Nonorthogonal Halftone Screens

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Abstract

In color reproduction, the most troublesome moiré pattern is the second-order moiré, or the three-color moiré, usually produced by mixing of cyan, magenta and black halftone outputs. A classical 3-color zero-moiré solution is using three identical cluster halftone screens with different rotations: 15º, 45º and 75º, respectively. However, for most digital printing devices, the size and shape of halftone screens are constrained by the "digital grid", which defines the locations of printed dots, and therefore, an exact 15º or 75º rotation of a cluster screen is impossible. Although there are many alternative approaches for moiré-free color halftoning, most of them only provide approximate solutions and/or have a tendency to generate additional artifacts associated with halftone outputs. The difficulty to achieve moiré-free color halftoning is greatly relieved by using nonorthogonal halftone screens, i.e., screens in general parallelogram shapes. As a matter of fact, there exist many practical solutions by combining three simple nonorthogonal halftone screens. In this paper, a general condition for 3-color zero-moiré solutions is derived. A procedure using integer equations to search practical solutions for different applications is also described.

Introduction

Most halftone screens used in color reproduction are orthogonal screens, or screens in rectangular shapes, more likely in squares. Nonorthogonal screens refer to screens in general parallelogram shapes. In 1970, Holladay developed an efficient way of encoding the halftone screens.[1] The method gives a unique halftone description with the advantages of simple implementation and a small memory requirement. The algorithm is based on geometry of general parallelograms; therefore, as explicitly pointed by Holladay, any nonorthogonal halftone screens can be produced and implemented by his method exactly as orthogonal screens. Obviously, orthogonal screens are only a small subset of the complete set of all nonorthogonal screens. However, using general nonorthogonal halftone screens had not shown any major advantage over orthogonal ones and, as a result, not much interest had been previously brought in the halftoning field.

In a typical screening process, a halftone screen is applied repeatedly in a way similar to a tiling process indicated by the gray lines in Fig. 1.

With any constant input, the halftone output by this halftone screen will be a two-dimensional spatial periodical function. From Fourier analysis, it is clear that the spectrum of the halftone output is composed of, and only of, the two fundamental frequencies and their higher-order harmonics from the Fourier transform of the halftone screen. There is no component with a frequency lower than the two fundamentals. For most of moiré analysis in halftone screen design, we may concentrate on fundamental frequencies of halftone screens, because the dominating moiré is most likely due to the interaction between fundamental frequencies of individual halftone screens for different color channels. First, let us consider a typical rotated orthogonal screen, as shown in Fig. 1. The geometry of this halftone screen can be specified by the dual representation of the Fourier transformation. In the spatial domain, the rectangular screen is specified by two orthogonal vectors, $\mathbf{v}_1$ and $\mathbf{v}_2$, shown in Fig. 2; while in the Fourier transform domain, it is represented by two orthogonal frequency vectors, $\mathbf{V}_1$ and $\mathbf{V}_2$, shown in Fig. 3. As properties of the Fourier transformation, the two frequency vectors, $\mathbf{V}_1$ and $\mathbf{V}_2$, are perpendicular to the two spatial vectors, $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively, and the moduli, or the absolute values, $|\mathbf{V}_1|$ and $|\mathbf{V}_2|$, are equal to $1/|\mathbf{v}_2|$ and $1/|\mathbf{v}_1|$, respectively.

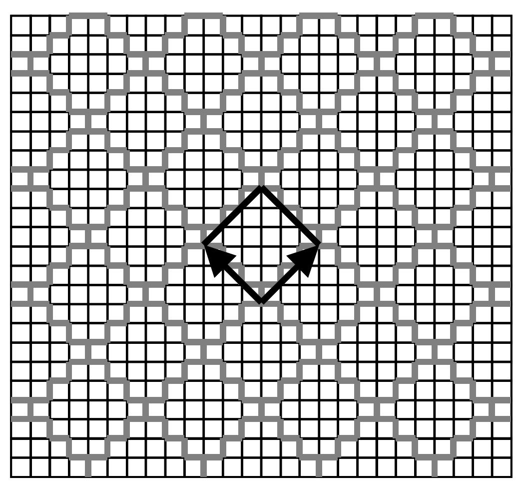

Figure 1. Digital grid with an orthogonal halftone screen outlined

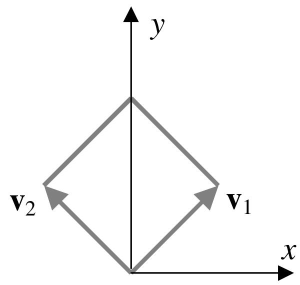

Figure 2. Spatial vector representation of an orthogonal halftone screen

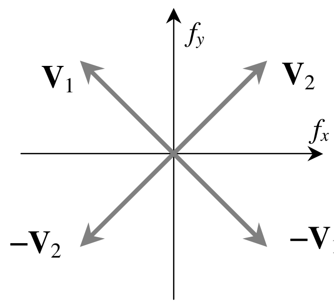

Figure 3. Frequency vector representation of an orthogonal halftone screen

To eliminate or reduce moirés caused by interaction between different color channels, it is often critical to have precise rotation angles of halftone screens. For example, to avoid three-color moirés, traditional analog halftoning uses an identical dot screen for cyan, magenta and black channels with 15°, 75° and 45° rotation, respectively. Unfortunately, in digital halftoning, the selection of possible rotations for halftone screens is greatly restricted by a digital grid, or raster, defined by the location of physical pixels. The tangential of the rotation angle, specified by the argument of the spatial vector **v**, or the argument of the frequency vectors **V**, has to be a rational number, because the two Cartesian-coordinate components of a spatial vector **v** have to be integers. For example, a 45° rotation of a halftone screen, as shown in Fig. 1, may be achieved in digital halftoning, because tan(45°) = 1. However, neither 15° nor 75° rotation of a halftone screen can be implemented digitally. Various digital halftoning methods have been proposed for precisely or approximately reaching certain desired rotation angles of halftone screens[2-4]. Perhaps, the most popular approach is the supercell, which is a rational tangent screen composed of many smaller subcells which are not uniform in size and shape[3]. The drawback of supercells is that a large halftone screen contains low fundamental frequencies. If the fundamentals or their spatial harmonics fall in the range of the sensitivity function of the human visual system, additional effort has to be made during screen design process to avoid possible low-frequency artifacts which might be shown in the halftone outputs. In general, designing a supercell is much more difficult than designing simple halftone screens with single or few centers. On the other hand, the selection of small orthogonal screens with different rotation angles is quite limited, especially for devices with relatively low resolutions, which makes almost impossible to find a moiré-free solution using rotated simple orthogonal halftone screens for color halftoning.

In this paper, we suggest using nonorthogonal screens for moiré-free color halftoning by searching integer equation solutions.[5] In the following sections, a general analysis on nonorthogonal screens will be discussed, which will lead to a method for searching moiré-free solutions using simple nonorthogonal halftone screens.

Dual Representation of Nonorthogonal Screens

A general nonorthogonal halftone screens is outlined by the red lines in Fig. 4. The shape of this parallelogram screen can be specified by two vectors, $\mathbf{v}_1(x_1, y_1)$ and $\mathbf{v}_2(x_2, y_2)$, shown in Fig. 5. The two fundamental frequencies of the Fourier transform of this screen can be represented by two frequency vectors, $\mathbf{V}_1(f_{x1}, f_{y1})$ and $\mathbf{V}_2(f_{x2}, f_{y2})$, shown in Fig. 6.

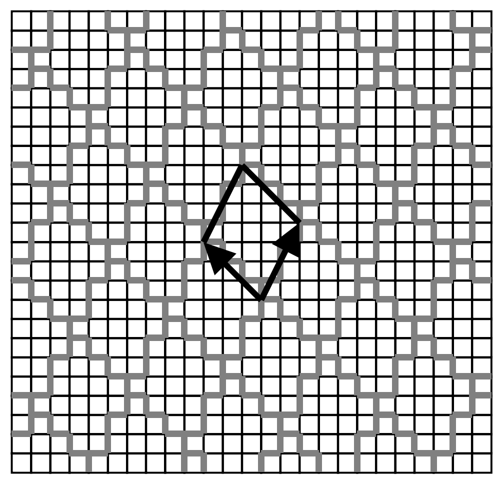

Figure 4. Digital grid with a nonorthogonal halftone screen outlined

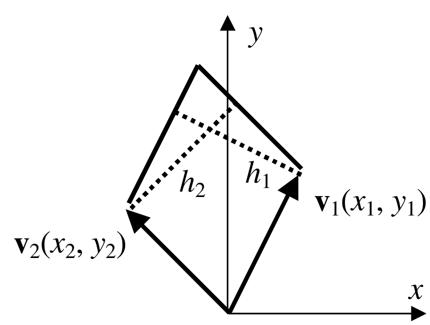

Figure 5. Spatial vector representation of a nonorthogonal halftone screen

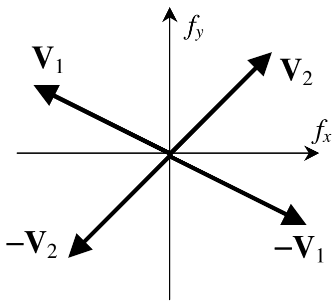

Figure 6. Frequency vector representation of a nonorthogonal halftone screen

Similar to the orthogonal case, $\mathbf{V}_1$ and $\mathbf{V}_2$ are perpendicular to $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively. However, the moduli of the frequency vectors $|\mathbf{V}_1|$ and $|\mathbf{V}_2|$ are not given by the reciprocals of $|\mathbf{v}_2|$ and $|\mathbf{v}_1|$, as for the orthogonal screens. Instead, $|\mathbf{V}_1|$ and $|\mathbf{V}_2|$ are equal to the reciprocals of h_1 and h_2, which are the heights, or the pitches, shown by the dot lines in Fig. 5. Since the product, $|\mathbf{v}_1|*h_1 = |\mathbf{v}_2|*h_2 = A$, is the area of the specified parallelogram, we may write the moduli of the frequency vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ as the following equations:

$$|\mathbf{V}_1| = \frac{|\mathbf{v}_1|}{A}, \tag{1a}$$

$$|\mathbf{V}_2| = \frac{|\mathbf{v}_2|}{A}, \tag{1b}$$

where A is given by the absolute value of the cross product of two vectors, $\mathbf{v}_1$ x $\mathbf{v}_2$, i.e.,

$$A = |x_1 y_2 - x_2 y_1|. \tag{2}$$

Since the spatial vector $\mathbf{v}_1(x_1, y_1)$ and the frequency vector $\mathbf{V}_1(f_{x1}, f_{y1})$ are perpendicular to each other, so are $\mathbf{v}_2(x_2, y_2)$ and $\mathbf{V}_2(f_{x2}, f_{y2})$, from Eqs. 1a and 1b, it is not difficult to prove that

$$f_{x1} = \frac{-y_1}{A}, \tag{3a}$$

$$f_{y1} = \frac{x_1}{A}, \tag{3b}$$

$$f_{x2} = \frac{-y_2}{A}, \tag{3c}$$

$$f_{y2} = \frac{x_2}{A}. \tag{3d}$$

Under above-mentioned digital grid constraint, i.e., all spatial vectors are specified by integers in the Cartesian coordinate, the area A given by Eq. 2 must be a nonnegative integer. Consequently, all frequency-vector components in the Cartesian coordinate are rational numbers. Imagine a parallelogram specified by the two frequency vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, one may see that it would be a rotated and scaled version of the parallelogram halftone screen with 90° rotation and $1/A$ scaling.

If the following condition,

$$x_1/y_1 = -y_2/x_2. \tag{4}$$

is satisfied, the general parallelogram becomes a rectangular. Furthermore, if

$$x_1 = \pm y_2, \text{ and} \tag{5a}$$

$$y_1 = \mp x_2, \tag{5b}$$

the parallelogram becomes a square.

It is interesting to notice that changing one spatial vector, say $\mathbf{v}_1$, of an orthogonal screen will only affect the frequency $\mathbf{V}_1$, while changing $\mathbf{v}_1$ of a nonorthogonal screen will affect, in general, both frequencies, $\mathbf{V}_1$ and $\mathbf{V}_2$. Since the two spatial vectors can be specified independently for a parallelogram, the number of different frequencies by using nonorthogonal screens is approximately N^2, comparing with N by using orthogonal screens. Practically, the outputs of many nonorthogonal screens may look just like ones by orthogonal screens by choosing the two vectors with an angle between them close to 90°. This might be not difficult to implement especially from devices with relatively high resolutions in either one or two dimensions. In addition, a diamond-shape parallelogram can produce halftone outputs with hexagon-like halftone texture, which might be also interesting.

Another possible extension of above analysis is also quite interesting. We may consider line screens as a special type of nonorthogonal screens, which contain at least one of the Cartesian components equal to unity. For example, a 45° rotated line screen can be treated as a special parallelogram specified by two spatial vectors, $\mathbf{v}_1$ (1, 1) and $\mathbf{v}_2$ (w, 0), where w is the period between lines measured horizontally. The two corresponding frequency vectors are given by $\mathbf{V}_1$ ($-1/w$, $1/w$) and $\mathbf{V}_2$ (0, 1), respectively. By the sampling theory, the frequency component one might be also interpreted as zero, and the frequency vector $\mathbf{V}_2$ might be considered as null and ignored for further moiré analysis.

Moiré-Free Conditions

One particular application of nonorthogonal halftone screens is to provide perfect solutions for moiré-free color halftoning. Moiré patterns may appear in the printed documents for several possible reasons. In color printing, the most unwanted moiré is due to the superposition of the halftone screens of the different process colors. Using Fourier analysis provided for halftone screens, we can express the result caused by superposition of two different colors as their frequency-vector difference, $\mathbf{V}_{cm} = \mathbf{V}_c \pm \mathbf{V}_m$, where $\mathbf{V}_c$ and $\mathbf{V}_m$ are two frequency components from two different colors, e.g., cyan and magenta, and $\mathbf{V}_{cm}$ is the difference vector. Since each Fourier component has its conjugate, i.e., there is always a frequency vector $-\mathbf{V}_c$ represents the conjugate component of $\mathbf{V}_c$, the sign definition of frequency vectors is rather arbitrary. For each parallelogram screen, there are two fundamental frequency vectors, therefore, the color mixing of two screens for two different colors yields four difference vectors, illustrated by Fig. 7.

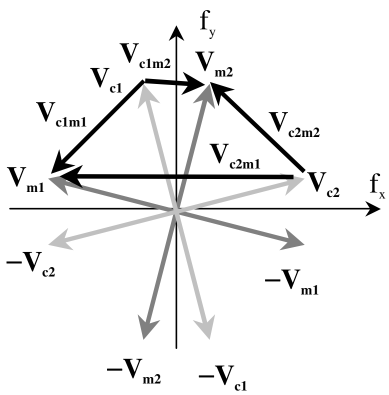

Figure 7. Difference vectors by interaction of two colors with different halftone screens

If any one of these difference vectors is much shorter than the cut-off frequency of the sensitivity function of human visual system and not very close to zero, there is a possibility to have two-color moiré appearing on the halftone output at the frequency represented by the corresponding difference vector. The common strategy to avoid any two-color moiré is to make sure that no two-color difference vector will be too small. The two-color moiré-free condition can be summarized by

$$\left|\mathbf{V}_c \pm \mathbf{V}_m\right| > V_{\min}\,, \tag{6}$$

where $\mathbf{V}_c = \mathbf{V}_{c1}, -\mathbf{V}_{c1}, \mathbf{V}_{c2}, -\mathbf{V}_{c2}$; $\mathbf{V}_m = \mathbf{V}_{m1}, -\mathbf{V}_{m1}, \mathbf{V}_{m2}, -\mathbf{V}_{m2}$, and $V_{\min}$ is a frequency limit set at somewhere 50-70 line-per-inch for just-noticeable moirés.

It is well known that the most troublesome moiré is the three-color moiré, usually appearing as the cyan-magenta-black moiré in outputs by cmyk four-color printers. As an extension of the two-color case, the three-color moiré-free condition can be summarized by,

$$\left|\mathbf{V}_c \pm \mathbf{V}_m \pm \mathbf{V}_k\right| > V_{\min}\,, \tag{7}$$

where $\mathbf{V}_c = \mathbf{V}_{c1}, -\mathbf{V}_{c1}, \mathbf{V}_{c2}, -\mathbf{V}_{c2}$; $\mathbf{V}_m = \mathbf{V}_{m1}, -\mathbf{V}_{m1}, \mathbf{V}_{m2}, -\mathbf{V}_{m2}$; $\mathbf{V}_k = \mathbf{V}_{k1}, -\mathbf{V}_{k1}, \mathbf{V}_{k2}, -\mathbf{V}_{k2}$, and $V_{\min}$ is set similar to the two-color case. Since there are altogether sixteen different combinations of different color components, practically, to make all three-color difference vectors, as well as all two-color difference vectors, large enough to avoid any color moiré is very difficult, unless the halftone screens have very high frequencies fundamentals, say higher than 200 line-per-inch. An alternate, also common, approach is to make two of the three-color difference vectors null while keeping rest large. Given that both the signs and the indices of frequency vectors are defined somewhat arbitrarily, without losing the generality, the three-color moiré-free condition can be specified by the following two vector equations:

$$\mathbf{V}_{c1} + \mathbf{V}_{m1} + \mathbf{V}_{k1} = 0\,, \tag{8a}$$

$$\mathbf{V}_{c2} + \mathbf{V}_{m2} + \mathbf{V}_{k2} = 0\,, \tag{8b}$$

Once the two equations, 8a and 8b, are satisfied, the rest combinations of three color components are equal to linear combination of higher-order harmonics from two colors. In most practical applications, they will satisfy the inequality 7. An example is illustrated by Fig. 8.

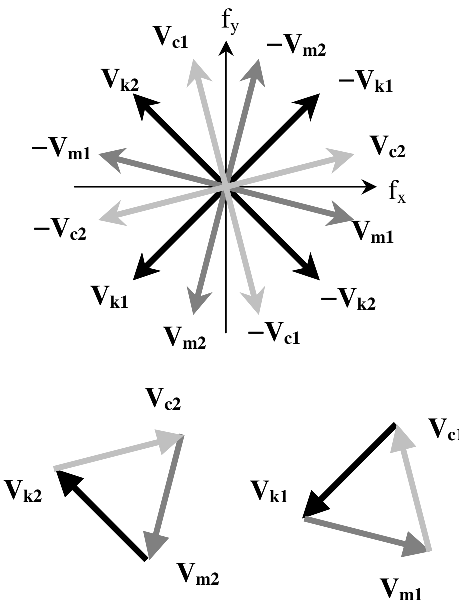

Figure 8. Three-color moiré-free conditions illustrated by two triangles formed by vector summations

Using scalar representation of above equations and Eqs. 3a-3d, one can easily rewrite the moiré-free condition as follows,

$$\frac{x_{c1}}{A_c} + \frac{x_{m1}}{A_m} + \frac{x_{k1}}{A_k} = 0\,, \tag{9a}$$

$$\frac{y_{c1}}{A_c} + \frac{y_{m1}}{A_m} + \frac{y_{k1}}{A_k} = 0, \qquad (9b)$$

$$\frac{x_{c2}}{A_c} + \frac{x_{m2}}{A_m} + \frac{x_{k2}}{A_k} = 0, \text{ and} \qquad (9c)$$

$$\frac{y_{c2}}{A_c} + \frac{y_{m2}}{A_m} + \frac{y_{k2}}{A_k} = 0. \qquad (9d)$$

where the values of parallelogram area are given by Eq. 2, or rewritten as

$$A_c = \left| x_{c1} y_{c2} - x_{c2} y_{c1} \right|, \qquad (10a)$$

$$A_m = \left| x_{m1} y_{m2} - x_{m2} y_{m1} \right|, \qquad (10b)$$

$$A_k = \left| x_{k1} y_{k2} - x_{k2} y_{k1} \right|. \qquad (10c)$$

Alternately, two vector equations in spatial vectors can be derived from Eqs. 9a-9d, i.e.,

$$\frac{\mathbf{v}_{c1}}{A_c} + \frac{\mathbf{v}_{m1}}{A_m} + \frac{\mathbf{v}_{k1}}{A_k} = 0, \text{ and} \qquad (11a)$$

$$\frac{\mathbf{v}_{c2}}{A_c} + \frac{\mathbf{v}_{m2}}{A_m} + \frac{\mathbf{v}_{k2}}{A_k} = 0. \qquad (11b)$$

If all Cartesian components of spatial vectors are integer specified, area A_c, A_m and A_k are also integers. So, by rearranging Eqs. 9a-9d, the three-color moiré-free condition can be fully specified by four integer equations:

$$x_{c1} A_m A_k + x_{m1} A_k A_c + x_{k1} A_c A_m = 0, \qquad (12a)$$

$$y_{c1} A_m A_k + y_{m1} A_k A_c + y_{k1} A_c A_m = 0, \qquad (12b)$$

$$x_{c2} A_m A_k + x_{m2} A_k A_c + x_{k2} A_c A_m = 0, \qquad (12c)$$

$$y_{c2} A_m A_k + y_{m2} A_k A_c + y_{k2} A_c A_m = 0. \qquad (12d)$$

Obviously, other moiré-free conditions by inequalities 6 and 7 can be also converted to integer inequalities in a similar matter.

Searching Moiré-Free Solutions

From the previous section, it is apparent that although the original moiré analysis is mostly based on frequency calculation, all moiré-free conditions for nonorthogonal screen halftoning can be completely specified by spatial vectors, which define the shape and size of parallelograms. Furthermore, for digital halftoning, all these moiré-free conditions can be stated by either integer equations or integer inequalities. Therefore, the number of moiré-free solutions is finite and all solutions can be searched by a computer, even though there are not enough equations and/or inequalities for analytic solutions. We have also learnt that due to the digital grid constraint the choices of different parallelograms are quite limited, which, on the other hand, allows us to do a quick searching for possible combinations for moiré-free color halftoning. For example, if we restrict the halftone screens to a frequency range above 120 line-per-inch for a 1200x1200 dot-per-inch printer, the searching is limited to all integers less than 10. Briefly, a possible searching routine might be described by the following steps:

1. Search all possible parallelograms, which meet the requirement of screen sizes and shapes, and store them as two integer-specified spatial vectors, (x_1, y_1; x_2, y_2) into a screen list.
2. For each set of three parallelograms, say (x_{c1}, y_{c1}; x_{c2}, y_{c2}), (x_{m1}, y_{m1}; x_{m2}, y_{m2}) and (x_{k1}, y_{k1}; x_{k2}, y_{k2}), from the stored screen list, check
   - 2a. if three-color moiré-free conditions, integer equations 12a-12d, are satisfied;
   - 2b. if all other two- or three-color moiré-free conditions, integer inequalities derived from Eqs. 7 and 8, are satisfied;
   - 2c. if other possible additional constraints, e.g., symmetry appearance, are satisfied.
3. Save the result if all constraints are satisfied, otherwise, stop checking, and continue to the next combination.
4. Evaluate the result. If it is necessary, change the requirements for screen size, shape, moiré frequency limit, or others, and redo the searching.
5. Conduct final evaluation by designing halftone screens based on selected geometries and generating and printing testing patterns.

Of course, the description above only provides a skeleton of a possible search routine. Many possible variations might be implemented to speed up the searching and/or accomplish additional requirements.

Example of Moiré-Free Nonorthogonal Screens

If we assume that all three nonorthogonal screens have the same area, i.e., $A_c = A_m = A_k = A$, the integer equations 12a-12d can be significantly simplified and the searching for moiré-free solutions could be done even without a computer. As an example, by selecting A = 60, a possible moiré-free solution is given by

$\mathbf{V}_{c1}$ = (8, 2), $\mathbf{V}_{c2}$ = (-2, 7);
$\mathbf{V}_{m1}$ = (2, 7), $\mathbf{V}_{m2}$ = (-8, 2); and
$\mathbf{V}_{k1}$ = (6, 5), $\mathbf{V}_{k2}$ = (-6, 5).

The three parallelograms and their spectral representations are shown in Fig. 9. When this set halftone screens is applied to a color printer with 1200x1200 dot-per-inch resolution, the halftone output will have the following frequency properties:

Cyan: 75.96º, 164.9 lpi and -15.95º, 145.6 lpi;
Magenta: 15.95º, 145.6 lpi and -75.96º, 164.9 lpi;
Black: -50.2º, 156.2 lpi and 50.2º, 156.2 lpi.

Figure 10 shows the output by this set of nonorthogonal halftone screens with certain constant input for cyan, magenta and black channels. There are two scaled-up versions of the output with different scale factors in Fig. 10. It is certainly interesting to notice that the rosette pattern generated by this set of nonorthogonal screens has clear repeated structure, which is quite different from rosette patterns generated by classical rotated orthogonal screens.

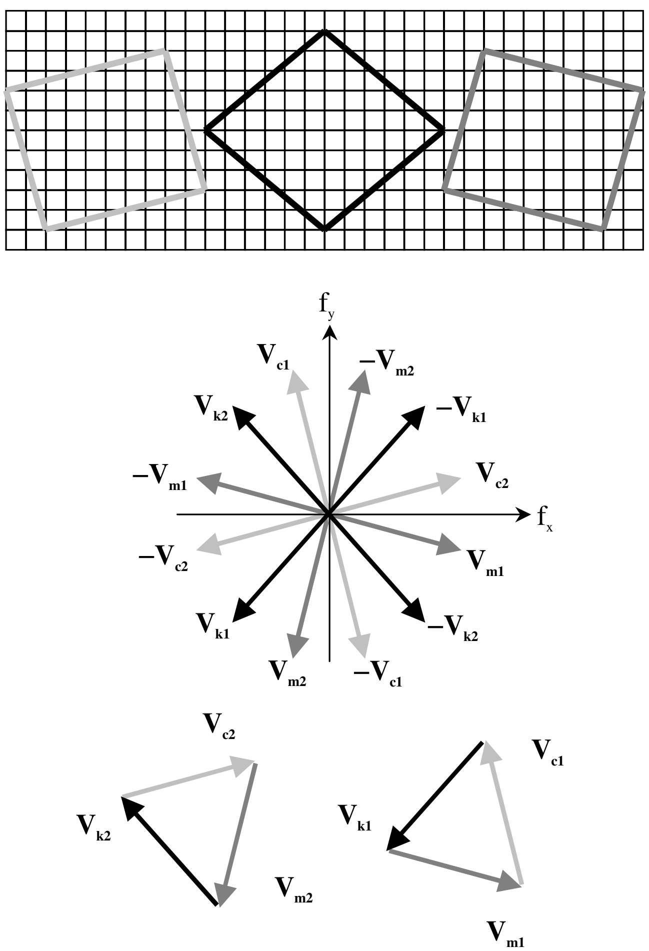

Figure 9. Three nonorthogonal halftone screens satisfying moiré-free conditions

Conclusion

Due to the "digital grid" constraint, selecting different frequencies, including angles and moduli, for regular orthogonal halftone screens is quite limited for digital halftoning. Instead, using nonorthogonal screens unfolds another dimension for the frequency choices. Therefore, for most printing devices with relatively high resolutions or high addressabilities, the three-color moiré can be completely eliminated by employing three single-cell nonorthogonal halftone screens.

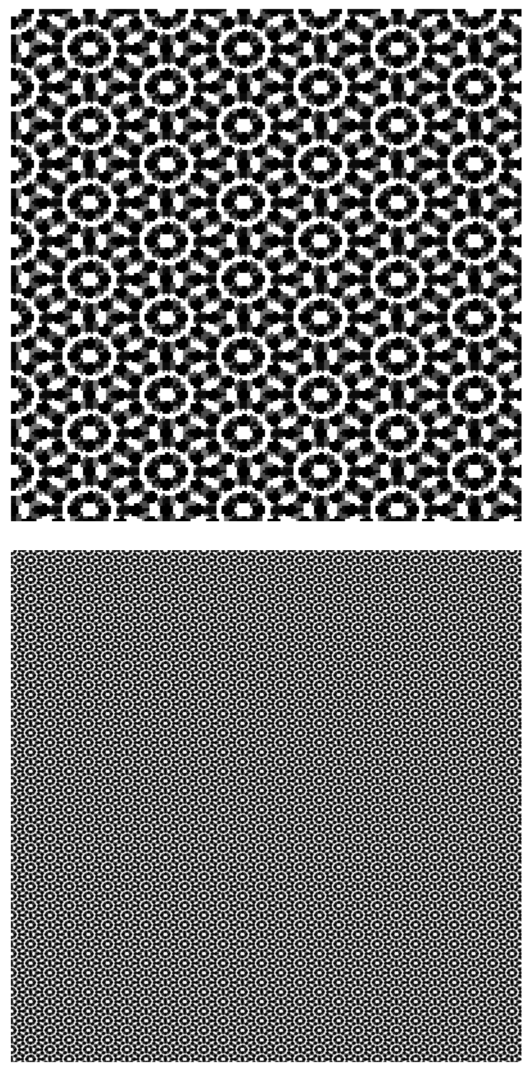

Figure 10. Sample halftone outputs by a set of moiré-free nonorthogonal halftone screens displayed in different scales

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Biography

Shen-ge Wang is a principal scientist with Xerox Corporation. He received a BS degree in Instrumental Mechanics from Changchun Institute of Optics, China, in 1970 and a Ph.D. degree in Optics from University of Rochester in 1986, respectively. His current research includes image processing, halftoning and printer modeling.

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