Representation of Primary Color Tone Reproduction Curves for their Stabilization

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Abstract

This paper proposed and compared different methods of representing the color tone reproduction curve (TRC) for stabilization of the monochorome xerographic printing process. TRC stabilization is vital in maintaining high consistency in color printout. Colors are typically represented by a 3-dimensional representation such as the CIELAB or CIEXYZ colorspaces. For stabilization control, the 3-dimensional representation is redundant due to the fact that limited actuators authority would not account for all the degree of freedom in the data representation. A 1- dimensional reparameterization of the 3-dimensional representation is better suited for the stabilizing control. The proposed parameterization methods involved projection of the 3- dimensional representation onto a parameterization curve. An empirical-based and model-based approach was used to describe this curve. Simulation and experimental results are presented to demonstrate the performance of the different parameterization method subject to measurement noise.

Introduction

In color printing, color reproduction with high consistency and fidelity is very important. Unlike typical black and white printers, image defects in color image composition is highly noticeable. In term of the color reproduction, the xerographic color printer is well represented by the color reproduction function, CRC :$C \rightarrow C$, desired-color $\mapsto$ output-color, where C is a 3-dimensional color space. It is desirable to have CRC to be an identity map.

The printing process of a particular color separation is characterized by the tone reproduction curve, TRC:[0,1] $\rightarrow$ C, desired tone $\mapsto$ output-color, where a solid colorant is represented by 1, and 0 represent the background without any colorant. Current concepts of sensing the xerographic process generally involve printing a small number of small patches of single primary color tone images which are subsequently read. The reading can be performed using a toner area coverage sensor (TAC) which monitor the development of the color tones on the organic photoconductor belt. With the advances of spectrophotometers and calorimeters, we expect that these patches can be similarly read to give the 3-dimensional color measurement data. Such sensing concept allows sensing of the entire color gamut and this represents a critical step in achieving color printout with high fidelity. Our work on TRC stabilizing control center around this sensing concept where the output-color, C is given by a 3-dimensional representation.

In this paper, we proposed and compared different methods of parameterizing the 3-dimensional representation to a 1-dimensional representation such that the TRC:$[0, 1] \rightarrow \Re$, maps the input-tone to the output-tone. The 3- dimensional representation can be reparameterized to a 1- dimensional representation because the primary color curve is invariant to changes in the xerographic printing process due to disturbances or actuator inputs. Therefore all the primary color tones are well defined by a single color curve in the 3-dimensional color gamut. This fundamental property is the key in reducing the dimensionality of the 3- dimensional representation because it allows a fix parameterization curve to be defined as a measure of the 3- dimensional representation. In this paper, we evaluate the effectiveness of the different parameterization methods subject to measurement noise. The most effective parameterization method would be the one that is least sensitive to the measurement noise.

The resulting TRC representation can then be effectively stabilized using the TRC stabilization controller as proposed in [1][2].

Colorspace Parameterization

Lets define a space, X with a color difference function $d_X(C_1, C_2) = \Delta E_C \in \Re$, where $C_1, C_2 \in X$. The colorspace reparameterization gives a 1-dimensional representation defined on a space, Y. Then the colorspace parameterization function, $\pi : X \rightarrow Y$ assign each 3-dimensional colorspace measurement $C \in X$ an element $\pi(C) = \kappa \in Y$. κ denotes the 1-dimensional parameterization. π can be any function that maps X into $Y, \pi X = \{\pi(C)|C \in X\} \subset Y$ (injective or 1-1 mapping). Figure 1 shows the mapping of different parameterization functions.

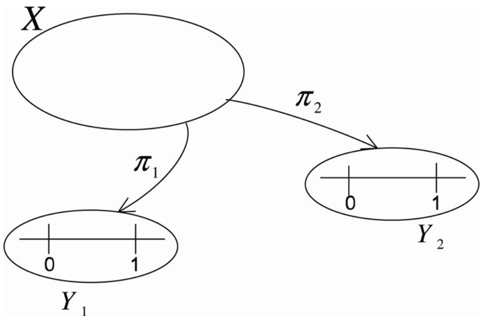

Figure 1: Parameterization map

Assume that the 3-dimensional color measurement is subject to measurement noise, $\varepsilon \in X$. Then $\pi : X \rightarrow Y$ assign each $(C + \varepsilon) \in X$ an element $\pi(C + \varepsilon) = \kappa_\varepsilon \in Y$. We propose that the best parameterization function π yields $min|d_X(\pi^{-1}(\kappa);\pi^{-1}(\kappa_\varepsilon))|$, where $\pi^{-1} : \pi X \rightarrow X$. Essentially the most effective parameterization method is least sensitive to measurement noise.

The map, π can be viewed as the projection of the measured 3-dimensional values onto a parameterization curve in the the 3-dimensional colorspace. The effectiveness of the parameterization is influenced by the parameterization curve itself and the distribution of the measurement noise. Different parameterization method can be obtained through different definition of the parameterization curve. We select the space (X, d_X) of CIELAB in our work because of its perceptually uniform property.

In this paper, we will investigate both empirical and physical model based parameterization methods for a single primary color.

Empirical Based Parameterization

The empirical based approach takes into account the availability of experimental data sets that can be used to characterize the parameterization curve. By considering a single primary color, a color wedge of different tones is printed and measured to obtained the set of experiment data in CIELAB colorspace. We denoted them by:

$$\hat{C}_{lab} = \left\{\hat{C}_{lab}\left[tone_1\right], \hat{C}_{lab}\left[tone_2\right] \ldots \hat{C}_{lab}\left[tone_{N_{exp}}\right]\right\} \in \Re^{N_{exp} \times 3}$$

The parameterization curve is in the form of a line where the concentration of the experimental data, $\hat{C}_{lab}$ is most significant (see Figure 2). An effective procedure for performing this operation is principal component analysis.

Let $v \in \Re^{3\times1}$ be the eigenvector associated with the largest eigenvalue from the eigenvalues and eigenvectors

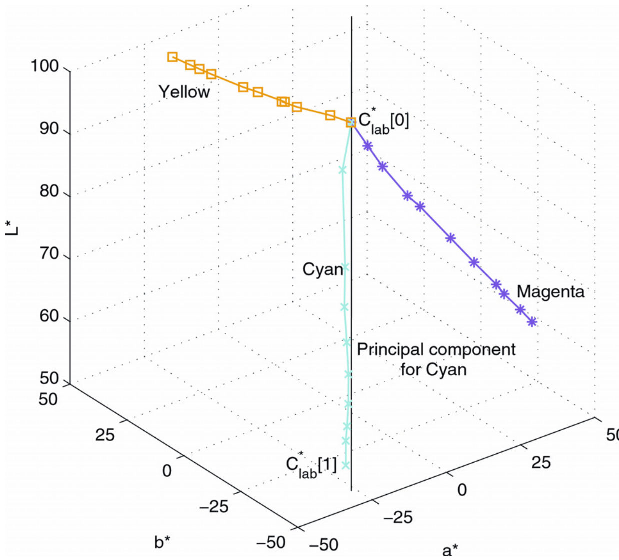

Figure 2: Parameterization by projection onto the principal component

of the covariance matrix. The covariance matrix, S is defined by:

$$S = E\left[\left(\hat{C}_{lab} - \mu_{\hat{C}_{lab}}\right)\left(\hat{C}_{lab} - \mu_{\hat{C}_{lab}}\right)^T\right] \in \Re^{3\times3}$$

where $\mu\hat{C}_{lab} = E\{\hat{C}_{lab}\} \in \Re^{3\times1}$ is the mean of the experimental data. The components of S, denoted by s_{ij}, represent the covariances between the random variable components $\hat{C}_{lab}[tone_i]$ and $\hat{C}_{lab}[tone_j]$. The component s_{ii} is the variance of the component $\hat{C}_{lab}[tone_i]$ and indicates the spread of the component values around its mean value. Hence by definition, v gives the direction of largest variance of the data – it gives the vector direction of the principal component where the experimental data set has the most significant amounts of energy.

Lets further define a nominal, TRC* : $[0,1] \rightarrow C^*_{lab}$. The nominal color representation corresponding to $tone = 0$ is $C^*_{lab}[0] \in \Re^{3\times1}$ and at $tone = 1$ is $C^*_{lab}[1] \in \Re^{3\times1}$. Figure 2 shows the principal component and the associated parameters in the CIELAB colorspace for the primary color separation of cyan. This parameterization function, $\pi_1 : X \rightarrow Y$ is obtained by projection of the 3-dimensional measurement vector, $C_{lab} \in X$ in the CIELAB colorspace onto the principal component. By normalizing this projection such that the range of $\kappa \in [0,1]$, we have:

$$\kappa = \frac{\langle v, C_{lab}\rangle - \langle v, C^*_{lab}[0]\rangle}{\langle v, C^*_{lab}[1]\rangle - \langle v, C^*_{lab}[0]\rangle} \quad (1)$$

where <·> gives the inner product of two vectors.

Physical Model Based Parameterization

For a CMYK color printer, with 4 number of colorants ($n_{color} = 4$) there will be $m = 2^{n_{color}} = 16$ primaries that will be produced through subtractive overlapping of 1, 2, 3 or 4 colorants (dot-on-dot or probabilistic mixing equations or hybrid mixing equation). These colors are called the Neugenbauer primaries and their reflectance are given by $r_i(\lambda) \in \Re^{N \times 1}$; i = 1, 2, . . . , 16. The spectral Neugenbauer model expressed the halftone print as weighted, w_i average of these 16 overlapping combinations. Taking into account the penetration and scattering effects on the substrate called the Yule-Nielsen effect [3], the Yule-Nielsen Spectral Neugenbauer (YNSN) model is given by:

$$r_p^{1/n}(\lambda, w) = \sum_{i=1}^{m} w_i r_i(\lambda)^{1/n} \qquad (2)$$

where $r_p(\lambda, w)$ is the predicted reflectance and n is the Yule-Nielsen correction factor. w_i is known as the fractional area covered by the colorants and $\sum_{i=1}^{m} w_i = 1$ holds (see Figure 3).

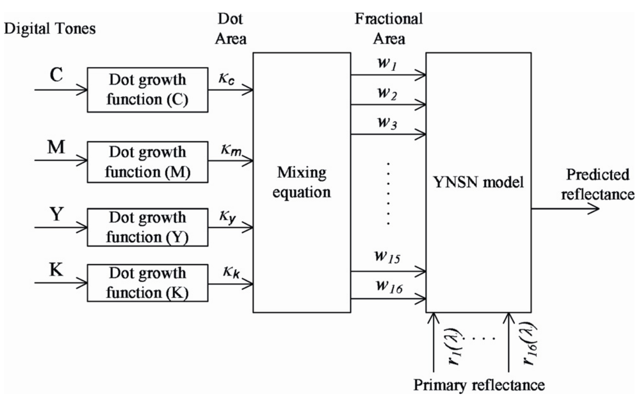

Figure 3: Schematic of Yule-Nielsen Spectral Neugenbauer Model

For a single colorant, $n_{color} = 1$, $m = 2$. By employing a probabilistic mixing model (Demichel equation), we have:

$$w_1 = \kappa$$
$$w_2 = 1 - \kappa$$

where w_1, w_2 give the fractional area of the solid colorant and the background respectively. κ is the dot-area of the colorant and represents the required 1-dimensional parameterization of the 3-dimensional primary color representation. The following derivation gives this relationship.

From (2) we have:

$$r_p(\lambda, \kappa) = \left[\kappa r_1(\lambda)^{1/n} + (1 - \kappa) r_2(\lambda)^{1/n}\right]^n \qquad (3)$$

where $r_1(\lambda)$ denotes the reflectance of the solid colorant (*tone* = 1) and $r_2(\lambda)$ is the background reflectance (tone = 0). The formulation relating the reflectance, $r_p(\lambda)$ to the 3-dimensional CIELAB values is given by [3]:

$$[X(\kappa), Y(\kappa), Z(\kappa)]^T = A \cdot r_p(\lambda, \kappa)$$
$$\tilde{C}_{lab}(\kappa) = \begin{bmatrix} 116 f(Y(\kappa)/Y_n) - 16 \\ 500(f(X(\kappa)/X_n) - f(Y(\kappa)/Y_n)) \\ 200(f(X(\kappa)/X_n) - f(Z(\kappa)/Z_n)) \end{bmatrix} \qquad (4)$$

where X_n, Y_n, Z_n are the tristimuli of the white stimulus and,

$$f(x) = \begin{cases} x^3 & \text{for } x > 0.008856 \\ 7.8787x + 16/116 & \text{for } x \leq 0.008856 \end{cases}$$
$$A := \gamma \begin{bmatrix} S_{\lambda_0}\bar{x}_{\lambda_0} & S_{\lambda_2}\bar{x}_{\lambda_2} & \ldots & S_{\lambda_{N-1}}\bar{x}_{\lambda_{N-1}} \\ S_{\lambda_0}\bar{y}_{\lambda_0} & S_{\lambda_2}\bar{y}_{\lambda_2} & \ldots & S_{\lambda_{N-1}}\bar{y}_{\lambda_{N-1}} \\ S_{\lambda_0}\bar{z}_{\lambda_0} & S_{\lambda_2}\bar{z}_{\lambda_2} & \ldots & S_{\lambda_{N-1}}\bar{z}_{\lambda_{N-1}} \end{bmatrix} \in \Re^{3 \times N}$$
$$\gamma = \frac{100\Delta\lambda}{\sum_\lambda (S_\lambda \bar{y}_\lambda \Delta\lambda)}$$

$\{\lambda_i\}_{i=0}^{N-1}$ are uniformly spaced wavelength with interval of $\Delta\lambda$ covering the visible region of the spectrum. S_λ is the power spectral density of the illuminant used and $\{\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda\}$ is the relative spectral sensitivity of a standard observer. We will be using CIE illuminant D50 and the 2° 1964 CIE standard observer.

Equation (4) gives an inverse parameterization function, $\pi_2^{-1} : \pi_2 X \rightarrow X$ that assign each 1-dimensional representation, $\kappa \in \pi_2 X$ an element $\pi_2^{-1}(\kappa) = \tilde{C}_{lab} \in X$. Related parameters $\{r_1(\lambda), r_2(\lambda), n\}$ in (3) can be estimated using least-square (LS), total least-square (TLS) [4] or robust estimation algorithm (REA) [5] method to color printer calibration. These parameters define the parameterization curve. The total least-square (TLS) approach will be used in this paper.

To obtain the parameterization of a particular CIELAB color measurements, $C_{lab} \in X$, that is to find the parameterization map, $\pi_2 : X \rightarrow Y$ the nonlinear least square minimization method is employed. We seek to minimize:

$$min\{r(\kappa) : \kappa \in \Re\}$$

where $r(\kappa) := ½\|C_{lab} - \tilde{C}_{lab}(\kappa)\|_2^2$. This problem can be solved using the nonlinear minimization function from the Matlab Optimization Toolbox [6].

Simulation and Experimental Results

We considered the performance of the parameterization subject to the following measurement noise characteristics:

$$\varepsilon = \delta\Delta_n(\ )k \qquad (5)$$

where $\Delta_n(k) \in \Re^{3\times 1}$ is randomly chosen with $|\Delta_n(k)| = 1$ and $\delta \in \Re$ is varied in the range of [0,2]. We consider sampling of $M = 21$ points on the TRC. To evaluate the effectiveness of the proposed parameterization methods, $\pi : X \to Y$ subject to measurement noise, ϵ, we consider:

$$d_X(\pi^{-1}(\kappa), \pi^{-1}(\kappa_\epsilon)) = \Delta E_{CEI2000-DE}, \{\kappa, \kappa_\epsilon\} \in Y$$

where $\Delta E_{CIE2000-DE}$ denotes the CIE 2000 color difference formulation [3] of $C_1 = \pi^{-1}(\kappa) \in X$ and $C_2 = \pi^{-1}(\kappa_\epsilon) \in X$. C_1 and C_2 are 3-dimensional vector is the CIELAB colorspace obtained from the parameterized 1-dimensional representation (the empirical or the model-based parameterization method) by a look-up table approach. For every noise level, δ, we evaluate the ℓ_∞ norm of $\Delta E_{CIE2000-DE}$. The evaluated parameterization methods with a short description is given in Table 1.

Notation	*Description*
EP	Empirical base parameterization method
YNSNP	Parameterization based on YNSN model
LAB	Using L^* from CIELAB colorspace

Table 1: *Evaluated parameterization methods*

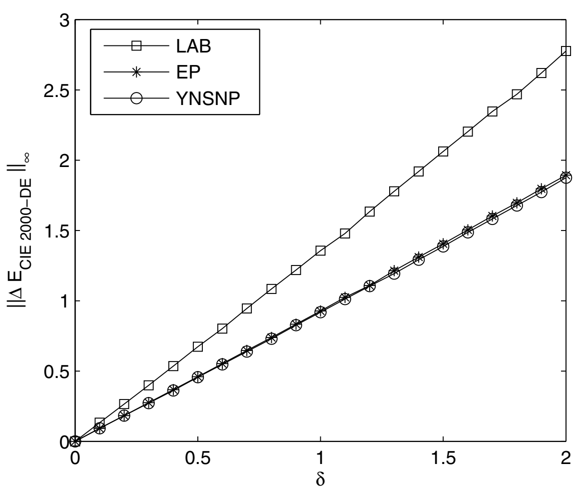

Figure 4: *Comparison of* $\|\Delta E_{CIE2000-DE}\|_\infty$ *at varying noise level* δ *for different parameterization methods*

Figure 4 shows the performance of the evaluated parameterization methods based on measurement noise of the form given by (5). The result shows comparable performance between the model-based and empirical-based parameterization approach. The empirical-based approach is computationally fast and is simple to apply. For a single primary color it is the preferred choice. In contrast, the model-based approach requires high computational requirements for minimization but will be useful when extending the parameterization to cover color combinations. The model-based approach is also preferred when we have limited number of experiments to define the parameterization curve. The YNSN model takes advantage of the physics behind the subtractive coloring process to predict the color behavior in region where the empirical data is sparse. This will be especially critical for parameterizing the 3-dimensional color gamut. As expected is it undesirable to arbitrarily use a single axis of the CIELAB colorsapce(in this case L^*) to stabilize the TRC.

The simulation on the effectiveness of the parameterization methods are based on the assumption that the measurement noise is of the ideal form of (5). This model assumed that the measurement noise of the 3-dimensional representation is randomly distributed in a sphere as shown in Figure 5(a). Experiments were carried out to characterize the measurement noise for a scanner (CanoScan N 650U) and spectrophotometer (GretagMacbeth™ Spectrolino). 100 repeated measurements were taken on the same primary cyan wedge of different tones using both devices. The nominal curve, C^*_{lab}, parameterization parameters and scanner calibration parameters are obtained from a separated training set. Figure 5 shows the noise characterizations of the simulated model, the scanner and the spectrophotometer. The result shows the uniformity of noise distribution with the actual color measuring devices. As expected, the measurements from the scanner are more noisy compared to that taken from the spectrophotometer. We conclude that the noise model of (5) is a valid characterization of the actual measurements noise. Hence, the parameterization performance using the measurement data from the scanner and spectrophotometer as shown in Figure 6 and 7 respectively, are similar to that obtain through simulation.

Conclusion

This paper addressed the problem of parameterizing the 3-dimensional colorspace into a 1-dimensional representation such that the influence of measurements noise can be minimized. The results presented here shows that it is undesirable to arbitrarily select a coordinate axis of the CIELAB colorspace for the TRC stabilizing control. Instead, the empirical-based and the model-based parameterization approaches are shown to be more effective in minimizing the effect of measurement noise for the TRC stabilizing control. Although the empirical-based approach can yield a more accurate parameterization curve and is computationally simple, this method cannot be easily extended for parameterizing the 3-dimensional color gamut. The number of experiments required to define the parameterization manifold i.e. the space where the 3-dimensional representation is projected onto, would be prohibitive for

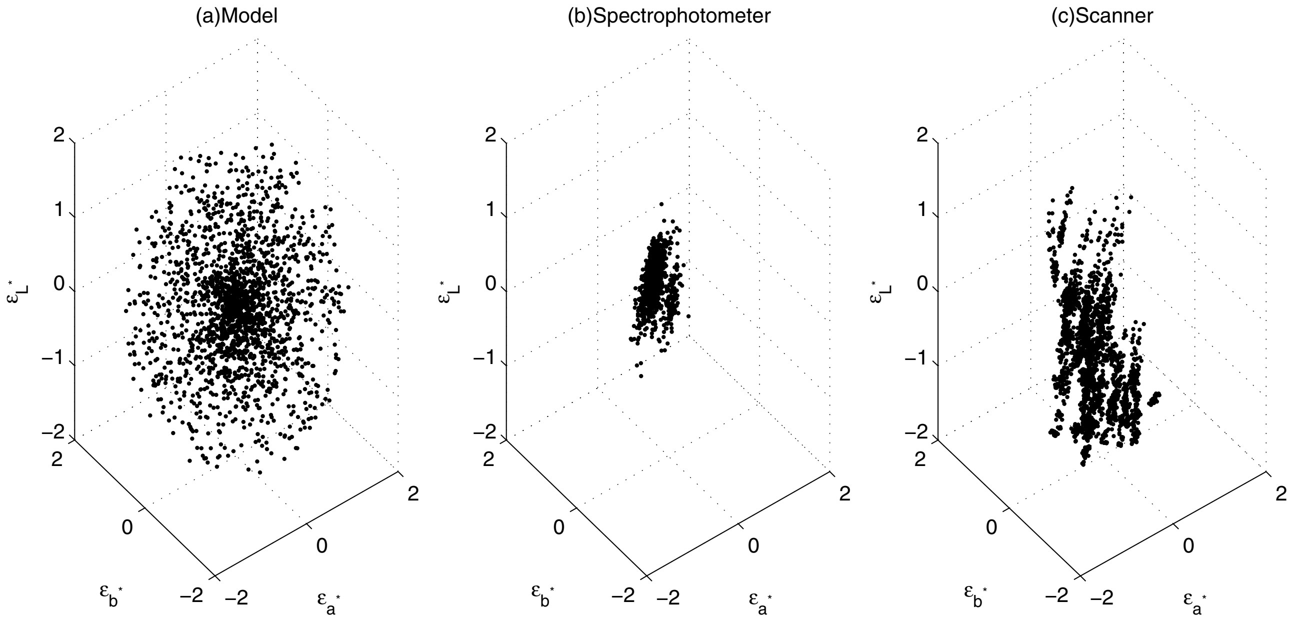

Figure 5: *Noise characterization for the modeled and actual measurement noise in the CIELAB colorspace*

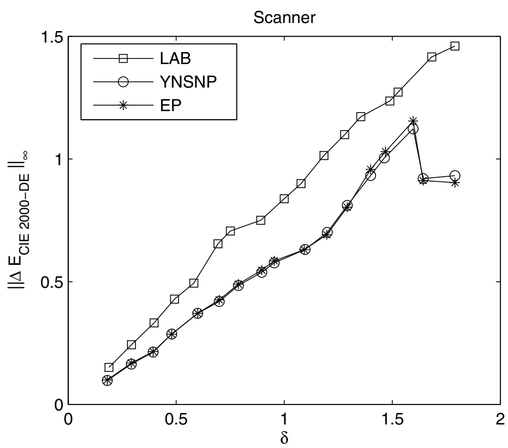

Figure 6: *Comparison of* $\|\Delta E_{CIE2000-DE}\|_\infty$ *at varying noise level* δ *for different parameterization method of the 3-dimensional measurement data using the scanner.* δ *is estimated from* $|C_{lab} - C^*_{lab}|$ *where* C^*_{lab} *is the nominal curve*

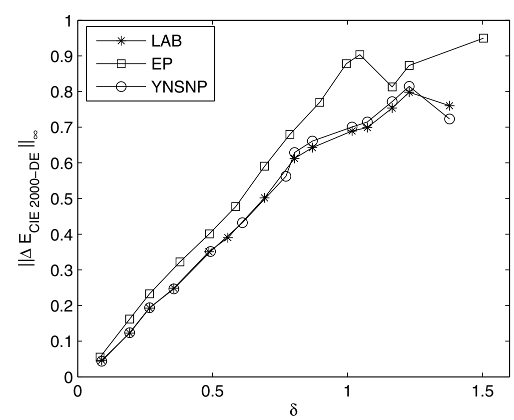

Figure 7: *Comparison of* $\|\Delta E_{CIE2000-DE}\|_\infty$ *at varying noise level* δ *for different parameterization method of the 3-dimensional measurement data using the spectrophotometer.* δ *is estimated from* $|C_{lab} - C^*_{lab}|$ *where* C^*_{lab} *is the nominal curve*

practical purpose. The model-based approach gives good parameterization results albeit higher computational requirements. The extension of this method to cover the entire color gamut requires less empirical data. This extension will be further explore in the future.

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Biography

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