# **Theory of Toner Adhesion**

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#### **Abstract**

A theory of toner adhesion to a conductive plane is suggested which takes into account the electrostatic proximity force. The proximity force is due to the attraction of charges on the toner particle in close proximity to the conductive plane to their respective image charges. In this paper the origin of the proximity force is discussed and experimental verification of this theory of toner adhesion is presented using a 16 micron diameter, ground toner with silica additives. The theory is fit to complete curves of the development and residual mass per unit area, charge-to-mass ratio, and size distribution in an electric field detachment experiment. The observed adhesion and ratio of the measured toner adhesion to theory is the lowest every observed (to the authors' knowledge).

#### I. Introduction

Toner adhesion is active in the subsystems of electrophotography in which toner particles are moved between surfaces, including development, transfer, and clean. Yet the source of toner adhesion remains under discussion, despite intensive investigations starting with the work of Goal and Spencer, 1 a review by Hays, 2 papers by Gady and co-workers 3 and others. 4

The charge on the surface of a toner particle is sometimes modeled as a spherically symmetric charge distribution. A theoretical model of the adhesion of a spherically symmetric charge distribution to a conductive plane is examined, one of the components of adhesion. People often assume that a spherically symmetric charge distribution can be equivalently replaced with a single point charge in the center of the sphere. This is true only in the case of an isolated sphere. It relies on spherical symmetry to apply Gauss' Law. However, in the situation in which the spherically symmetric charge distribution is in contact with a conductive plane, the spherical symmetry no longer exists and no simple integral can be found to apply Gauss' Law. Since the conductive plane is an equipotential, the method of images can be used. We will show that the simple model (which assumes that the spherically symmetric charge distribution can be replaced by a point charge in the middle of the sphere) underestimates the force of adhesion because it ignores the force due to the charges in the proximity of the conductive plane.<sup>5</sup> Therefore the force of adhesion has three

terms<sup>6</sup>, the usually assumed image force, the proximity force times the number of contact points, and the van der Waals force. Experimental verification<sup>7</sup> of this new theory is presented for a 16 micron diameter, ground toner with silica added to its surface.

# **II. Proximity Force Theory**

A spherically symmetric charge distribution is modeled by using finite element analysis both analytically and with numerical calculations. In Fig. 1 we place the toner charge in charge points around N annuli on a sphere of diameter d resting on the conductive plane at z=0. The charge points are chosen using polar coordinates to maintain a constant arc length  $d\Delta \phi/2$  between charge points in the two orthogonal directions. Therefore, the vertical angle between two adjacent annuli,  $\Delta \phi$ , is simply given by  $\pi/N$ . Since, the circumference of each annulus is different, they each hold a different number of charge points. The number of charge points for the i<sup>th</sup> annulus is given by

$$k_i = 2N \sin(\pi i / N + \pi / 2N)$$
  $i = 0,...,N-1$  (1)

The  $\pi/2N$  term maintains the correct charge density at the tip. This equation is derived by dividing the circumference  $(\pi d \sin(i\Delta \phi + \Delta \phi/2))$  by the arc length  $d\Delta \phi/2$  where  $\Delta \phi = \pi/N$ . The total number of charge points, K, can be derived by dividing the total surface area of the sphere by the area that a charge point occupies,  $(d\pi/2N)^2$  giving  $K=4N^2/\pi$ . The charge q in each charge point is the total charge on the sphere Q divided by K or  $q=Q/K=Q\pi/4N^2$ .

The number of charge points on the first annulus nearest the conductive plane is given, in the limits for a large N, by

$$k_0 = 2N\sin(\pi/2N) \approx \pi \tag{2}$$

The plane of the i<sup>th</sup> annulus crosses the z-axis at

$$z_i = \frac{d}{2} 1 - \cos(\frac{\pi}{2N} + \frac{\pi i}{N})$$
.  $i = 0,..., N-1$  (3)

Using the first two terms of the Taylor series expansion, the separation  $z_0$  of the first annulus from the reference plane z = 0 is given by:

$$z_0 = \frac{d}{4} \left(\frac{\pi}{2N}\right)^2 \tag{4}$$

Consider the electrostatic forces due to the interactions between the charge points located in proximity to the conductive plane (which are on the first annulus) with their image charges located symmetrically across the conductive plane. Using Coulomb's Law, the force on a single charge point q in the first annuli by its own image charge point  $F_{11}$  located symmetrically across the conductive plane is

$$F_{11} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{(2z_0)^2} = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{d^2} \frac{4}{\pi^2}$$
 (5)

using Eqs. 4 and the expression of q. The functional dependence of q and  $z_0$  on the number of annuli N cancels out. Since there are approximately  $\pi$  charge points in the first annuli (Eq. 2), the contribution to the electrostatic force by these charges, which we will call the proximity force  $F_p$ , is

$$F_{p} = \pi F_{11} = \frac{1}{4\pi\varepsilon_{0}} \frac{Q^{2}}{d^{2}} \frac{4}{\pi}.$$
 (6)

This is a remarkable result. Only a few point charges with a fractional charge q, which are located in the vicinity of the contact point can generate an attractive force 1.27  $(4/\pi)$  times greater than a charge point of charge Q located in a center of a sphere. We note that this result is independent of the number of annuli, N, in the limits of large N, which suggests that this result is independent of the particular charge distribution chosen for this calculation.

In principal, all of the other image charges contribute to forces on the charges in the first annuli, which are identified as  $F_{12}$ ,  $F_{13}$ , etc. However, for a large N, the separation  $2z_0$  is small compared to the distance to all other charge points.

Since the number of charge points in the proximity annulus is much smaller than the total number of charge points, i.e.  $k_0 << K$ , the rest of the charge points can be considered as a complete sphere of charge. This can be modeled by the usual method of placing a single charge in the center of the sphere, giving for the force for the bulk of the charges  $F_b$ 

$$F_b = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{d^2}.$$
 (7)

The total electrostatic force is just the sum of the electrostatic forces due to the bulk of the charges Q from Eq. 7 and due to the proximity charges from Eq. 6. This simple derivation shows that a closed form solution for the electrostatic force of a spherically symmetric charge distribution in contact with a conductive plane can be derived in a straightforward way and provides a useful and universal result.

Numerical calculations, in which the number of annuli and the distance of the sphere from the conductive plane are varied are shown in Fig. 2. At contact, the numerical calculations agree with analytical theory.

## **III. Theory of Toner Adhesion**

Having shown that a spherically symmetric charge distribution has a force of adhesion to a conductive plane that is composed of two parts, one of which is due to a newly identified proximity force, we now apply this result to toner. Assume that toner particles are not perfect spheres and consequently have many contact points. We suggest that at each contact point the proximity force is active. If there are  $n_p$  contact points, the force of adhesion is

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{d^2} + n_p \frac{4}{\pi} \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{d^2} + n \frac{3}{2} \omega_A \pi R \quad (8)$$

where the last term takes into account van der Waals forces.  $^3$   $\omega_A$  is the thermodynamic work of adhesion and R is the effective radius of the asperities of the particles in contact the plane (either toner or silica asperities) and n is the number of toner (or silica) asperities which contribute to the adhesion.

The large discrepancy between reported measurements<sup>2</sup> and calculated adhesion based on the image force of Eq. 7 is easily resolved if n<sub>p</sub> is on the order of 10 to 40 contacts, which is not unreasonable. Further, large distribution of forces, which are also observed, can be ascribed to large distributions of contact points among the toner particles. Quantitative experimental verification of this theory is presented in the next section.

## IV. Experimental Verification

An electric field detachment experiment was carried out in a single component development system in which the toner is charged against an aluminum development roller after passing under a counter rotating supply roller and a polyurthethane doctor blade (all tied together electrically). A metal cylinder was spaced 150 microns from the development roller moving at the same speed (usually 55 mm/s, although experiments at 2x, 0.5x and at zero speed gave the same results). A bias voltage between the metal cylinder and the development roller allowed for the electric field detachment experiment. The toner mass per unit area M/A and charge to mass ratio Q/M can be measured with standard vacuum pencil techniques both on the cylinder and

remaining on the development roller ("residual"). Collected toner in the filter of the vacuum pencil can be analyzed by the Coulter Counter for toner size distributions.

Toner "develops" from the development roller to the metal cylinder when the Coulomb force QE exceeds the force of adhesion where E is the applied field. Expressing E as V/L where L is the gap, toner develops when V is larger than (using Eq. 8).

$$V = \frac{L\rho}{24\varepsilon_0} (1 + n_p \frac{4}{\pi}) \frac{Q}{M} d + 9 \frac{L}{\rho} n\omega_A \frac{R}{(Q/M)d^3}$$
(9)

where  $\rho$  is the toner density and Q/M is the charge-to-mass ratio

Our procedure is to measure the size distribution of the toner on the development roller and then to use the experimental charge distributions reported in Ref. 8 and 9 to guide a choice of charge distribution which will give the measured average Q/M. This results in a set of particles each with its own Q/M and d, which can be tested against Eq. 9 to determine at what voltage the particle develops. The developed and residual M/A, Q/M and size distribution can then be predicted in a Monte Carlo simulation.

The experiment and theoretical fit for a 16 micron diameter, ground toner with a monolayer of silica on the surface, with Q/M =  $5.3 \mu$ C/g are shown in Fig. 3 and Fig. 4. The data are shown in Fig. 3. On the top left is development efficiency, scaled to 100% of the toner on the development roller. The top right is a plot of developed and residual O/M. The bottom left is the developed particle size distribution (derived from Coulter Counter data by converting the diameter axis to uniform increments to make it easier to compare to theory). The bottom right is the residual particle size distribution. The best fits to the data are given in Fig. 4. With very few parameters, all of these curves were fit and the values of the parameters obtained were all reasonable: (1) the two parameters which characterize the charge distribution are highly constrained by the requirement that the charge distribution be similar to published curves and the charge distribution had to give the measured O/M. (2) The value of the van der Waals force is small, which results directly from the observation of a finite but small intercept in the top left curve of Fig. 3. The result that the van der Waals force is small is almost surely due to the addition of silica to the surface of the toner and is in agreement with prior work.<sup>4</sup> (3) n<sub>p</sub>, the number of contact points, for most of the toner was between 1 and 2.8, which can be rationalized as follows: as 3 points define a plane, most toner is balanced on the planer metal surface by three points, probably three silica particles. Some, however are balanced against neighboring toner, so n<sub>p</sub> is slightly lower. The adhesion predicted based by Eq. 7 is 4.5 nN; the adhesion observed at the 50% point (190 volts) in Fig. 3 is 14.2 nN. The value of the adhesion and the ratio, 3.2, are the lowest ever reported (to the authors' knowledge). (4) About 15% of the toner has much higher adhesion, as might be expected of a ground, manufactured toner.

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## **Biography**

Lawrence B. Schein received his Ph.D. in solid state experimental physics from the University of Illinois in 1970. He worked at the Xerox Corporation from 1970 to 1983 and at the IBM Corporation from 1983 to 1994. He is now an independent consultant. He has helped implement development systems in IBM laser printers, has proposed theories of most of the known electrophotographic development systems, and has contributed to our understanding of toner charging and charge transport mechanisms in photoreceptors. He is the author of "Electrophotography and Development Physics," a Fellow of the Society of Imaging Science and Technology, recipient of the Carlson Memorial Award in 1993, a Senior Member of the IEEE, and a member of the American Physical Society and the Electrostatics Society of America.

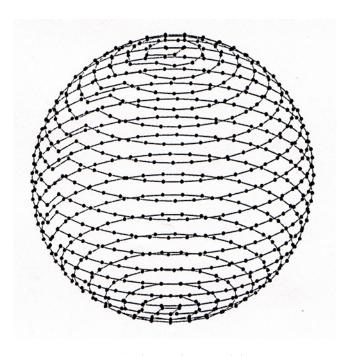


Figure 1. Sphere with points of charge.

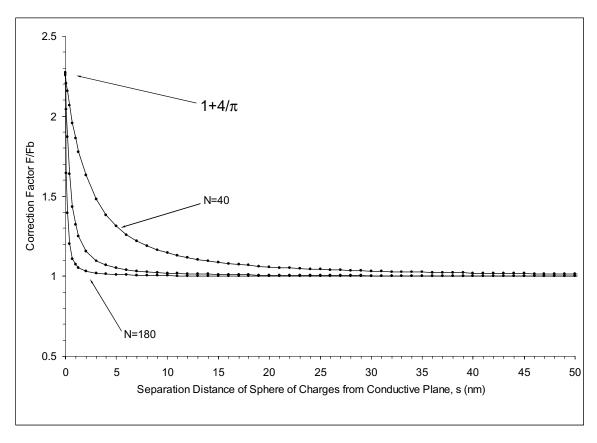


Figure 2. Correction factor, which is the total electrostatic force (Eq. 6+Eq. 7) normalized to  $F_b$  (Eq. 7) vs. separation distance s between the sphere and the conductive plane, and N, the number of annuli, for a 6 micron radius sphere. The three curves are for N=40, 90, and 180.

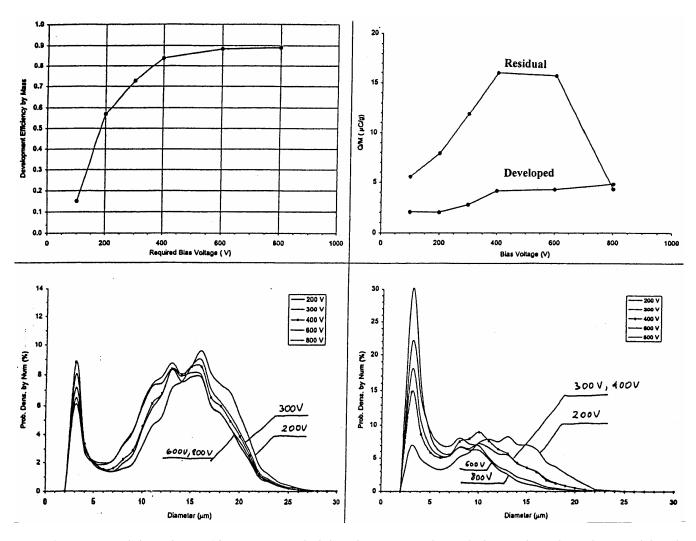


Figure 3. Experimental data taken on 16 micron toner which has about one monolayer of silica on the surface. The upper left is the development efficiency (developed mass per unit area normalized to the mass per unit area on the development roller). The upper right is the developed and residual Q/M. The lower left is the size distribution of the toner developed at each voltage. The lower right is the size distribution of the toner left behind (residual) at each voltage.

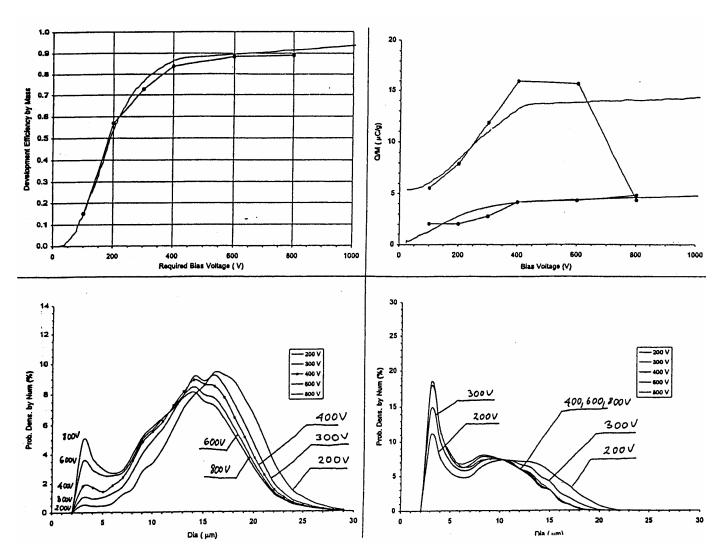


Figure 4. Best fit to the data. The theoretical curves are superimposed on the data in the upper two graphs. The predicted size distribution of the developed (left) and residual (right) toner can be compared with the data given in Figure 3.