Stabilization of Color Tone Reproduction Curves Using Time-Sequential Sampling

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Abstract

In xerographic printing, stabilization of the color reproduction function (CRC) or the tone reproduction curve (TRC) for the color separations are critical for achieving color consistency. This problem is challenging because: 1) there exist only a small number of actuators to stabilize a potentially infinite dimensional TRC; 2) only a small number of tone / color test patches can be printed and measured at a time. The first issue is addressed by a curve-fitting TRC stabilization controller based on Linear Quadratic (LQ) control with integral dynamics is proposed. It specifies particular tones or color that to be precisely controlled, while allowing other tones or colors to be close to the desired value. To address the second problem, time-sequential sampling is proposed to enable the time varying TRC or CRC to be reconstructed based on small number of samples. Experimental and simulation results verify the adequacy of the approach.

Introduction

A Xerographic color printer can be represented by the color reproduction function; CRC : $\mathcal{C} \to \mathcal{C}$, $\mathcal{C}$ is a 3-dimensional color space, that maps the desired color into the output color. This map is the identity for an ideal printer that produces consistent color. The printing of each of the CYMK color separation is characterized by the tone reproduction curve for that color TRC : $[0, 1] \to \mathcal{C}$, mapping the desired tone ($\in [0, 1]$) to the printed color. The 3-dimensional color representation for the output-color can be reparameterized into a 1-dimensional representation based on the procedure described in [1] resulting in a representation of the TRC : $[0, 1] \to \Re$, mapping the input tone to the output tone.

In this paper, we consider the problem of stabilizing TRCs for individual color separations using time sequential sampling. The proposed approach can be extended to the stabilization of the CRC as well. CRC and TRC stabilizations are concerned with maintaining constant the CRC and TRC mappings, i.e. the input color or tone consistently produce the same output in the presence of disturbances such as humidity, temperature, and material age. This makes use of xerographic actuators such as laser power, charge and development voltages etc. Once this is achieved, the CRC and the TRC can be inverted via pre-filtering to achieve the ideal identity map.

Challenges in the CRC or TRC stabilization are largely to related to the limited number of actuators and limited sensing capabilities in the face of TRC and CRC being high dimensional objects. The dimensionality M of the TRC is determined by the level of tone discretization; and the dimensionality of the CRC is of the order of $M = c_r^3$ where c_r is the discretization of each color coordinate (e.g. $M = 16^3 = 4096$ for $c_r = 16$). In contrast, the number of xerographic actuators is of the order of $m \approx 3$ per color separation. Sensing of the TRC and CRC is currently achieved by printing and measuring small patches (n = 3 to 5 per belt cycle) of uniform tone or color. Increasing the number of test patches increases hardware needs as well as consumables (toner) and productivity.

The limited actuator issue is addressed by a curve-fitting approach based on linear quadratic control with integrator dynamics. This approach allows the designer to specify q particular tones or colors ($q < m$) which will be precisely regulated while allowing the TRC or CRC to stay close to the desired values at the other tones or colors.

The limited sensing issue is addressed by time-sequential sampling to increase the utility of available feedback information [6]. In this approach, test patches of *different* tones or colors are printed at different times. Previous sensing schemes use test patches that are fixed. Time-sequential sampling was investigated in the 1980's and 1990's for video and time varying imaging applications [3][4][5]. The derivation and analysis of time-sequential sampling and reconstruction of the TRC is given in [6]. This paper present results in which the TRC stabilization controller is combined with time-sequential sampling.

Problem Formulation

The time-varying $TRC(k)$: [0,1] $\to \Re$ gives the input tone to output tone of the xerographic printing process. The TRC is a mapping, it is potentially infinite dimensional. In this paper, we assume that the TRC can be adequately described by its values at M points, i.e.

$$TRC(k) = \begin{pmatrix} TRC(k)[tone_1] \\ \vdots \\ TRC(k)[tone_M] \end{pmatrix} \in \Re^M,$$

where M can be fairly large. As noted in [2], the possibly nonlinear TRC can be represented by the static, linear time varying, uncertain model as follows:

$$TRC(k) = \hat{\phi}(I + \Delta(k)W_u)\bar{u}(k) + TRC^* + \bar{d}(k) \quad (1)$$

where $u(k) \in \Re^m$ are the xerographic actuators, $d(k) \in \Re^M$ are the disturbances, and $TRC^* \in \Re^M$ is the nominal TRC and $\bar{d}(k) \in \Re^M$ is a slowly time varying disturbance. Also, $\bar{u}(k) := u(k) - u_o$, where u_o is the nominal control input. $\hat{\phi} \in \Re^{M \times m}$ is the nominal sensitivity function, $\Delta(k) \in \Re^{m \times m}$ is the multiplicative uncertainty, $W_u \in \Re^{m \times m}$ is the matrix of given uncertainty weights. In this paper, we will assume that there is no uncertainty in the model (i.e. $\Delta(k) = 0$ in (1)).

Sensing of the $TRC(k)$ at time instant k is achieved by printing and measuring $n << M$ tones in the form of small test patches. Typically, the same set of n tones is printed at each k and n will be determined by the number of available sensors, as well as the productivity and materials cost of printing the test patches.

In [6] and here, we propose to print n different tones at different time k according to the $M-$ periodic time-sequential (TS) sampling pattern: given by $\alpha(k) = \alpha(k + M) = [\alpha_1(k), \alpha_2(k), \ldots, \alpha_n(k)]$ so that at time k, tones determined by $\alpha_i(k)$, $i = 1, \ldots, n$ are printed and measured. In this paper, we focus on $n = 1$. This allows each of the M-tones to be sampled at some time. The sampling sequence $\alpha(k)$ defines an indicator matrix sequence $C_\alpha(k) \in \Re^{n \times M}$ such that on (i, j) element of $C_\alpha(k)$ is a 1 when $j = \alpha_i(k)$ and it is there 0 otherwise.

The time sequentially (TS) sampled TRC is therefore given by:

$$TRC_s(k) = C_\alpha(k)TRC(k) \in \Re^{n=1}. \quad (2)$$

Two sequences that will be pursued in this paper are (Figure 1): lexicographic sequence, $\alpha(k) = \text{mod}(k, M)$, and the bit-reversed sequence where $\alpha(k)$ is given by reversing the order of the significants bits for the binary representation of the index-k. The idea behind the bit-reversed sequence is that it is roughly a uniform sampling of the time-tone space (see Figure 1).

The general control objective is to control the TRC so that it matches the desired nominal TRC at each $tone_i$, $i = 1, 2, ..., M$, as $k \to \infty$:

$$TRC(k)[tone_i] \to TRC^*(k)[tone_i] \quad (3)$$

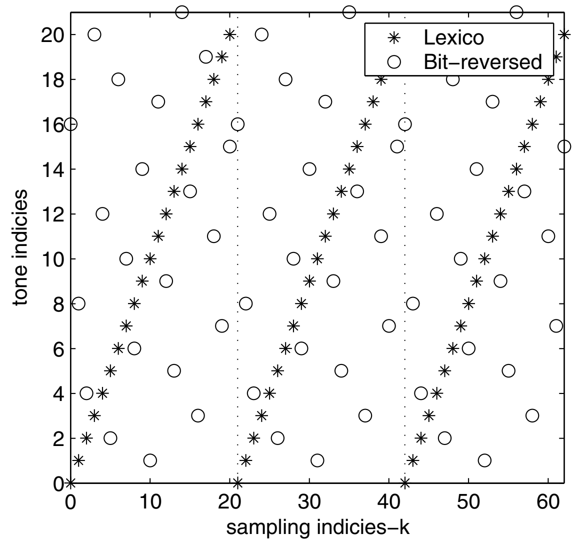

Figure 1: Lexicographical and bit-reversed time sequential sampling sequence with $M = 21$ tones.

where $TRC^*(k)$ is the desired TRC. Since there are fewer actuators m than the number of tones M to be controlled, it is typically not possible for Eq.(3) to hold for all M tones in the presence of disturbance $\bar{d}(t)$. One possibility is to require that Eq.(3) is satisfied only at m instead of all M tones. Theoretically, this can be achieved using integral control for constant or slowly varying disturbances. However, as shown in [2], the integral control approach may lead to mis-behavior at the unspecified tones and poses robustness problem in the presence of model uncertainty. Instead, a curve-fitting approach is proposed in [2] to minimize the 2-norm error of the TRC over the entire tone range. In this paper, the curve fitting approach also allows $q < m$ tones to be precisely controlled.

Reconstruction of TS sampled TRC by Kalman Filter [6]

Let $\Delta TRC(k) := TRC(k) - TRC^*$ be the TRC error. Neglecting model uncertainty ($\Delta(k) = 0$), the xerographic plant given in (1) becomes:

$$\Delta TRC(k) = \hat{\phi}\bar{u}(k) + \bar{d}(k). \quad (4)$$

To exhibit its tonal spectral contents, the TRC disturbances is modeled by its DFT so that:

$$\bar{d}(k) = G \cdot x_d(k) \quad (5)$$

where $k \in \mathcal{Z}^+$ is the index, $G \in \Re^{M \times M}$ is a matrix of Fourier basis function and $x_d(k) \in \Re^M$ is the vector of Fourier coefficients representing the tonal frequency content of the disturbance. Substituting (5) into (4), we have:

$$\Delta TRC(k) = \hat{\phi}\bar{u}(k) + G \cdot x_d(k)$$

The disturbance dynamics, $x_d(k)$ is modeled as pink noise having a compact tonal-temporal spectral support as generated by the following dynamics:

$$\begin{aligned} x_w(k+1) &= A_w x_w(k) + B_w w(k) \\ x_d(k) &= C_w x_w(k) + D_w w(k) \end{aligned} \tag{6}$$

where $w(k)$ is a white process noise. The matrix A_w, B_w, C_w and D_w are obtained from a bank of low-pass butterworth filters that filter each spatial channel(each Fourier coefficients) with temporal cutoff frequencies corresponding to an ellipse. This gives an approximation of a compact ellipsoidal tonal-temporal spectral support. From the definition given in (2), the time-sequentially sampled signal subject to measurement noise $n(k)$ is given by:

$$\begin{aligned} \Delta TRC_s(k) - C_\alpha(k)\hat{\phi}\bar{u}(k) = C_\alpha(k)GC_w x_w(k) + \\ C_\alpha G D_w w(k) + n(k) \end{aligned} \tag{7}$$

where both $w(k)$ and $n(k)$ is zero-mean white noise sequences with covariance R_{ww} and R_{nn} respectively.

By treating the LHS of (7) at the measurement, the M-periodic linear system given by (5), (6) and (7) admits a M-periodic Kalman filter for estimating the disturbance $\bar{d}(k)$:

$$\begin{aligned} \hat{x}_w(k+1) =& A_c(k)\hat{x}_w(k) + B_c(k)\Delta TRC(k) - \\ & B_c(k)\hat{\phi}\bar{u}(k) \\ \widehat{d}(k) =& \tilde{G} \cdot \hat{x}_w(k) \qquad (8) \\ \Delta \widehat{TRC}(k) =& \widehat{d}(k) + \hat{\phi}\bar{u}(k) \qquad (9) \end{aligned}$$

where

$$\begin{aligned} A_c(k) &= A_w(I - L(k)C_\alpha(k)\tilde{G}) \\ B_c(k) &= A_w L(k) C_\alpha(k) \\ \tilde{G} &= GC_w \end{aligned}$$

and $L(k)$ is the periodic Kalman filter gain obtained by solving the periodic Riccati equation:

$$\begin{aligned} \bar{P}(k+1) =& A_w[\bar{P}(k) - L(k)C_\alpha(k)\tilde{G}\bar{P}(k)]A_w^T + \\ & B_w R_{ww} B_w^T \\ L(k) = \bar{P}(k)\tilde{G}^T C_\alpha^T(k) & \left[R_{nn} + C_\alpha(k)\tilde{G}\bar{P}(k)\tilde{G}^T C_\alpha^T(k)\right]^{-1} \\ \bar{P}(k) =& \bar{P}(k+M) \qquad (10) \end{aligned}$$

For further analysis and derivation of time-sequential sampling with reconstruction using Kalman filter, the readers are referred to [6].

TRC Stabilization Controller

The high dimensionality of the TRC coupled with limited actuation does not permit us to keep track of all the color tones. Moreover, we have to consider uniformly sampling of only a finite M-tones. We consider an optimal control approach to ensure each of these M-tones achieve the nominal TRC in a least-squared sense. We also impose an integrator dynamics on the optimal control formulation to ensure certain q-tones ($q \leq m$) achieve (3). This feature would be extremely useful when it is desirable to have certain q-tones to coverage exactly to the desired tones. This however will come at the expense of reducing the freedom to curve fit the TRC onto the desired TRC. Hence the optimal control problem is to find the control $u(k)$ based on the measured TRC, $TRC(k)$, such that the following quadratic performance index(QPI), J is minimized:

$$\begin{aligned} J = \frac{1}{2}\sum_{k\in\mathcal{Z}} \Delta TRC_i^T(k) Q_i \Delta TRC_i(k) + \\ \frac{1}{2}\sum_{k\in\mathcal{Z}} \Delta TRC^T(k) Q \Delta TRC(k) \end{aligned}$$

where the integrator dynamics are given by

$$\Delta TRC_i(k+1) = \Delta TRC_i(k) + C_i \Delta TRC(k) \tag{11}$$

where $C_i \in \Re^{q\times M}$ is the indicator matrix for the selected q-tones to fulfill (3). $Q_i \in \Re^{q\times q}$ and $Q \in \Re^{M\times M}$ are the weighting matrices. The linear-quadratic state feedback follower-controller for the given QPI for system (4) can be solved by using the backward-sweep solution [7]. Assuming the disturbance $\bar{d}(k)$ is available, the optimal control is given by:

$$\bar{u}(k) = -K_1 \Delta TRC_i(k) + K_2 \bar{d}(k) \tag{12}$$

where

$$\begin{aligned} K_1 =& Z_{ww}^{-1} Z_{xw}^T \\ K_2 =& -Z_{ww}^{-1}(\hat{\phi}^T C_i^T S_B C_i + \hat{\phi}^T Q - \hat{\phi}^T C_i^T F_B) \end{aligned}$$

are the feedback and feedforward gains respectively, with $Z_{ww} := R_u + \hat{\phi}^T C_i^T S_B C_i \hat{\phi}$; $Z_{xw} := S_B C_i \hat{\phi}$; $R_u = \hat{\phi}^T Q \hat{\phi}$. S_B is obtained from the solution of the discrete algebraic Riccati equation.

$$\begin{aligned} & I^T S_B I - S_B - S_B C_i \hat{\phi} \cdot \\ & (R_u + \hat{\phi}^T C_i^T S_B C_i \phi)^{-1} (S_B C_i \hat{\phi})^T + Q_i = 0 \end{aligned}$$

and,

$$\begin{aligned} F_B =& (Z_{xw} Z_{ww}^{-1} \hat{\phi}^T C_i^T)^{-1} \cdot \\ & (Z_{xw} Z_{ww}^{-1} (\hat{\phi}^T C_i^T S_B C_i + \hat{\phi}^T Q) - S_B C_i) \end{aligned}$$

To implement Eq.(12), the Kalman filter estimate of the TRC disturbance based on time-sequential sampling, $\hat{d}(k)$ in Eq.(9), is used in lieu of the actual disturbance

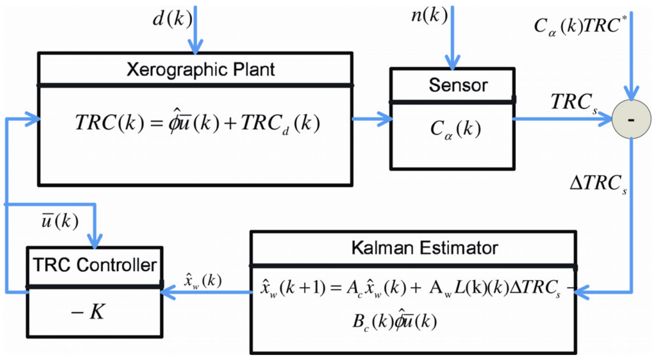

Figure 2: Kalman estimator of time-sequentially sampled TRC and TRC stabilization controller mechanization

$\bar{d}(k)$. $\Delta TRC_i(k)$ is obtained from Eq. 11 where the Kalman estimate of the TRC error based on time-sequential sampling, $\Delta\widehat{TRC}(k)$ in (9), is used in lieu of the actual TRC error, $\Delta TRC(k)$.

Figure 2 shows the schematic of the TRC stabilization controller with time sequential sampling. The well known separation principle [8] allows the controller and estimator to be designed separately yet used together.

A robust static controller was previously proposed in [2] which also uses the curve-fitting approach (i.e. it tries to minimize the overall TRC error). A major issue addressed in [2] is that fixed TRC sampling is assumed, and the number of tones n that are measured is small compared to the dimensionality of the TRC (M). The proposed control ensures that the TRC behaves adequately even at unmeasured tones in the presence of uncertainty. With the use of time-sequential sampling, the apparent number of measured tones can be significantly increased, thus relaxing this difficulty. Nevertheless, the time-sequential sampling can be used with the robust static control law in [2] to take advantage of its robustness feature.

Simulation and Experiments

Simulation

The behavior of the xerographic plant is simulated by taking the model of the form of (1) based on TRC experimental data from a commercial printer using different xerographic inputs. The nominal sensitivity matrix $\hat{\phi}$ in (1) is obtained by least-square fitting of the experimental data into the linear model. The behavior of the system without plant perturbation is considered i.e. $\Delta(k) = 0$. The disturbance $\bar{d}(k)$ dynamics is simulated from the pink noise model given in (6) with the temporal cutoff frequency, f of the butterworth filter at each spatial channel, u given by an ellipse:

$$(u/U)^2 + (f/W)^2 = 1$$

where U and W gives the highest tonal and temporal frequencies in $\bar{d}(k)$. In our study, we used a sampling interval of $T = 0{:}4s$ and the tonal range is $tone_i \in [0,1]$. With $M = 21$, this gives a tonal temporal Nyquist frequencies of $(u_N, f_N) = (10.5\text{cycles/tone}, 0.06\text{Hz})$.

We consider two cases of sampling: full sampling where all M-measurement points are used at each sampling instant, k and time sequential sampling where only one tone is sampled at each sampling instant according to a prescribed sampling pattern i.e. lexicographical or bit-reversed sampling sequences. As our primary interest is to analyze the effect of disturbances with different sampling schemes, we assume the measurement noise $n(k) = 0$.

The controller that minimizes (11) with one ($q = 1$) integral fix points at $tone_2$ is obtained from (12). The performance weighting matrices Q_i and Q in (11) are identities.

The root mean square of the TRC errors,

$$\left[\sum_k \|\Delta TRC(k)\|_2^2\right]^{1/2}$$

for all M-points is taken as the measure of performance of the TRC stabilization controller. Simulations were carried out at different tonal-temporal support frequencies (U;W) for $\bar{d}(k)$ within the range of $\{(U,W)|1 \le U \le u_N, 0{:}01 \le W \le 6f_N\}$ using the optimal TRC stabilization controller. The comparable RMS of time-sequential sampling (lexicographical sampling sequence) and full sampling as shown in Figure 3 means that we are able to achieve good TRC stabilization by just sampling one TRC tone at each time step. Figure 3 also shows the expected response in increasing the tonal-temporal support frequencies of the disturbance signal – the higher the tonal-temporal support frequencies, the higher the RMS of TRC error. The higher tonal-temporal disturbance frequencies the harder it is for the TRC controller to curve-fit the measured TRC to the desired TRC. Figure 4 compares the difference in RMS TRC error between lexicographical and bit-reversed sampling sequences. This shows the clear advantage of using the bit-reversed time-sequential sampling sequence over the lexicographic sequence. The better signal reconstruction of the bit-reversed sampling sequence as reported in [6] translates to better TRC stabilization control performance. Hence the bit-reversed sampling sequence is preferred for our application where sensing is expected to be highly sparse.

Experiment

The proposed TRC stabilization system was also experimentally tested on a Xerox Phaser 7700 xerographic printer.

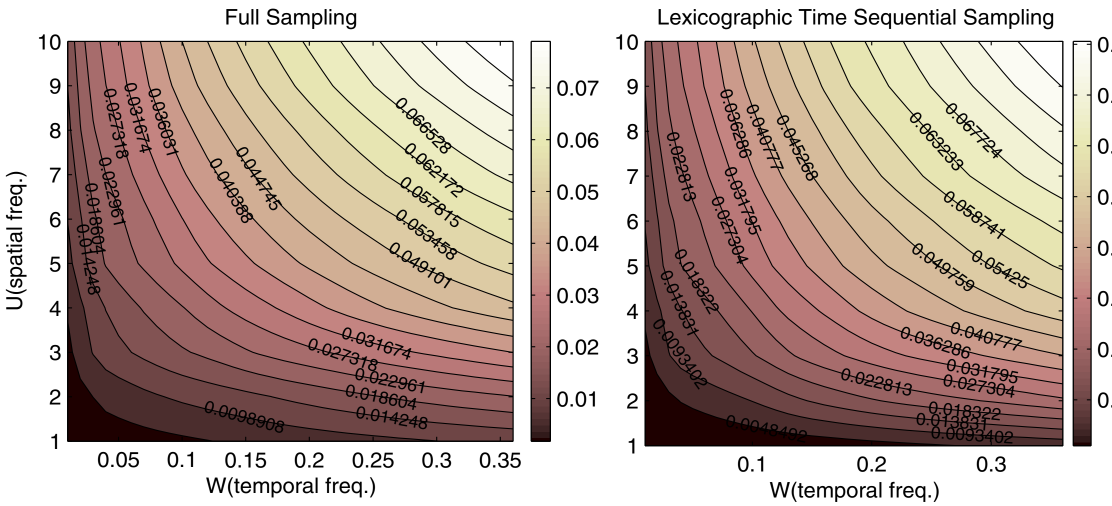

Figure 3: *RMS of TRC error using full and lexicographic time-sequential sampling at different tonal-temporal frequencies support* (U, W)

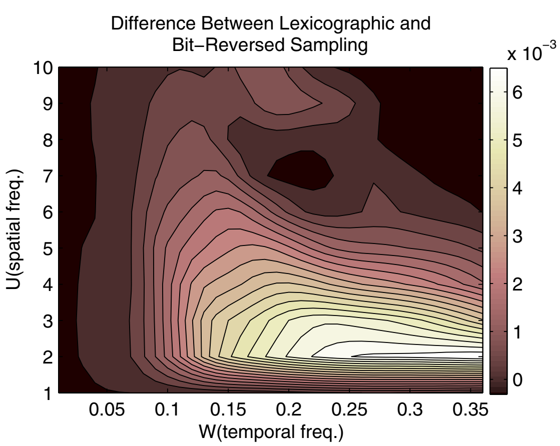

Figure 4: *Difference in RMS of TRC error* ($RMS_{Lexico} - RMS_{Bit\text{-}reverse}$) *of lexicographical and bit-reversed time sequential sampling for different tonal-temporal support frequencies* (U, W).

Currently we do not have direct access to the xerographic actuators. To evaluate the TRC stabilization controller with full and time sequential sampling, a virtual printer model is used to generate the response (color image) due to changes in the actuator inputs. The virtual printer model is the one used in the simulation study above. The output response from the virtual printer is then printed using the physical printer. By calibrating the printer such that it is an identity map at the nominal condition, we can capture the effect of the actual disturbances on the performance of the TRC stabilization system. The output response is in the form of a single colorant wedge of 21 different tones i.e. $M = 21$. The disturbances was artificially induced by introducing a transparency in the optical path of the laser. Sensing of the color wedge is performed using a scanner that has been calibrated using a spectrophotometer.

Figure 5, 6 and 7 show the effectiveness of the proposed TRC stabilization system using both full and time sequential sampling. The TRC stabilization using all the sampling approaches result in the convergence of the TRC to the nominal TRC with each time step. Considering that only one tone is sampled at each time step, the time sequential sampling perform well compared to that achieved using full sampling($M = 21$).

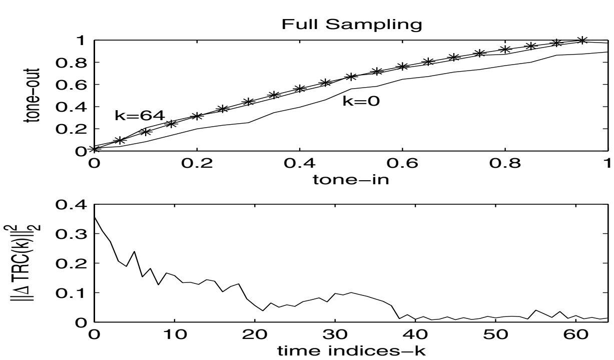

Figure 5: *Response of TRC stabilization control subjected to induced disturbance with full sampling. The curve marked with * is the desired TRC*

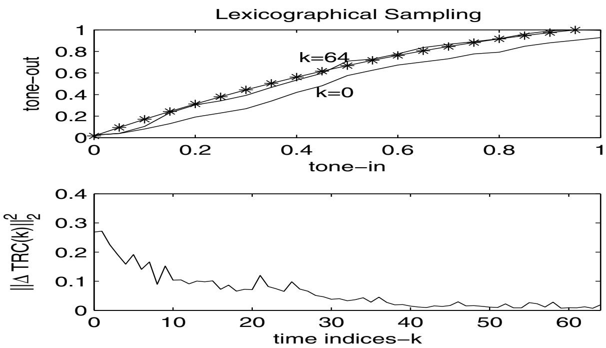

Figure 6: *Response of TRC stabilization control subjected to induced disturbance with time sequential sampling (Lexicographical sequence). The curve marked with * is the desired TRC*

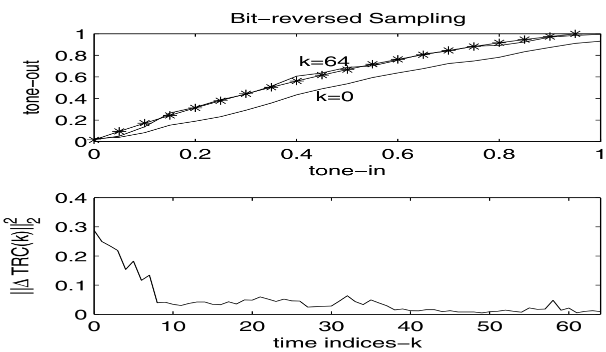

Figure 7: *Response of TRC stabilization control subjected to induced disturbance with time sequential sampling (Bit-reversed sequence). The curve marked with * is the desired TRC*

Conclusion

This paper addressed two main problems in realizing a practical TRC stabilization controller in maintaining consistency in color reproduction. The first problem of under actuation is resolved using a curve-fitting optimal control approach. The proposed TRC stabilizing control makes use of all available measurement/reconstruction data and allows certain tones to be regulated to converge to the corresponding desired tones. The second problem relating to limited sensing capability is resolved using time sequential sampling. Both simulation and experimental results shows the effectiveness of the proposed approach. In particular we demonstrated the comparable stabilization property in using time sequential sampling in place of full sampling. The bit-reversed sequence is also found to yield better signal reconstruction as has been reported in [6]. This result translates to better TRC stabilization control performance as reported here. Time sequential sampling substantially lower the TRC sensing requirements and this is important in actual implementation where available print area should be devoted to customer images and not in printing sensor patches.

The next step would be to expand the idea to cover stabilization of not only one single color separation as addressed here, but to all different color combinations. The same basic idea as proposed here should apply.

Acknowledgment

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Biography

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