

Robust Spatial-sampling Controller Design for Banding Reduction in Electrophotographic Process

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Abstract

An improved controller design and implementation technique for electrophotographic process (EP) was proposed. The new controller was modified from a previous design to address two additional issues for generic EP platforms, i.e. reducing position-dependent disturbances and reducing system sensitivity to manufacturing variations in EP engine and consumables. To handle position-dependent periodic disturbances, a digital repetitive controller was developed and implemented using spatial sampling. The result is a control algorithm that will take into account the variation of the nominal operating speed. Second, system variations due to manufacturing variations as well as consumable changes were incorporated into the design of a two degree of freedom (TDOF) robust controller. The controller is optimal in the sense that it minimizes the size of the sensitivity function from a set of disturbance signals to a set of measurable signals critical to print quality, e.g. photoconductor drum velocity or scan line spacing. A suitable trade-off between system performance and robustness to system modeling uncertainties was considered in the synthesis and optimization formulation. The effectiveness of the proposed controller design and implementation technique was numerically and experimentally verified. Printed samples demonstrated significant reduction in visible banding that was verified by reflectance measurement.

Introduction

Electrophotography (EP) or xerography (XP) has been the technology of choice for high-speed office printing, e.g. laser printers and copiers. For laser EP process, halftone banding, which affects the uniformity of a printed image, is a well-known artifact that appears as periodic light and dark streaks across a printed page perpendicular to the process or print direction. A lot of literature has devoted to the modeling and reduction of banding in laser EP process. Most of them can be found in patents and past IS&T conferences, e.g. the paper by Chen et al.¹ and the reference therein. Periodic nature of banding usually points its source

towards the imaging subsystem which consists of a motor, gear trains, and an imaging component (e.g. the photoconductive imaging drum). As demonstrated by Chen et al.¹ and many other researchers, transmission degradation due to photoconductive imaging drum velocity variation, gear eccentricity, and tooth-profile error directly affected scan line spacing variation, which in turn induced banding.

Typically, design of EP engines has relied on tight-tolerance components and open-loop design principles. System parameters and operating conditions are designed and maintained so that the physical process is least affected by disturbances and uncertainties, i.e. a sweat spot. For example, exposure level is determined to saturate on the photoreceptor; large inertia of the photoreceptor motion system is used to reduce motion sensitivity to load variation; very high manufacturing precisions are kept for the key components in the development system. These solutions are limited in the sense that a change of consumables and the wear and tear of normal operation will significantly affect the system characteristics, e.g. the stacking of the tolerances, and ultimately the stability of the print quality during the life of the product. In addition, as engine architecture changes, the sensitivity of modifying one component is difficult to manage and often relied on exhaustive environmental testing that are often expensive and time consuming.

Closed-loop control approach has been shown to be effective in sensitivity reduction in component level controls e.g. speed regulation of the polygon mirror and main drive motor. However, regulation of each individual subsystem based on local sensor signal feedback, e.g. velocity from the motor, usually does not guarantee good regulation on the component where the image is formed, e.g. photoconductor drum. We propose to close the loop by controlling the actuator using signal feedback from sensors placed on the imaging component of which certain physical quantity is to be regulated. For example, in a monochrome platform, a suitable choice for the actuator and sensor placement would be the main drive motor and photoreceptor, respectively. Next, the control algorithm

should be able to reduce the effect of disturbances, i.e. sensitivity reduction, while stabilizing the system even with nonlinear physical limitation on the control input, i.e. actuator saturation.

Another objective of using closed-loop control is to obtain system robustness, which, by definition, is to reduce system sensitivity to uncertainties such as consumable change and component aging. This goal along with disturbance rejection can actually be achieved by formulating and treating the control problem in a unified linear design framework, e.g. linear fractional transformation (LFT). Extensive theoretical results have been developed for solving robust control problems formulated within this design framework (e.g. the book by Zhou and Doyle⁵).

A digital repetitive control system has been shown to work well for tracking periodic reference commands or for rejecting periodic disturbances in regulation applications. Typical digital repetitive controller is synthesized in temporal domain. For example, to synthesize the prototype repetitive controller proposed by Tomizuka,² one of the key steps is to determine the number of delay taps based on the temporal frequency of the tracking or disturbance signal. To ensure effectiveness of the design, an underlying assumption is that the tracking or disturbance signal is stationary, i.e. the frequency value of the tracking or disturbance signal does not vary with time. In many cases, where the objective of design is to track repetitive trajectory, this assumption can easily be satisfied. However, it might not be valid for most disturbance rejection problems where the disturbances might have time-varying frequencies. For instance, for a motor/gear transmission subsystem in the EP process, disturbances due to gear eccentricity or tooth profile error are inherently position-dependent. Gear eccentricity usually induces disturbances that repeat once per revolution, and tooth profile error usually induces disturbances with fundamental frequency equal to the number of teeth per revolution. This implies that the temporal frequency values for these two types of disturbances will be proportional to the nominal angular velocity and vary accordingly. Thus a repetitive controller designed for a motor/gear transmission system and with its parameters determined based on the temporal information of disturbances will not perform well as the angular velocity fluctuation is one of the source of the disturbance. Recent researches have looked into rejecting position-dependent disturbances or tracking periodic time-varying references.^{3,4} Nakano et al.³ suggested the idea of transforming the original linear time invariant (LTI) system into position domain and linearizing the resulting nonlinear system around the nominal angular velocity. Thus a linear position invariant (LPI) system was obtained for controller design.

This paper extended our previous work¹ and proposed a more general closed-loop control design framework for EP process. First it was suggested that the control system should be formulated and operated with respect to spatial coordinate (e.g. angular position) rather than time coordinate. The immediate advantage is that the new spatial-based repetitive control system is robust to those

time-varying spatial-periodic disturbances, which are commonly seen in the motor/gear transmission system within an EP process. Moreover, the control problem was formulated using a unified design framework, i.e. linear fractional transformation (LFT), which can be solved with existing numerical tools.⁷ The advantage of using the LFT formulation is that many practical issues such as modeling uncertainties due to consumable change, component aging, and actuator saturation can be easily integrated into the design framework. Some design and numerical details were left out to avoid repetition and for succinctness. The proposed scheme was implemented on a typical 600-dpi EP engine. Results from both the temporal-based and the spatial-based design are presented. As expected, the spatial-based repetitive controller design performed better than temporal-based design under load variation during printing.

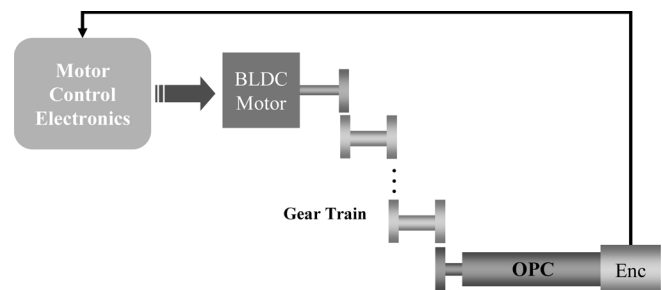


Figure 1. Motor/Gear transmission subsystem for a typical EP printing system.

System Description

The imaging part of an EP process can be viewed as combination of two subsystems: motor/gear transmission subsystem and laser/polygon optical subsystem. This paper focuses on the control of the motor/gear transmission subsystem.

Motor/Gear Transmission Subsystem for Laser EP Process

The motor/gear transmission subsystem within a typical laser printer is shown in Figure 1. The system consists of a main brushless dc (BLDC) or stepper motor with its onboard driver and a gear train that connects to the photoconductor (PC), in this case is an organic photoconductive (OPC) drum. The task of the main motor is to supply torque to preserve constant angular velocity of the photoconductor. The driver controls the velocity of the motor by adjusting the amount of winding currents (in BLDC motors) or the stepping rate (in stepper motors). The motor shaft drives the gear train that connects with the OPC drum. Phase locked loop (PLL) and pulse width modulation (PWM) are two of the most popular methods utilized motor drivers for achieving the desired velocity regulation.

Position-dependent Disturbances

Many researches have shown that velocity fluctuation of the PC is one of the main causes of banding'. In a typical laser printing process, the low-pass filtering of the load inertia and high-speed operation of the main motor drive usually render the effect of motor torque ripples on the PC velocity negligible. In fact, most of the velocity fluctuations come from gear eccentricity and tooth profile error, both are periodic and position-dependent. They can be further characterized by analyzing the PC velocity signals sampled at equal position interval, also known as order analysis. Order spectrum obtained from order analysis provides a direct link between a frequency and the corresponding gear component, as will be demonstrated in the following example. The frequency values will also be used for specifying parameters of the repetitive controller. Figure 2 shows the motor/gear transmission configuration for a typical 600-dpi monochrome laser printer. The order spectrum was obtained by taking the OPC position/velocity sampling at 2500 samples/rev. The largest peak at 384 cycles/rev corresponds to the meshing between the large gear on shaft 1 and the pinion on the motor shaft. The peak at 96 cycles/rev corresponds to the meshing between the large gear on shaft 2 and the small gear on shaft 1. The peak at 48 cycles/rev was due to the eccentricity of the motor shaft. The peak at 32 cycles/rev, which is equal to 8.65 cycles/in under the nominal speed, was caused by the OPC drum. It is worth noticing that gear meshes far away from the OPC drum could also cause noticeable OPC velocity variation.

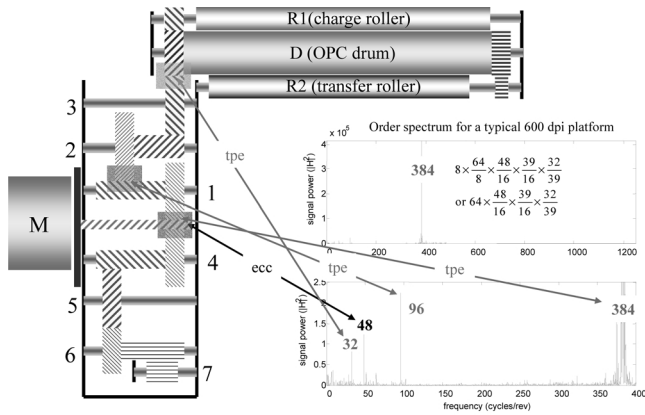


Figure 2. Configuration of a motor/gear transmission subsystem of a monochrome laser printer.

Closed-loop Control Structure

Suppose linear control design approach is applied. We can start the design by first looking at the linear fractional transformation (LFT) representation of the desired closed loop control system depicted in Figure 3. Several variables and components need to be identified here. First, the generalized plant $P(z)$ includes the plant and all weighting

filters. Note that only motor actuators are considered in the framework and driven by the control input u calculated by the controller $K(z)$. Other actuators such as lasers can also be implemented. Second, the inputs to the controller y are output signal measurement, e.g. velocity error from the OPC drum. Third, the variable w includes those external signals such as periodic disturbances while the variable z includes those physical quantities which are important to image quality, e.g. velocity error, scan line spacing variation, or color plane misregistration. Furthermore, p and q represent the input and output of the generalized uncertainty which is formed by all the uncertainty blocks from the generalized plant. The uncertainty blocks usually come from modeling error and plant nonlinearity.

Based on the LFT representation, a discrete-time state space realization can be written as

$$\begin{aligned} x_{k+1} &= Ax_k + [B_{M1} \ B_{M2}]u_k + B_w w_k, \\ y_k &= \begin{bmatrix} C_{IM} \\ C_{ID} \end{bmatrix} x_k + D_{yw} w_k, \\ z_k &= C_{IQ} x_k + D_{zW} w_k, \\ q &= \Delta p, \end{aligned} \tag{1}$$

and the optimal stabilizing controller $K(z)$, which should minimize the size of the transfer function (e.g. H_∞ norm) from w to z in the presence of the generalized uncertainty, can be represented as

$$K(z) = \begin{bmatrix} C_{K1} \\ C_{K2} \end{bmatrix} (zI - A_K)^{-1} [B_{K1} \ B_{K2}] + D_K. \tag{2}$$

Theoretical details regarding LFT formulation and synthesis of stabilizing controllers can be found in the book by Zhou and Doyle.⁵

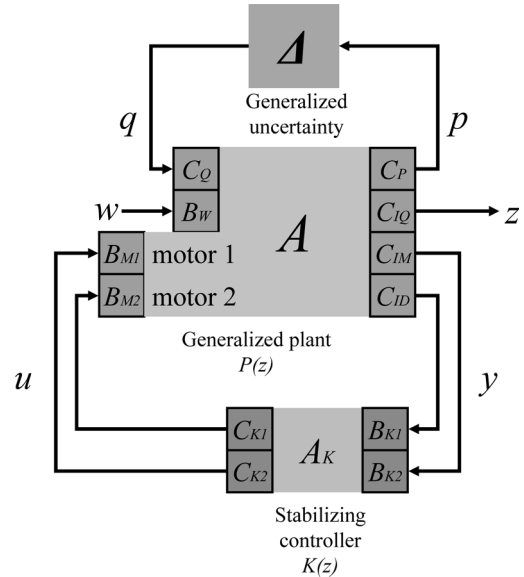


Figure 3. LFT representation of the EP closed-loop control system using motor actuators.

In the case of controlling the motor/gear transmission subsystem using OPC drum velocity feedback ($C_{k2} = 0$ and $B_{M2} = 0$), u will be the motor control voltage while y and z simply represent the velocity error of the OPC drum.

Spatial-sampling Repetitive Controller Design

Discrete-Position Model of the System

To obtain the discrete-position model of the corresponding linear time invariant (LTI) system, we first inspect the temporal model $P(s)$ for the motor/gear transmission system. Suppose $P(s)$ has a state space realization, i.e.

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + B_u u(t) \\ y(t) &= C_y x(t) + D_{yv} v(t), \end{aligned} \quad (3)$$

where $v(t)$ denotes disturbances at the plant output. Instead of using time t as the independent variable, it is possible to consider an alternative independent variable $\phi = f(t)$, which is a strictly monotone function of t such that its inverse ($t = f^{-1}(\phi)$) exists. Thus in the ϕ -domain Eq. (3) can be expressed as

$$\begin{aligned} \frac{d\phi}{dt} \frac{d\tilde{x}(\phi)}{d\phi} &= A\tilde{x}(\phi) + B_u \tilde{u}(\phi) \\ \tilde{y}(\phi) &= C_y \tilde{x}(\phi) + D_{yv} \tilde{v}(\phi), \end{aligned} \quad (4)$$

where

$$\tilde{x}(\phi) = x(f^{-1}(\phi)), \quad \tilde{u}(\phi) = u(f^{-1}(\phi)), \quad \tilde{y}(\phi) = y(f^{-1}(\phi)),$$

and $\tilde{v}(\phi) = v(f^{-1}(\phi))$.

We can pick angular position, $\theta(t)$, as the independent variable, i.e. $\phi = \theta(t)$. Since by definition

$$\theta(t) = \int_0^t w(\tau) d\tau, \quad (5)$$

where $w(t)$ is the angular velocity, the following condition

$$w(t) = \frac{d\theta}{dt} > 0, \quad \forall t > 0 \quad (6)$$

will guarantee that $\theta(t)$ is strictly monotonic. In practice, this condition can be easily satisfied in an EP process, since the imaging component such as the OPC drum always rotates in one direction. Substituting Eq. (6) into Eq. (4) and linearize the equation around the nominal angular velocity w_0 , we have

$$\begin{aligned} \frac{d\tilde{x}(\theta)}{d\phi} &= \frac{A}{w_0} \tilde{x}(\theta) + \frac{B_u}{w_0} \tilde{u}(\theta) \\ \tilde{y}(\theta) &= C_y \tilde{x}(\theta) + D_{yv} \tilde{v}(\theta). \end{aligned} \quad (7)$$

Equation (7) is a linear position invariant (LPI) system with the angular position $\theta(t)$ as the independent variable. Note that this transformation will render those position-dependent disturbances within \tilde{v} periodic and stationary.

The performance of a repetitive controller synthesized in the θ -domain will not be compromised. Properly choosing spatial sampling frequency T_θ (in number of samples per revolution), we can discretize Eq. (7) and acquire a discrete-position model, i.e.

$$\begin{aligned} \tilde{x}_{k+1} &= e^{\frac{A}{w_0} T_\theta} \tilde{x}_k + \left(\int_0^{T_\theta} e^{\frac{A}{w_0} \tau} d\tau \right) \frac{B_u}{w_0} \tilde{u}_k \\ \tilde{y}_k &= C_y \tilde{x}_k + D_{yv} \tilde{v}_k. \end{aligned} \quad (8)$$

The procedures summarized in our previous design⁶ can now be applied to the plant model expressed in Eq. (8) for synthesizing a two degree of freedom (TDOF) discrete-position robust repetitive controller.

TDOF Robust Repetitive Controller

To reduce system sensitivity or increase system robustness to unmodeled dynamics or nonlinearity (i.e. actuator saturation), we can formulate the control problem within a unified linear design framework, i.e. using LFT. The proposed TDOF control structure is depicted in Figure 4. The actual plant is represented as a saturation element $0.5(1+\Delta_1)$ with $|\Delta_1| \leq 1$ followed by a nominal model $P(z)$ with output multiplicative uncertainties $W_2 \Delta_2$. W_2 is the frequency-dependent uncertainty weighting filter such that $\|\Delta_2\|_\infty \leq 1$. It can be picked to be any stable filter with its magnitude upper bounding the multiplicative error between the model and the actual plant, i.e.

$$\left| \frac{\hat{P}(e^{jw}) - P(e^{jw})}{P(e^{jw})} \right| \leq |W_2(e^{jw})|, \quad \forall w \quad (9)$$

Furthermore, the kernel of the repetitive controller $q(z)z^{-N}$ is replaced by a fictitious uncertainty Δ_3 . Also another fictitious uncertainty Δ_f is connected between the disturbance input and plant output. W_1 is the frequency-dependent weighting filter that approximates human contrast sensitivity function.¹ Thus, a TDOF controller is obtained by solving the following mixed-sensitivity optimization problem given by

$$\gamma_{opt} = \inf_{K \text{ stabilizing}} \left\| \begin{array}{c} W_1(1+PC_4)^{-1} \\ C_4 P(1+C_4 P)^{-1} \\ -W_2 PC_4(1+PC_4)^{-1} \\ 1-P(1-C_4 P)^{-1} C_3 \end{array} \right\|_\infty, \quad (10)$$

where

$P \triangleq$ Motor/Gear transmission system

$W_1 \triangleq$ Performance weighting

$W_2 \triangleq$ Uncertainty weighting

$K \triangleq [C_3 \ C_4]$ The TDOF controller

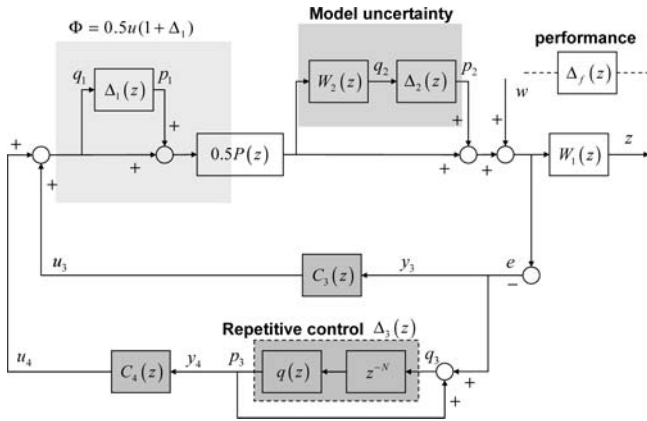


Figure 4. The proposed TDOF robust repetitive control system.

With upper and lower LFT denoted by $F_u(\bullet, \bullet)$ and $F_l(\bullet, \bullet)$, respectively,⁵ the robust performance of the designed control system can further be evaluated by looking at the structure singular value of $F_l(F_u(M, R), K)$ with respect to the uncertainty block $\Delta = \text{diag}(\Delta_1, \Delta_2, \Delta_f)$, i.e. $\mu_\Delta(F_l(F_u(M, R), K))$. Note that $R(z) = q(z)z^{-N}$ is the kernel of the repetitive controller.

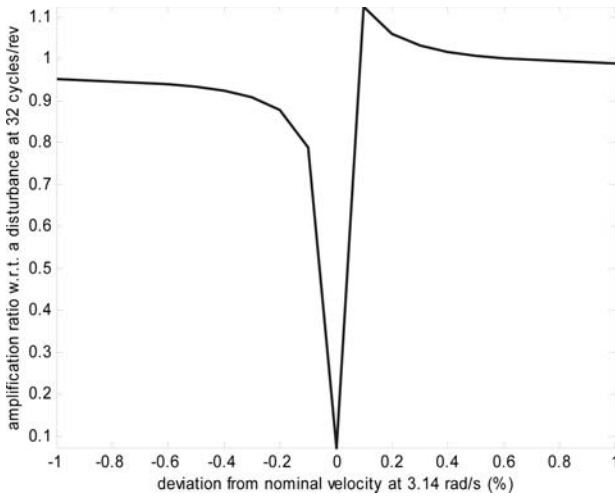


Figure 5. Effect of nominal angular velocity variation on performance of the repetitive controller.

Effect of Nominal Angular Velocity Variation on Temporal-based Repetitive Control

A repetitive control system creates comb-like notches in the system sensitivity function at periodic disturbance frequencies. For a motor/gear transmission system where significant disturbance sources come from gear eccentricity or tooth profile error, temporal frequencies of those disturbances will be proportional to the nominal angular velocity. Thus the performance of temporal-based repetitive control systems will deteriorate as the nominal angular velocity varies. The velocity variation can be caused by

friction or medium in use in a laser printing system, which is usually time-varying and difficult to be taken into account during design of the controller. Based on the proposed TDOF repetitive controller design, Figure 5 shows the effect of nominal velocity variation on the performance of the sensitivity reduction. Parameters of the repetitive controller were specified to reject a disturbance located at 16 Hz when the system is operating at a nominal angular velocity of 3.14 rad/s. It can be seen that as the nominal velocity deviates from the desired value, the ability of the repetitive controller to reject the disturbance at 16 Hz degrades significantly. As shown in Figure 5, a 0.2% variation in the nominal speed has an order of magnitude effect in the effectiveness of disturbance rejection. This high sensitivity to operating velocity is the motivation for pursuing the spatial-based repetitive control.

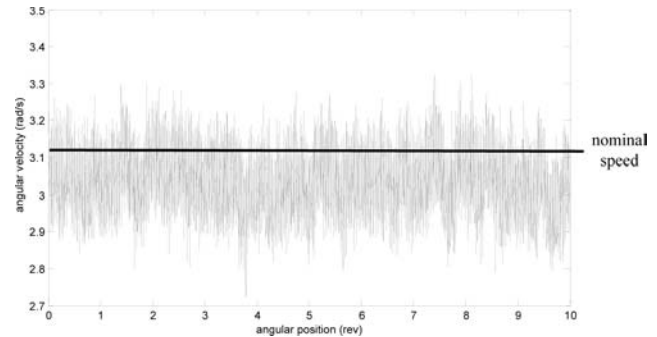


Figure 6. Measured OPC angular velocity during printing.

Experimental Results

Spatial-based Repetitive Control

The proposed discrete-position repetitive controller was implemented on a typical 600-dpi laser printing system. An optical encoder was mounted on the OPC drum. A spatial sampling scheme that uses the encoder pulses (instead of a master clock) to trigger the interrupt of the control algorithm at intervals of equal angular position was implemented. Instead of counting number of pulses within a sampling period, the angular velocity was determined by monitoring the amount of time elapsed for fixed number of encoder pulses. This method actually enables low-cost encoders to achieve high-resolution velocity measurement. The spatial sampling frequency was set at 2000 samples/rev such that the discrete-position repetitive controller has a period of $N=2000/16=125$. The engine started printing when velocity data of 10 revolutions were collected from the OPC drum for analysis. Figure 6 shows the measured angular velocity from the OPC drum. Note that as the paper goes through the printing process, it slightly increased the load on the transmission system. This impact decreased the nominal angular velocity from 3.14 rad/s to 3.07 rad/s. However, the frequency spectrums, as shown in Figure 7, indicated that the performance of the discrete-position repetitive control system was not degraded by this variation

in the nominal velocity. It can be seen that capability of the repetitive controller was compromised due to frequency shifting of those periodic disturbances. This can further be verified by looking at actual printed images (see Figure 8).

Conclusions

In this paper, a TDOF robust repetitive control system was synthesized in spatial domain. The immediately obtained advantage was that the new spatial-based repetitive control system is robust to those time-varying spatial-periodic disturbances, which are commonly seen in the motor/gear transmission system within laser EP process. The proposed control system was also robust by integrating practical issues such as plant parameter change due to consumable change, component aging, and actuator saturation into the controller design. Using specific spatial-sampling scheme, high-resolution or low-quantized velocity feedback was possible and the compensated printing system was able to reduce banding artifact significantly even with load variation. Although in this study the scheme was verified only on monochrome platform, it can be modified to work on color platforms by incorporating multiple sensors and actuators.

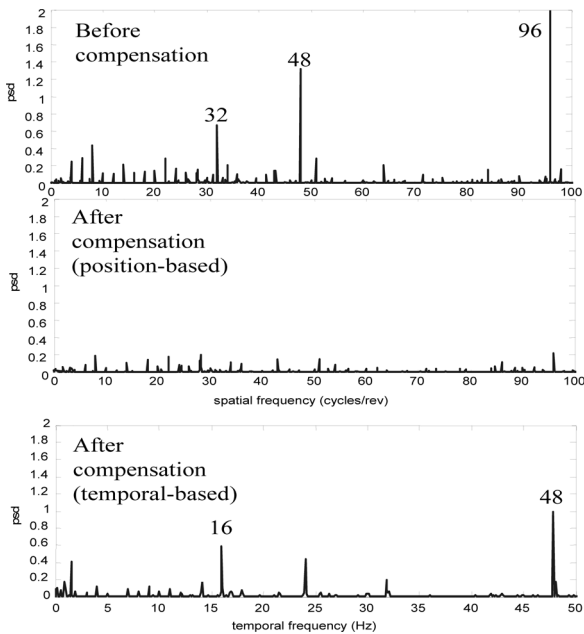


Figure 7. Experimental PC velocity variation spectrum

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Figure 8. Actual images printed from the uncompensated (top) and compensated (bottom) systems. The compensated system implemented the proposed spatial-based TDOF repetitive controller.

Biography

Cheng-Lun Chen received his BS and MS degrees from National Tsing-Hua University, Taiwan (Department of Power Mechanical Engineering) in 1993 and 1995,

respectively. Since 1998 he has been studying towards his Ph.D degree in the School of Mechanical Engineering, Purdue University at West Lafayette. His current research interest is on modeling and closed-loop control of printing systems.