

# Gamut Boundary Determination by Sampling an Inverse Printer Model

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## Abstract

In this paper we present a novel method to calculate the gamut boundary of a color printing device using a sparse number of measured values. This method belongs to the SMGBD (Segment Maxima Gamut Boundary Descriptor) family. At first, we construct an inverse printer model based on a set of printed color patches. This model allows us to find the colorant values that have to be sent to the printer in order to obtain a given color appearance, or to show that such a colorant value does not exist. Then we construct a GBD (Gamut Boundary Descriptor) with the measured color patches, using the segment maxima method. After calculating the GBD every segment contains exactly one color value, which might not necessarily be part of the device gamut boundary. Therefore we sample along the line determined by the contained color value and the middle grey value  $(L, a, b) = (50, 0, 0)$  for every segment, using a bisection method based on the inverse printer model, and store the printable color value on the line with the largest distance from the middle grey value in the GBD. Due to this approach practical gamut boundaries can be calculated based on a sparse number of measured colors. Measurement results and a comparison with other methods are given in the text.

## Introduction

The reproduction of colors by a printer is a two stage process. The first stage consists of characterizing the device, in order to determine the colorant values which obtain a desired color appearance (inverse printer model). In the second stage colors which are physically not reproducible by the printer must be transformed to printable colors in a faithful way (gamut mapping). The mapping of not printable colors into the amount of physically reproducible colors requires the knowledge of the gamuts boundary. Additionally, it is important to store the gamut boundary information in a compact form to allow an easy and fast access for gamut mapping methods.

In recent years different methods have been developed to calculate the gamut surface. Some of these approaches are based on the assumption that the hull of the hyper-cube of the colorant space maps onto the gamut boundary (physical gamut boundary), e.g. Braun and Fairchild [1] have constructed

their algorithm as follows: Printed and spectrophotometrically measured colorant values from the colorant hyper-cube surface are first transformed into cylindrical *CIELab* coordinates, then projected on the  $L^*h^*$  plane and triangulated using neighboring information from the colorant space.

Mahy [7] has examined the "Neugebauer model" (Neugebauer [9]) and find out that for some printing processes the physical boundaries are not the real gamut boundaries. Colorants which are inside the colorant hyper-cube are mapped out of the physical gamut. He called the boundary determined by the printer model "natural gamut boundaries".

Cholewo [2] used alpha-shapes for determining the gamut surface. The alpha shape of a finite point number is a sub-graph of the delaunay triangulation. He calculated the alpha shape for a set of printed and measured colorants, but it is difficult to choose the control parameter  $\alpha$  to get accurate results.

Herzog [4] has distorted a "kernel gamut cube" by analytical functions which were fitted to a set of measured values. He called this method "gamulyt". Morovic [8] has developed the "Segment Maxima Gamut Boundary Descriptor (GBD)" method which will be explained below.

All these methods do not take the mapping properties of the appropriate printer model into account. Even if the set of spectrophotometrically measured colorant values is located also inside the hyper-cube, only the printer model which will be used for characterization determines the shape of the gamut (natural gamut boundaries). The presented method exploits the properties of the printer model mapping and provides a GBD which allows easy access to shape information.

This paper is organized as follows: At first, we will give an overview about printer characterization and define the indicator function  $\mathcal{I}_{\tilde{\mathcal{F}}} : CIELab \rightarrow \{-1, 1\}$  based on the inverse printer model  $\tilde{\mathcal{F}}$ .  $\mathcal{I}_{\tilde{\mathcal{F}}}(\vartheta)$  returns 1 if there exist colorant values that reproduce  $\vartheta$ , otherwise  $-1$ . In the next section the SMGBD method of Morovic [8] will be sketched. Then we will describe the Segment Maxima Sampling GBD (SMSGBD) method. This bisecting algorithm enhances the SMGBD method by sampling an inverse printer model using the above indicator function  $\mathcal{I}_{\tilde{\mathcal{F}}}$ . Finally some measurement results and conclusions will be given.

## Inverse Printer Model

A colorimetric printer model  $\mathcal{F}_P : \Omega_{ink} \rightarrow CIELab$  is a mapping which approximately describes the behavior of a printer for a given paper type ( $\Omega_{ink}$  specifies the colorant values of the printer, eg. *RGB, CMY, CMYK, CcMmYK*). For every colorant value  $\chi \in \Omega_{ink}$  which will be sent to the printer  $\mathcal{F}_P$  returns the appropriate color appearance  $\vartheta \in CIELab$ , which will be generated by the printer. In general  $\mathcal{F}_P(\Omega_{ink}) \neq CIELab$  applies. We denote the printer media gamut by  $\mathcal{F}_P(\Omega_{ink}) \subset CIELab$ , depending on the appropriate printer model. The better the model describes the printer the better  $\mathcal{F}_P(\Omega_{ink})$  describes the printer media gamut.

The inverse question is even more interesting: What colorant value has to be sent to the printer in order to obtain a given color appearance?

As in general there is more than one colorant value which maps to the desired color appearance, the printer model is not invertible ( $\mathcal{F}_P$  is not injective). This applies especially for printers using more than 3 ink types. To solve this problem special criterions will be established to select one specific colorant value out of the set of possible colorant values (e.g. black generation). The inversion of a printer model is in general not unique. Therefore there exists a set of "inverse" printer models for every printer model  $\mathcal{F}_P$

$$\mathbb{F}_{\mathcal{F}_P} := \left\{ \tilde{\mathcal{F}} : \mathcal{F}_P(\Omega_{ink}) \rightarrow \Omega_{ink} \mid \forall \vartheta \in \mathcal{F}_P(\Omega_{ink}) : \mathcal{F}_P(\tilde{\mathcal{F}}(\vartheta)) = \vartheta \right\}. \quad (1)$$

Because for color appearances  $\vartheta \in CIELab \setminus \mathcal{F}_P(\Omega_{ink})$  which are out of gamut the inverse printer models cannot return an appropriate colorant value, we can define the following indicator function for every inverse printer model  $\tilde{\mathcal{F}} \in \mathbb{F}_{\mathcal{F}_P}$

$$\mathcal{I}_{\tilde{\mathcal{F}}} : \begin{cases} CIELab \rightarrow \{-1, 1\} \\ \vartheta \mapsto \begin{cases} 1, & \exists \chi \in \Omega_{ink} : \chi = \tilde{\mathcal{F}}(\vartheta) \\ -1, & \text{else.} \end{cases} \end{cases} \quad (2)$$

For a given printer model  $\mathcal{F}_P$  all indicator functions are equal  $\mathcal{I} := \mathcal{I}_{\tilde{\mathcal{F}}_i} = \mathcal{I}_{\tilde{\mathcal{F}}_j}$ , with  $\tilde{\mathcal{F}}_i \neq \tilde{\mathcal{F}}_j$ .

In recent years different models (e.g. Neugebauer [9], Yule-Nielson [11] and their derivatives, Urban-Grigat [10]) have been developed to characterizing printers. The printer models will be constructed using a set of printed and measured color patches with known colorant values  $\Omega_{ink}^{test} \in \Omega_{ink}$  (eg. IT8.7/3 for  $\Omega_{ink} = CMYK$ ).

Techniques to invert this models can also be found in the literature (Cholewo [3], Mahy-Delabastita [5], Urban-Grigat [10]).

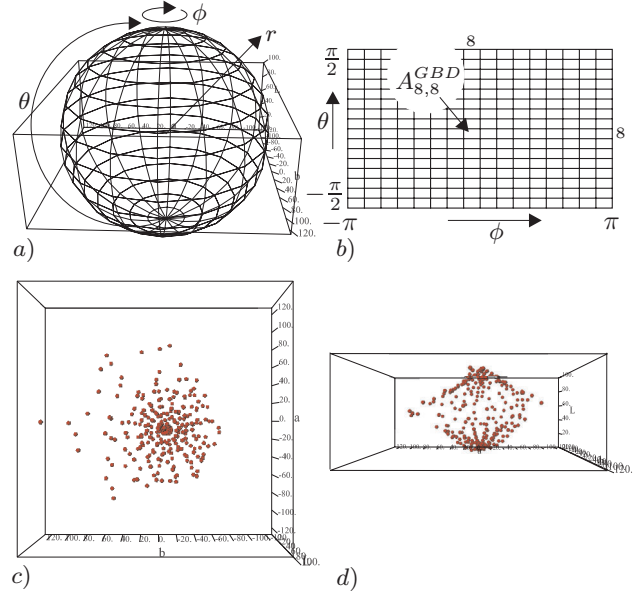


Figure 1: a)  $16 \times 16$  spherical segments b) Appropriate GBD matrix  $A^{GBD}$  c) and d) Segment maximal points for the HP Deskjet 990Cxi printer on premium paper

## Segment Maxima Sampling GBD Method (SMSGBD)

### Segment Maxima GBD Method (SMGBD)

This method was proposed by Morovic [8] and enables the determination of a 3D printer media gamut in the *CIELab* color space by calculating a GBD as follows:

1. The *CIELab* color space will be transformed into spherical coordinates with center in the middle gray value  $(L_M, a_M, b_M) = (50, 0, 0)$ , using following formulae

$$r = \sqrt{(L - L_M)^2 + (a - a_M)^2 + (b - b_M)^2} \quad (3)$$

$$\phi = \arctan\left(\frac{b - b_M}{a - a_M}\right) \quad (4)$$

$$\theta = \arctan\left(\frac{L - L_M}{\sqrt{(a - a_M)^2 + (b - b_M)^2}}\right) \quad (5)$$

$\phi \in [-\pi, \pi]$  is the hue angle and  $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  is the angle in a plane of constant  $\phi$ .

2. The *CIELab* color space will be divided into  $n \times n$  segments

$$S_{ij} := \left\{ \vartheta \mid \vartheta_\phi \in \left[-\pi + \frac{2(i-1)\pi}{n}, -\pi + \frac{2i\pi}{n}\right], \vartheta_\theta \in \left[-\frac{\pi}{2} + \frac{(j-1)\pi}{n}, -\frac{\pi}{2} + \frac{j\pi}{n}\right] \right\}$$

$i, j = 1, \dots, n$  (Figure 1a). The segment  $S_{ij}$  corresponds to the entry  $A_{ij}^{GBD}$  of the GBD matrix  $A^{GBD}$  (Figure 1b).

- Each of the measured reference colors  $\Omega_{ink}^{test} \in \Omega_{ink}$  will be transformed into spherical coordinates using equations 3 - 5. For every segment  $S_{ij}$  the point  $\vartheta \in S_{ij}$  with the largest r-value  $\vartheta_r = \max_{\sigma \in S_{ij}} \sigma_r$  will be stored in the  $A_{ij}^{GBD}$  entry of the GBD matrix  $A^{GBD}$ .
- GBD matrix entries for segments without colors will be interpolated from nearest GBD matrix entries.

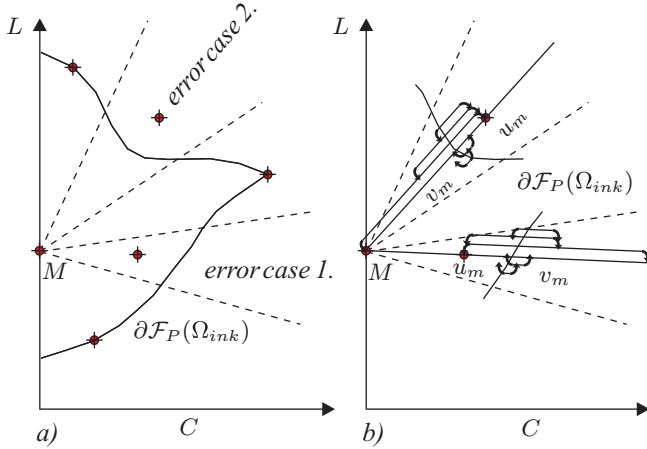


Figure 2: a) Possible error cases after SMGBD method  
b) Bisection sequences for the two error cases

### Sampling the SMGBD by an Inverse Printer Model

The SMGBD depends on the number of reference colors and the number of segments. Therefore the following error cases can occur if the printer model is not taken into account (Figure 2a):

- The segment maximum point is in the printer media gamut, but not on the gamut border  $\partial\mathcal{F}_P(\Omega_{ink})$ , determined by the printer model  $\mathcal{F}_P$  ("natural" boundary, Mahy[6])
- The interpolation in step 4 makes it possible, that the segment maximum point is outside the printer media gamut.

To correct these errors for each segment an inverse printer model can be sampled along the line determined by the segment maximum point and the middle gray value  $M := (L_M, a_M, b_M) = (50, 0, 0)$ , using a bisection method. (Figure 2b) In the following we denote the segment maximum point of segment  $S_{ij}$  in cartesian coordinates by  $C_{ij}$  and the largest radius of all segment maximum points by

$$r_{max} := \max_{i,j \in \{1, \dots, n\}} (A_{ij}^{GBD})_r. \quad (6)$$

The following equation describes the line for segment  $S_{ij}$

$$\begin{aligned} \mathcal{B}_{ij}(\lambda) &= M + \lambda \frac{C_{ij} - M}{\|C_{ij} - M\|_2} \\ &= M + \lambda \frac{C_{ij} - M}{(A_{ij}^{GBD})_r} \end{aligned} \quad (7)$$

And the following sequence  $(u_m, v_m)$  will define the bisection iteration for segment  $S_{ij}$  using the indicator function  $\mathcal{I}$  from equation 2:

$$(u_0, v_0) := \begin{cases} ((A_{ij}^{GBD})_r, 0) & , \mathcal{I}(\mathcal{B}_{ij}(A_{ij}^{GBD})_r) = -1 \\ ((A_{ij}^{GBD})_r, r_{max}) & , \mathcal{I}(\mathcal{B}_{ij}(A_{ij}^{GBD})_r) = 1 \end{cases}$$

For  $m = 1, 2, \dots$  and  $x := \frac{u_{m-1} + v_{m-1}}{2}$ :

$$(u_m, v_m) := \begin{cases} (x, v_{m-1}), & \mathcal{I}(\mathcal{B}_{ij}(x)) \cdot \mathcal{I}(\mathcal{B}_{ij}(v_{m-1})) = -1 \\ (u_{m-1}, x), & \text{else.} \end{cases}$$

The iteration stops, if the distance between  $u_m$  and  $v_m$  falls below a threshold  $\epsilon$ :  $|u_m - v_m| < \epsilon$  ( $\epsilon = 1$  is mostly sufficient).

After termination the new sampling maximum point

$$C_{ij}^{new} := M + u_m \frac{C_{ij} - M}{\|C_{ij} - M\|_2} \quad (8)$$

will be stored in the GBD matrix in spherical coordinates using equations 3 - 5.

Using the same GBD structure as the SMGBD algorithm the same methods can be also used to access the surface information, like CLGB (Constrained Line Gamut Boundary Method) or FSLGB (Flexible Sequential Line Gamut Boundary Method) developed by Morovic [8].

## Results

We have reviewed the algorithm on a HP Deskjet 990Cxi RGB-printer printing at first 17576 test color patches on normal paper. We have selected the set of color patches as follows

$$\Omega_{all}^{test} := \{0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100\}^3 \subset RGB.$$

To calculate the inverse printer model we have chosen the "Printer Characterization Using Adaptive Constraint Optimization" method from Urban-Grigat [10]. Measurements of color patches have been made with an X-Rite DTP41 spectrophotometer, which used D50 illuminant, a  $0^\circ/45^\circ$  measuring geometry and the  $2^\circ$  standard observer. The following 4 subsets of  $\Omega_{all}^{test}$  have been chosen for gamut boundary calculation:

$$\begin{aligned} \Omega_{2744}^{test} &:= \{0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 100\}^3 \subset \Omega_{all}^{test} \\ \Omega_{729}^{test} &:= \{0, 12, 24, 36, 48, 60, 72, 84, 100\}^3 \subset \Omega_{all}^{test} \\ \Omega_{512}^{test} &:= \{0, 16, 32, 48, 64, 80, 96, 100\}^3 \subset \Omega_{all}^{test} \\ \Omega_{216}^{test} &:= \{0, 20, 40, 60, 80, 100\}^3 \subset \Omega_{all}^{test}. \end{aligned}$$

We have compared the SMSGBD method to the SMGBD method using  $32 \times 32$  segments. At first, two GBDs for  $\Omega_{all}^{test}$  have been calculated using both methods. These GBDs serve as the reference gamut boundary information for the appropriate method. After calculating the gamut boundary for a set of test colorants using one method the results have been compared with the reference GBD of the same method. For each segment center the gamut boundary point has been determined for both GBDs. The  $\Delta E_{94}$  distance between these two points has been calculated. For every set of colorants and for each method this result in  $32 \times 32$  distance values. Figure 3 shows the mean error and figure 4 the maximal error.

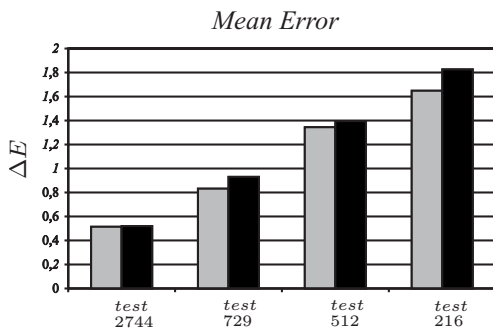


Figure 3: Left bars SMSGBD method, right bars SMGBD method

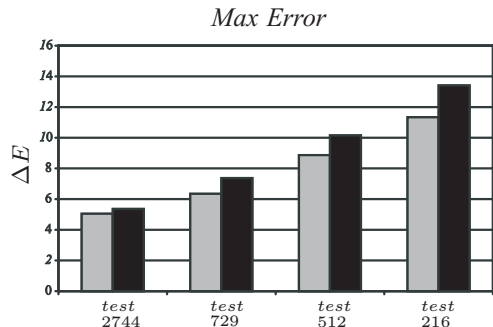


Figure 4: Left bars SMSGBD method, right bars SMGBD method

## Conclusion

We have presented a new method (SMSGBD) to determine gamut boundaries by sampling an inverse printer model. This approach takes the properties of the printer model into account and stores the surface information in an GBD which is easy to address by gamut mapping algorithms. The quality of the SMSGBD method's results can be improved by using a more accurate printer model. Increasing the number of segments allows a more precise representation of the gamut boundary.

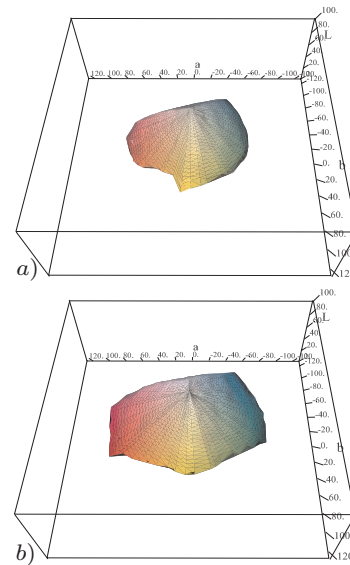


Figure 5: a) Gamut of the HP Deskjet 990Cxi printer on normal b) Gamut of the HP DesignJet 1055CM printer on glossy paper paper

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## Biography

Philipp Urban was born in 1975 in Wroclaw, Poland. He received the dipl. math at the University of Hamburg, Germany in 1999. Since 1999 he is part of the research group "Vision Systems" of the Technical University Hamburg-Harburg, Germany, and works for "Ratio Entwicklungen GmbH", ICC-member. His research interests are color management, gamut boundary calculation, device calibration and characterization.