Modelling Ink-jet Printing – Does Kubelka-Munk Theory Apply?

Li Yang
Campus Norrkoping (ITN), Linköping University
Nörrköping, Sweden

Abstract

We present a method to characterize the ink jet printing by means of spectral reflectance measurements and simulations, which includes optical properties of primary inks, the ink volume, scheme of color-mixing for generating secondary colors. The simulations are based on Kubelka-Munk theory whose availability to ink-jet printing is discussed.

1. Introduction

Ink-jet is a non-impact dot-matrix printing technology in which droplets of ink are jetted from a small aperture directly to a specified position on a substrate to create an image.[1] The color rendition of the printed image depends above all on the optical properties of the primary inks and the scheme for generating the secondary colors. The saturation of the printed colors on the other hand depends also on the volume of the printed inks or equivalently the thickness of the ink layers. Therefore characterizations of the output print in terms of ink volume, the scheme of ink mixing, light absorption and light scattering are of essential importance in controlling and understanding the quality of the color reproduction.

2. Methodology

The full tone samples were printed with primary- (cyan, magenta and yellow) and secondary-colors (red, green and blue). In order to prevent the inks from penetrating into the substrate, ink-jet films were used as the substrates. Therefore the sample consists of an ink layer and the plastic substrate. By varying ink-level specification in the printer driving software, one can obtain samples printed with up to 5 ink levels (the ink-volume increases from the ink-level 1 to 5). The spectral reflectance values of the printed samples were measured by using SpectroScan, which covers the a range of wavelength between 380 and 730 *nm* with an interval of 10 *nm*. To achieve high reflection from the samples, a white and opaque background (a bunch of white paper) has been lain under the samples.

According to Kubelka-Munk Theory,[2] the spectral reflectance value of an ink layer is a function of its scattering- and absorption-lengths, $s_q(\lambda)z_q$ and $k_q(\lambda)z_q$, i.e.

$$R_q(\lambda, z_q) = f(s_q(\lambda)z_q, k_q(\lambda)z_q) \tag{1}$$

where $s_q(\lambda)$ and $k_q(\lambda)$ are the scattering- and absorption coefficients and z_q is the thickness of the ink layer. The subscript is an index of color, and $q = c, m, y$ means cyan, magenta and yellow, respectively. When the interface reflection is negligible, the function in Eq. (1) may be written as[3,4]

$$f(sz,kz) = \frac{(R_\infty - R_g)e^{-(1/R_\infty - R_\infty)sz} - R_\infty(1 - R_g R_\infty)}{R_\infty(R_\infty - R_g)e^{-(1/R_\infty - R_\infty)sz} - (1 - R_g R_\infty)} \tag{2}$$

where $R_g(\lambda)$ is the spectral reflectance of the bare substrate while

$$R_\infty(\lambda) = 1 + \frac{k(\lambda)}{s(\lambda)} - \sqrt{\left(\frac{k(\lambda)}{s(\lambda)}^2 + 2\frac{k(\lambda)}{s(\lambda)}\right)} \tag{3}$$

stands for that of an infinitely thick ink layer $(z \to \infty)$. Therefore by fitting to two sets of the measured spectral reflectance values, one can obtain the scattering-absorption-lengths of the ink. In the present study, one of the data was obtained from samples that were printed in ink level 3 (defined by the printer driving program) primary inks and another set was from samples of twice printed with the same ink level.

The scattering- and absorption-length of the secondary colors was computed by applying additivity approximation. For example, color red is composed of ink magenta and ink yellow. Its scattering- and absorption-lengths, be calculated

$$s_r z_r = \beta_m s_m z_m + \beta_y s_y z_y \tag{4}$$

$$k_r z_r = \beta_m k_m z_m + \beta_y k_y z_y \tag{5}$$

where β_m and β_y are the relative ink thicknesses of magenta and yellow.

3. Results and Discussions

In this section we present the experimental measurements together with our simulations. As mentioned above, the spectral absorption- and scattering-lengths (*kz* and *sz*) of the primary inks were obtained by fitting to the experimental spectral reflectance values of samples printed with ink level 3.

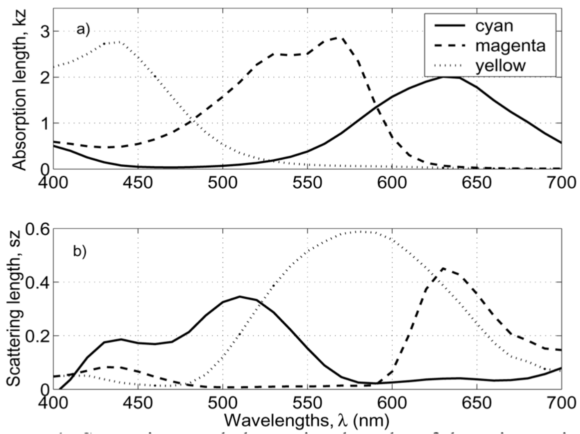

Figure 1. Scattering- and absorption-lengths of the primary inks obtained by fitting into the measured spectral reflectance values of samples printed on ink-jet films with (printer driving) program specified ink level 3.

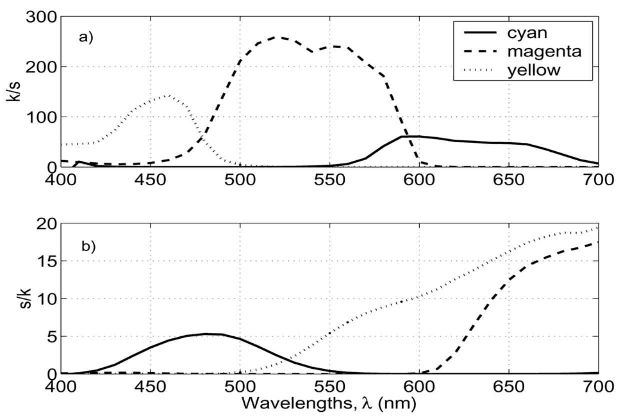

Figure 2. Spectral variation for the relative strength of the absorption and the scattering.

3.1. Spectral Reflectance Values of Primary Inks

The scattering- and absorption-lengths of the primary colors are shown in Fig. 1. The inks show clearly *absorption/transparent* band structure. Interestingly enough, the inks also show remarkable scattering strengths in their transparent regions, even though their peak values are about 5 or more times smaller than those of their absorption-lengths. Considering the fact that the reflectance depends not only on the absolute values of their scattering- and absorption-lengths, but more importantly on their relative strength (it can be clearly seen from the expression of R_∞), *k/s* and *s/k* ratios are presented in Fig. 2. Scattering may play a dominant role ($s/k >> 1$) in the non-absorption regions which makes the color visible even for a thick ink layer where reflection from the substrate background becomes negligible.

After obtaining the scattering- and absorption-lengths of the primary ink layers (ink level 3) one can then predict reflectance values for an ink layer of any given ink thickness, α_q, by applying

$$R_q(\lambda, \alpha_q z_q) = f(\alpha_q s_q(\lambda) z_q, \alpha_q k_q(\lambda) z_q) \quad (6)$$

where $\alpha_q (q = c, m, y)$ is the relative ink thickness of the ink layer. Inversely, for known spectral reflectance values, R_q, the relative ink thickness of the ink layer, α_q, can be estimated by fitting according to Eq. (6). Because the spectral reflectance values of each sample consist of reflectance values at 31 wavelengths (400 – 700 *nm*), the agreement between the computed spectral reflectance values and the measured ones can serve as a quantitatively test to the quality of the scattering- and absorption-values obtained from ink level 3. The agreement may also serve as a test to the availability of the present method. Finally the thickness of the ink layer is proportional to the printed ink volume. Therefore one can actually characterize the ink application controlled by the printing engine.

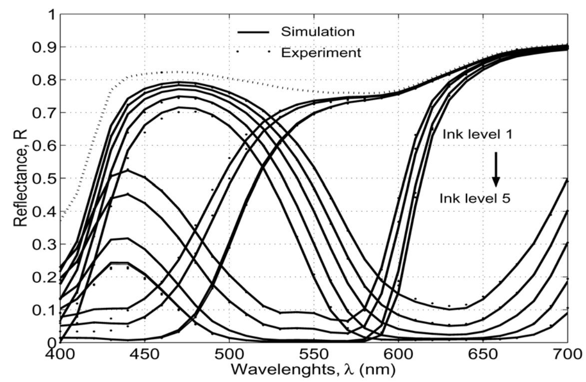

Figure 3. Simulated and measured spectral reflectance values for samples printed with different (printer driving) program specified ink levels. The dot line is the spectral reflectance values of the bare film.

In the present study, the test was made by applying the scattering- and absorption-coefficients to compute spectral reflectance values for samples printed with other 4 ink levels (ink level 1, 2 ,4 and 5) and to compare with their measured values as shown in Fig. 3. The excellent agreement between the computed and the measured spectral reflectance values over the whole range of visible light may be considered as a confirmation for the method validity and for the reliability of the $s_q(\lambda)$, $k_q(\lambda)$ values.

It is worth to notice that the 5-ink volume specification in the printer driving program does not always mean 5 different printing ink levels. For ink cyan there are 5 different ink levels indeed, but there are practical only 3 and 4 different ink levels for ink yellow and ink magenta, respectively. The variation of the actual ink volumes (αq) with respect to the (printer driving program) specified ink levels (1-5) is shown in Fig. 4. Additionally the practical ink volumes vary non-linearly with respect to the program specified ones (for simplicity, the actual volume of ink level 1 has been set as unit).

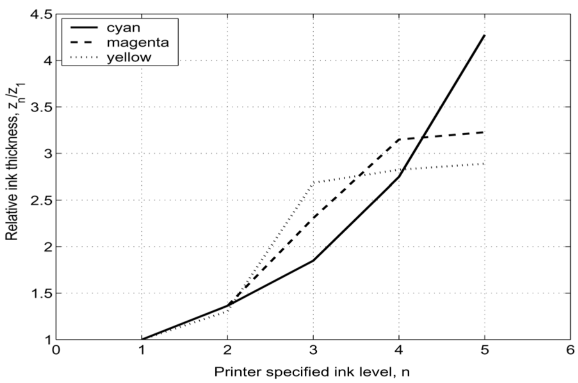

Figure 4. Actual ink volumes $\alpha = z_n/z_1$ *vs. printer driving program specified ink volumes (n=1-5 for ink level 1-5). The actual ink volume for the program specified ink level 1 (z_1) has been set as unit for each color.*

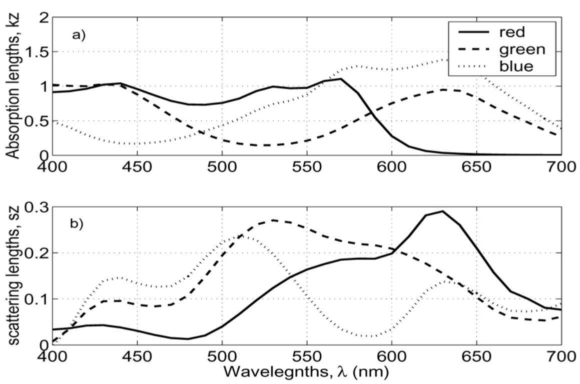

Figure 5. Scattering- and absorption-lengths of the secondary colors obtained by applying the assumption of additivity.

3.2. Spectral Reflectance Values of Secondary Colors

The scattering and absorption lengths of the secondary colors (ink level 1), were shown in Fig. 5. They were obtained by applying the additivity given in Eqs. (4, 5). Fig. 5 shows clearly existence of *absorption/transparent* band-structures that match well intuitiveness. Consulting to the Fig. 1, one can easily see correlations between these *absorption=transparent* structures of the secondary colors with those of their primary components.

The weighting factors (β_q, q=c,m,y) representing the contributions from the primary colors, were determined by fitting the simulations to the experimental spectral reflectance data (see Tab. 1). Similar to the case for the primary inks, the inks amounts vary nonlinearly with respect to the printer specified ink levels. Moreover the relative amount of the primary inks may even vary from one ink level to another. Color green, for example, the ratio $\beta_y{=}\beta_c$ varies only modestly with respect of different printer specified ink levels. However it varies significantly for red.

Table 1. Ink Composition for Secondary Colors From Their Primary Components. The Relative Ink Thicknesses Are Relative to the Ink Level 1 for Each Primary Ink.

Secondary Color	Printer Printer Ink Level	Relative Ink Thickness		
		β_c	β_m	β_y
Red	1		0.84	0.80
	2		1.18	1.05
	3		1.62	1.22
	4		1.92	1.36
	5		2.47	1.30
Green	1	0.85		0.90
	2	1.02		1.24
	3	1.41		1.93
	4	1.83		2.65
	5	2.27		2.78
Blue	1	1.24	0.55	
	2	1.71	0.67	
	3	2.87	0.87	
	4	4.13	1.13	
	5	4.27	1.42	

*The relative ink thickness of the primary inks, β_q (*q = c, m, y*), that contribute to the secondary colors. Note that β_q has the same definition as α_q in Fig. 4

The simulated spectral reflectance values together with the corresponding experiment values are shown in Fig. 6. As one can see that the simulations agree fairly well with the experimental data.

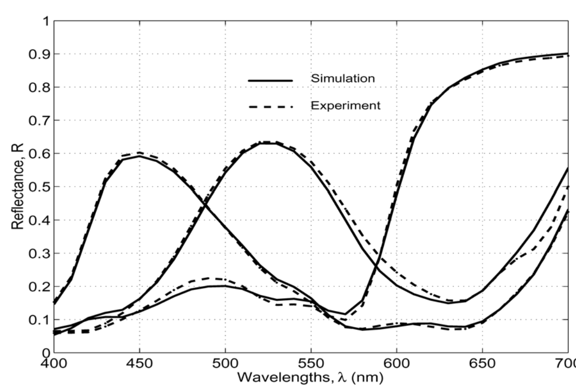

Figure 6. Simulated and measured spectral reflectance values for samples in secondary colors. The samples were printed with program specified ink level 1.

3.3. Some Remarks on Kubelka-Munk Theory

Kubelka-Munk (K-M) theory has been the most widely applied theory for the colorist in research and in industries since it was introduced in 1930's. Over the years the KM theory has been subjected to very close scrutiny. The result is an appreciation of the limitations and strengths of the theory.[5] Of the original assumption made by Kubelka and Munk, that of uniformly diffuse forward and reverse flux through the sample is the most possible source of imprecisions, especially it is applied to a system having strong absorption. The question is how well that K-M theory can be applied to ink jet printing.

As shown in Fig. 1 the light-ink interaction (or the ink spectrum) shows a clear multi-band characteristics which makes the statement that ink is of strong absorption only conditionally true, i.e. for the light whose wavelength lies in the absorption band of the ink. For the light whose wavelength lies well off the absorption band, the absorption is little important and the scattering plays a dominant role (see Figs. 1 and 2). Therefore for a day light type of illumination the portion of the illumination whose wavelengths are well off the absorption band of the ink, its light distribution will probably remain well diffuse if the original illumination is diffuse.

The reason behind the success for the K-M theory in the present study may lie on two factors. First the portion of light that stimulates human color vision has wavelength well off the absorption band of the printed inks. This portion is properly described by the K-M theory. Second the portion of illumination whose wavelength lies in the absorption band will mostly be filtered out by the ink absorption even though the light may not be accurately treated in the K-M theory.

The validity of the K-M theory may be characterized more quantitatively by the relative strength of the scattering over the absorption, or the s/k ratio. For the portion of illumination whose wavelength lies in the region where $s/k \gg 1$ (non-absorption band), the light propagation is well described by the K-M theory as being pointed out by other authors.[6] On the other hand, for the portion of illumination whose wavelength lies in the region where $s/k \ll 1$ (absorption band), this portion of light is filtered out by absorption after passing through the optically thick ink layer. But for the portion of illumination whose wavelength lies in the region where $s/k \sim 1$, the assumption of diffused light distribution will be broken down and may result in remarkable errors. Therefore the accuracy of predictions made by applying the K-M theory depends on the spectral structure of the ink. Explicitly speaking the narrower the band where $s/k \sim 1$ the higher the accuracy of the predictions for the spectral reflectance and vice versa.

4. Summary

We developed a method to characterize the printed ink volume and the properties of the ink by means of spectral reflectance measurements. The measured data were analyzed with the help of theoretical simulations. The scheme of color composition for generating the secondary colors (from the primary inks) was obtained. Simulations for the spectral reflectance values have been carried out for both primary- and secondary colors. The simulations have been in fairly good agreement with the measurements. This may suggest that K-M theory can even be applied in ink jet printing with satisfactory precision for color reproduction.

5. Acknowledgement

Many thanks to Professor Bjorn Kruse and Reiner Lenz, Senior Researcher Nils Pauler in M-real (Sweden), and Senior Research Associate Staffan Rydefalk at Swedish Pulp and Paper Research Institute (STFI) for suggestions, comments and discussions. This work has been supported by the Swedish Foundation for Strategic Research through Surface Science and Printing Program (S2P2).

References

1. H. P. Le, Progress and trends in ink-jet printing technology. *J. Imaging. Sci. Technol.*, **42**:49–62, 1998.
2. P. Kubelka and F. Munk, Ein beitrag zur optik der farbanstriche. *Z. Tech. Phys.*, **12**:593–601, 1931.
3. L. Yang and B. Kruse, Ink penetration and its effect on printing. In *Proceedings of SPIE, Color Imaging*, volume **3963**, pages 365–375, 2000.
4. N. Pauler, A model for the interaction between ink and paper. In *Advance in Printing Science and Technology,in Proceeding of the 19th International Conference of Printing Research Institute, Edited by W. H. Banks*. Pentech. Press, 1987.
5. J. H. Nobbs, Kubelka-Munk theory and the prediction of reflectance. *Rev. Prog. Coloration*, **15**:66–75, 1985.
6. P. S. Mudgett and L. W. Richards, Multiple scattering calculations for technology ii. *J. Collid and Interface. Sci.*, **39**:551–567, 1972.

Biography

Li Yang is a Ph.D Student in Group of Media Thecnology, at Campus Norrkoping, Linkoping University, Sweden. His research is mainly about quality related issues of ink-jet printing, such as optical dot gain, ink penetration, and their chromatic effects. He has proposed an unified framework dealing with the optical dot gain and the ink penetration and compared with experiments. He has about 10 publications in jounals and on international conferences. Hopefully he will finish his dissertation in the end of this year.