \tag{4}$$

Where, *Mod*[•] and *Floor*[•] denote the modulo-*M* operator and the round operator to integer omitting the fractions.

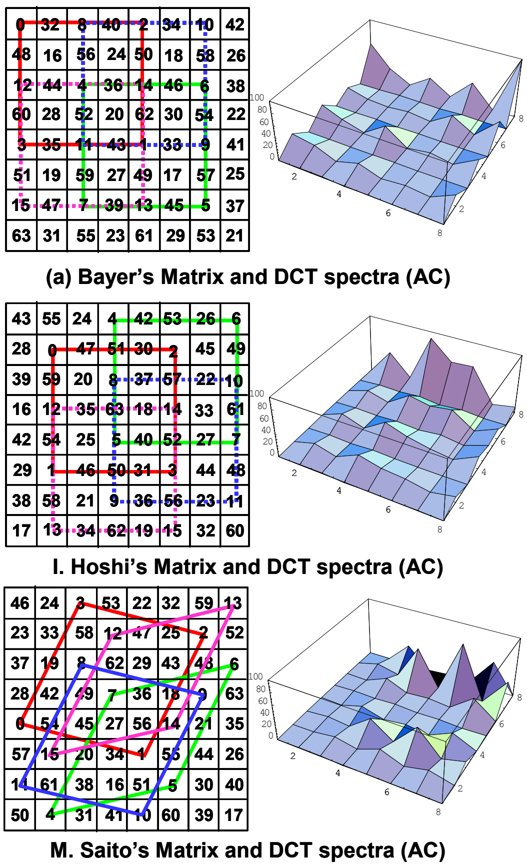

Figure 4. Matrices and DCT spectra of MS vs. Bayer

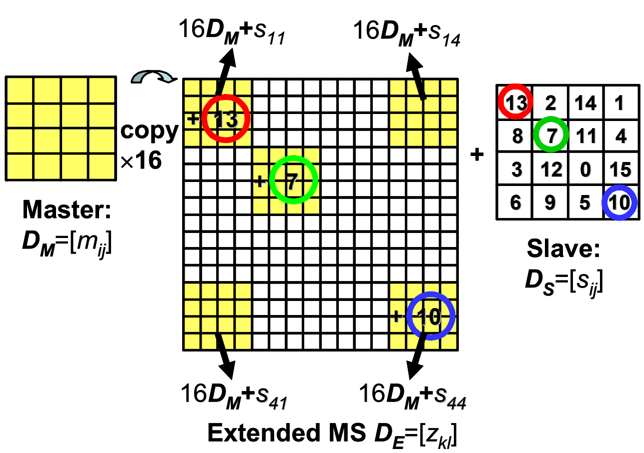

Figure 5. Extension of MS by "master-slave" system

DCT Spectra of Extended MS Matrices

Figure 6 shows some examples of matrices displayed as a gray pattern and their *DCT* spectra for *MN* =16 "*pan-perfect*" *MS* extended from 4 × 4 [E] or [F] in Fig. 5. [E][E] or [E][F] denotes master=slave=[E] or master=[E] and slave=[F].

In the proposed "*master-slave*" method, the spectral feature of "*master*" appears stronger than "*slave*" in its DCT.

It shows uniquely concentrated spectral distributions as compared with that of broadly spread *Bayer*'s.

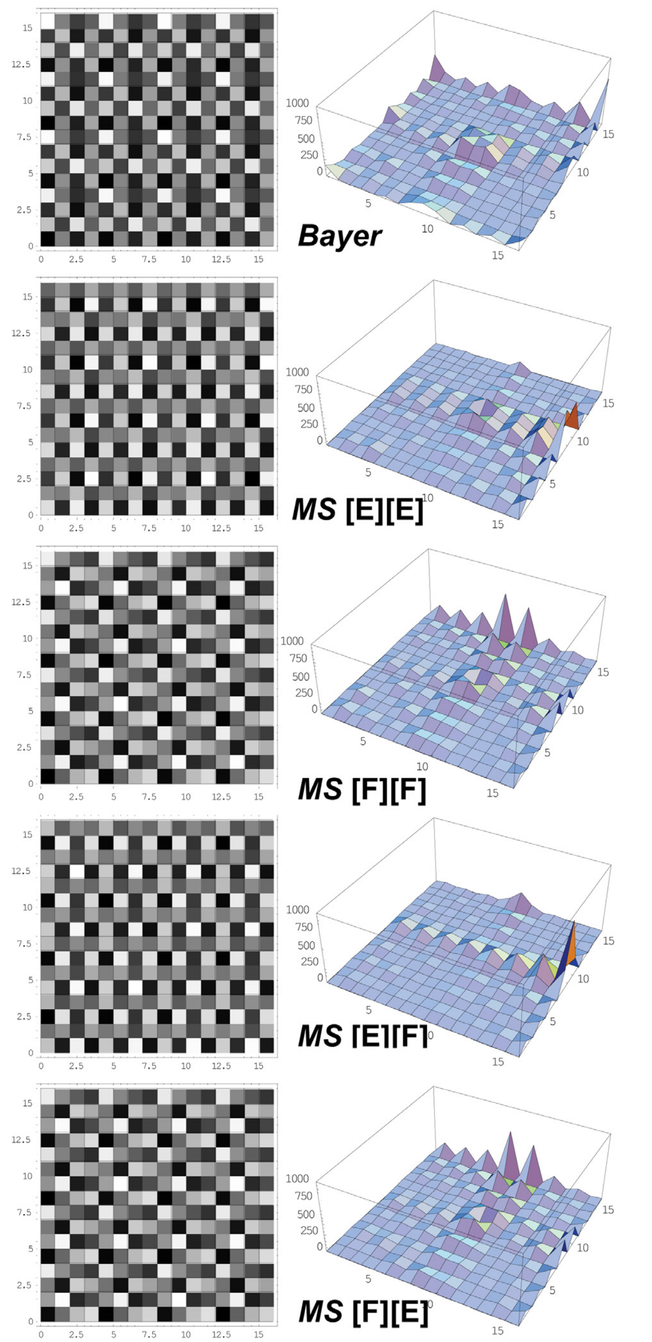

Figure 6. Example of extended MS and its DCT spectra

Experimental Results

Figure 7 shows a part of close-up in tonal rendition. Different textures are visible in *Bayer* (a) and specified regular and irregular patterns are striking in *ED* (b). Similar textures are existent in our proposed *MS* but less visible in (c) or (d). Our "*pan-perfect*" *MS* [F] worked excellent in gray scales.

Figure 8 shows a result for natural color images. When looking the sample (a) on display, *ED* may be best and the proposed *MS* [F][F] will come next, but better than *Bayer* in the background sky of "*Sphinx*". In sample (b), the proposed *MS* resulted in the better tonal rendition than *ED* with unwanted "*wormy pattern*" in "*red pimiento*".

However, we have not examined all of the possible "*pan-perfect*" *MS* yet and a program for searching the better *MS* systematically is under planning.

Discussion

Bayer's dither is designed under a clear criterion to have the maximum spatial frequency. This criterion is simply satisfied for 2 × 2 minimum matrix $\boldsymbol{D}_2$ by *Limb* and systematically extended to a large size $\boldsymbol{D}_N$ ($N=2^n$) by *Bayer*. Surely *Bayer*'s array gives the highly *"dispersed"* dots, but the texture changes in each gray level cause visible artifacts in tonal reproduction with wide-spread Fourier spectra.

Ostromoukhov et al[8] reduced these artifacts by applying a "*rotated Bayer mask*" replicated in discrete Cartesian coordinates. This simple idea has the same objective as our *MS* to generate "*small clustered*" dot patterns. We have approached to this objective from a different point of view.

1. The entries in *Bayer*'s matrix are strongly interlaced with a four pair of "alternating" large and small figures in row and column direction but not so well balanced locally.
2. The entries in perfect *MS* have the equal local sum in all row, column, and diagonal well-balanced entirely.
3. In addition, "*super MS*" has the extended equal sum to the nearest neighbor cells.
4. The "*pan-perfect MS*" maybe a subspecies of "*super MS*" and has a unique and a beautiful interlaced structure.

Conclusions

The paper discussed the application of *MS* to digital halftoning. We proposed a novel idea for choosing a master *MS* and a concrete method for extending the mask size enough to represent smoothed gray scales.

1. "*pan-perfect*" *MS* with the equal local sum in all 2 × 2 nearest neighbors was selected as a promising candidate for generating a spatially balanced "small-clustered & moderately dispersed" dots.
2. The extended "*pan-perfect*" *MS* worked better in tonal rendition than *Bayer* or *ED* depending on the images.

MS structured in "*perfect*" and "*interlaced*" neatly will be most hopeful. We have discovered only a few "*quasi-best*" candidates but more chances are still left for future works.

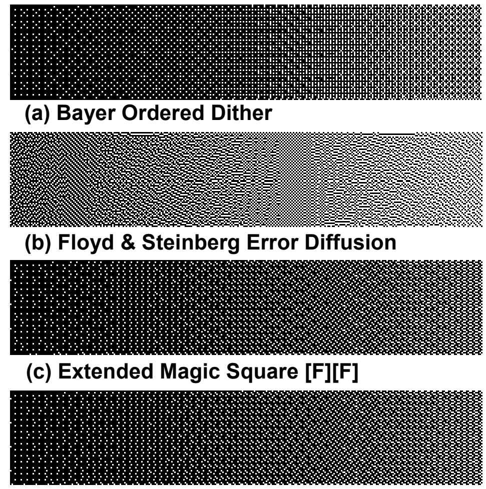

Figure 7. Tonal reproduction in a part of gray scales

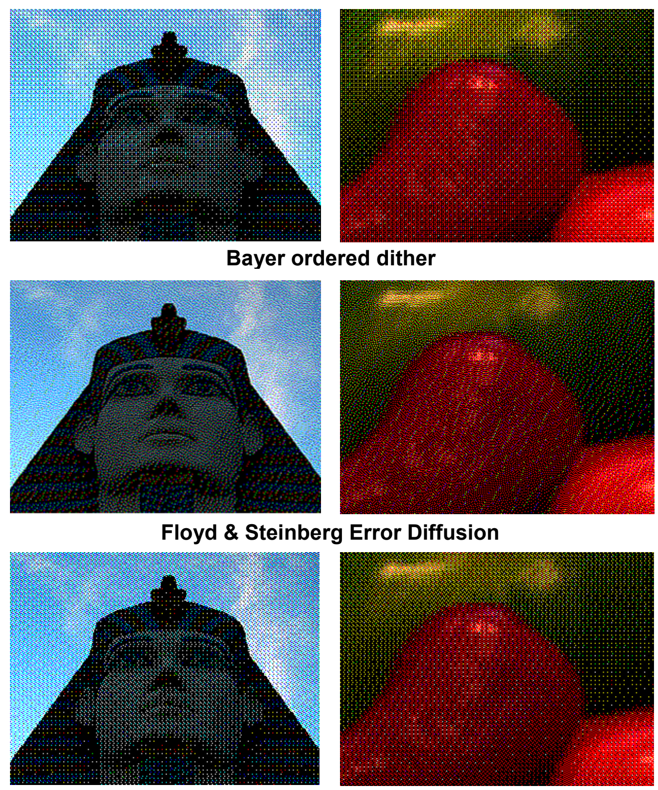

Figure 8. Reproduced images (a) "Sphinx" and (b)"Onion"

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Author Biography

Hiroaki Kotera received his B.S degree from Nagoya Inst. Tech. and Doctorate from University of Tokyo. He joined Matsushita Electric Industrial Co. Since 1973, he worked in digital image processing at Matsushita Res. Inst. Tokyo, Inc. In 1996, he moved to Chiba University. He is a professor at Dept. Information and Image Sciences. He received Johann Gutenberg prize from SID in 1995 and is a Fellow of IS&T.