Compact Description of 3D Image Gamut Surface by SVD

Hiroaki Kotera and Ryoichi Saito
Department of Information and Image Sciences, Chiba University, Japan

Abstract

This paper proposes a compact description method for 3D gamut shell. The real 3D image-to-device gamut mapping is a best way to make the appearance matching in between display and print images suppressing the source color information losses. The image gamut surface is shaped as polygon meshes by extracting the most outside points from the random color distributions of natural image. The gamut shape data should be attached to image in compact format. In our proposal, first, the image gamut shape is given by the 2D color distance array, called r-image, which is measured as radius distance $r(\theta, \phi)$ in discrete polar angles from the image color center. Next, the r-image is transformed into svd-image by SVD (Singular Value Decomposition). Finally, the compact parameters (the major singular values neglecting the small values and the orthogonal eigen vectors) are delivered to the users. The r-image can be approximately restored from the reduced SVD parameters and the image gamut surface is reconstructed and used for the users to map the image data into the inside of the device gamut referencing to the image gamut shape. The paper discusses how the r-image is compressed and the gamut shape could be well recovered with its surface colors. The SVD description method is compared with DCT based compaction in spatial frequency domain.

Introduction

A variety of works to describe the 3D device gamut[1] have been done, but most of them characterize the devices gamut not the images. Also a variety of GMAs[2] are under developments, they are mostly designed to work in 2D L-C planes, based on D-D, not I-D[6]. They don't reflect the image gamut exactly. A key factor to real 3D GMA[3] is to extract the 3D image gamut from the random color distributions quickly and to describe its boundary surfaces.[4] In the previous paper,[5] we proposed a gamut shell description method by partitioning the color space into sub-spaces to include constant number of samples. However, this method can't be applied to the ***r-image*** description because of uneven division in polar angle. Here a constant division in discrete polar angle (θ, φ) is introduced to extract the image gamut surface. The gamut shell is described as simple radial distances, ***r-image*** and is easily compressed by SVD.

Extraction of Image Gamut Shell

Letting the image center be

$$\left[L_0^*, a_0^*, b_0^*\right] = \left[\frac{1}{N}\sum_{i=1}^{N}\left(L_i^*\right), \frac{1}{N}\sum_{i=1}^{N}\left(a_i^*\right), \frac{1}{N}\sum_{i=1}^{N}\left(b_i^*\right)\right] \qquad (1)$$

A radial distance to the image center is measured by

$$r_i = \left[\left(L_i^* - L_0^*\right)^2 + \left(a_i^* - a_0^*\right)^2 + \left(b_i^* - b_0^*\right)^2\right]^{1/2} ; \; 1 \le i \le N \qquad (2)$$

The polar angle of each pixel in CIELAB color space is defined by

$$\theta_i = tan^{-1}\left[\frac{b_i^* - b_0^*}{a_i^* - a_0^*}\right]; \quad 0 \le \theta_i \le 2\pi \qquad (3)$$

$$\varphi_i = (\pi/2) + tan^{-1}\left[\frac{L_i^* - L_0^*}{\left\{\left(a_i^* - a_0^*\right)^2 + \left(b_i^* - b_0^*\right)^2\right\}^{1/2}}\right] ; \; 0 \le \varphi_i \le \pi \qquad (4)$$

Here, the gamut surface is formed by picking up the points with the maximum radial distance in every segment divided by ($\Delta\theta$,$\Delta\varphi$) as shown in Fig. 1.

$$\begin{aligned} &\boldsymbol{r} = [r_{jk}] = max\{r_i\} \\ &\quad for \; (j-1)\Delta\theta \le \theta_i \le j\Delta\theta \;\; and \;\; (k-1)\Delta\varphi \le \varphi_i \le k\Delta\varphi \\ &\quad \Delta\theta = 2\pi / M \; ; \; 1 \le j \le M \\ &\quad \Delta\varphi = \pi / N \; ; \; 1 \le k \le N \end{aligned} \qquad (5)$$

Figure 2 (a) and (b) shows a CG image and its color map in CIELAB space. The distribution of radial vectors [rjk] is illustrated in (c) and the gamut shell surface is represented by connecting these radial points as given in (d).

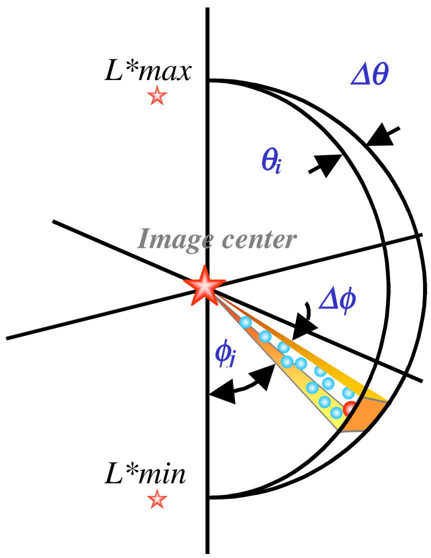
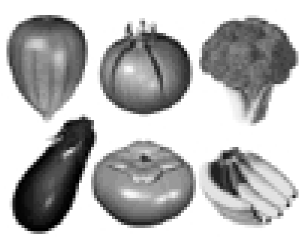
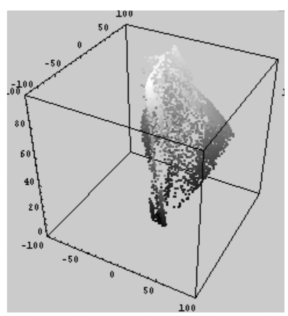


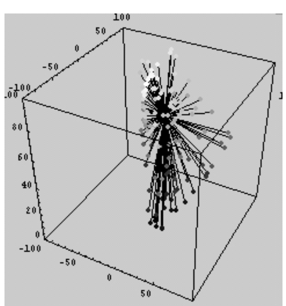
Figure 1. Segmentation of Image Gamut Space in Polar Coordinate



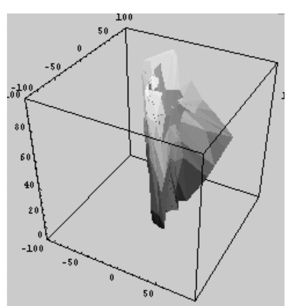
(a) Original CG image



(b) Color distribution in CIELAB



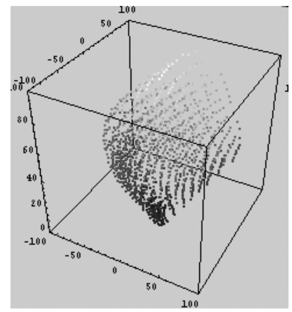
(c) Radial vectors to gamut surface



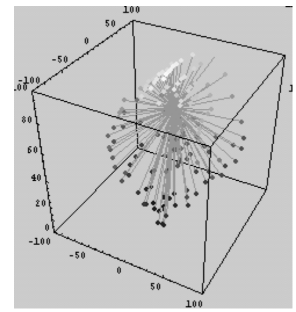
(d) Gamut shell surface

Figure 2. Formation of Image Gamut Shell from Radial Vectors

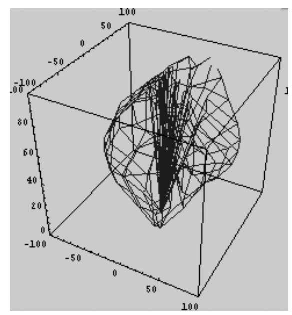
As well, the printer gamut shell is shaped from a random color distribution of measured color chips. Figure 3 shows the calculated shell shape for Epson PM800C inkjet prints on coated photo paper; (a): 11^3=1331 color chips distribution in CIELAB, (b): Radial vectors to segmented surface points in polar coordinate, (c): Gamut shell in wire frame, (d): Polygonal gamut shell surface.



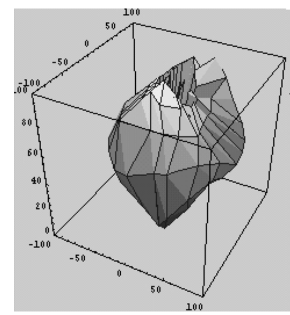
(a) Color distribution of Chips



(b) Radial vectors



(c) Gamut shell in wire frame



(d) Gamut shell surface

Figure 3. Formation of Device Gamut Shell (Inkjet printer)

Gamut Shell by r-image and Compression

The r-image is a gray scale image given by M × N matrix r=[rjk]. If the gamut shell has a smooth 3D surface, the array rjk will be highly correlated in spatial. The r-image can be compressed by applying a transform coding such as DCT.

Compression by DCT

The r-image is transformed into spatial frequency components R by forward M × M DCT.

$$\boldsymbol{R} = [R_{jk}] = \boldsymbol{A}^t \boldsymbol{r} \boldsymbol{A} \tag{6}$$

$$\boldsymbol{A} = [a_{jk}], \quad a_{jk} = \begin{cases} \dfrac{1}{\sqrt{M}}, & for\ k = 1 \\ \dfrac{2}{\sqrt{M}} cos\left(\dfrac{(2j-1)(k-1)\pi}{2M}\right), & for\ k = 2,\cdots,M, \\ & j = 1,2,\cdots,M \end{cases} \tag{7}$$

The inverse IDCT is given by the same formula

$$\boldsymbol{r} = \boldsymbol{A}\boldsymbol{R}\boldsymbol{A}^t \tag{8}$$

Since the spatial frequency energy is concentrated in low frequency components of R, the r-image is approximately reconstructed from the reduced m × m (m<M) matrix by

$$\hat{\boldsymbol{r}} \cong \boldsymbol{A}\boldsymbol{R}^m\boldsymbol{A}^t\ ,\ \ \boldsymbol{R}^m = \left[R^m_{jk}\right],\ \ R^m_{jk} = \begin{cases} R_{jk}\ , & for\ j,k \le m \\ 0 & for\ j,k > m \end{cases} \qquad (9)$$

Compression by SVD

DCT is easy to use because its basis function is prefixed independent of image. However, the image has its own shell shape then the image-dependent basis function may be better for the gamut description. Here SVD has been tested.

The ***r-****image* can be expressed by SVD as

$$\boldsymbol{r} = \left[r_{jk}\right] = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}^t \qquad (10)$$

where, the columns of U and V are the eigenvectors of $\boldsymbol{r}\boldsymbol{r}^t$ and $\boldsymbol{r}^t\boldsymbol{r}$, and $\boldsymbol{\Lambda}$ is the diagonal matrix containing the singular values of ***r*** along its diagonal.

Because ***U*** and ***V*** are orthogonal,

$$\boldsymbol{\Lambda} = \boldsymbol{U}^t\boldsymbol{r}\boldsymbol{V} = \begin{bmatrix} \lambda_1\ 0 & \cdots\cdots & 0 \\ 0\ \lambda_2\ 0 & \cdots\cdots & 0 \\ \vdots & & \vdots \\ \vdots & \ddots & \vdots \\ 0 & \cdots\cdots\ 0 & \lambda_M \end{bmatrix} \qquad (11)$$

The ***r-****image* is approximated by using the reduced numbers of singular values and eigenvectors as.

$$\hat{\boldsymbol{r}} = \left[\hat{r}_{jk}\right] \cong \boldsymbol{U}_m\boldsymbol{\Lambda}_m\boldsymbol{V}^t_m \qquad (12)$$

That is, the *M* x *M* matrix ***r*** can be restored from *m* (< *M*) singular values and the corresponding vectors of ***U*** and ***V***.

$$\boldsymbol{\Lambda}_m = \begin{bmatrix} \lambda_1\ 0 & \cdots\cdots & 0 \\ 0\ \lambda_2\ 0 & \cdots\cdots & 0 \\ \vdots\ \ \ddots & & \vdots \\ 0\ \cdots\cdots & \lambda_m & \cdots 0 \\ \vdots & & \vdots \\ 0 & \cdots\cdots\cdots & 0 \end{bmatrix} \qquad (13)$$

Reconstruction of Image Gamut Shell

The [*L*, a*, b**] value of ***r-****image* is recovered from the reconstructed $\hat{r}_{jk}$ values as follows.

$$\hat{a}^*_{jk} \cong \hat{r}_{jk}\cos(j - 0.5)\Delta\theta\sin(k - 0.5)\Delta\varphi + a^*_0$$

$$\hat{b}^*_{jk} \cong \hat{r}_{jk}\sin(j - 0.5)\Delta\theta\sin(k - 0.5)\Delta\varphi + b^*_0 \qquad (14)$$

$$\hat{L}^*_{jk} \cong L^*_0 - \hat{r}_{jk}cos(k - 0.5)\Delta\varphi$$

Experimental Results

Typical source images were converted into ***r****-image*, and the compression by DCT and SVD has been tested. Figure 4(a) is a sRGB test image "bride" in standard image database SHIPP. Figure 4(b) shows its radial vectors obtained by 32 × 32 segmentations in polar coordinate (θ, φ) angles.

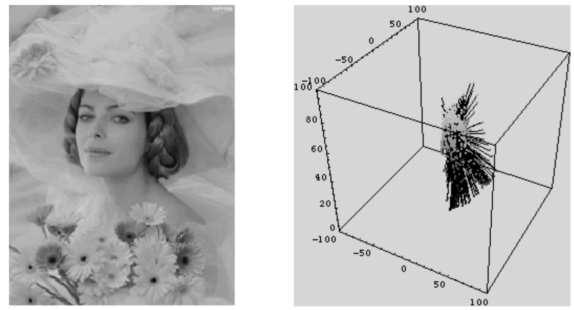


Figure 4, (a) Original image (b) Radial vectors

Figure 5 shows a comparison of reconstructed image gamut. Original r-image was created by 32x32 segmentations in (θ, φ) polar coordinate angles; (a): r-image, (b): reconstructed color map of gamut surface, (c): reconstructed gamut shell. The gamut shell shape reconstructed from reduced 4 × 4 DCT coefficients loses its details due to the lack of higher spatial frequency components and was still insufficient even by 8 × 8 DCT. On the other hand, the gamut shell shape by 4 × 4 SVD was very well recovered in details.

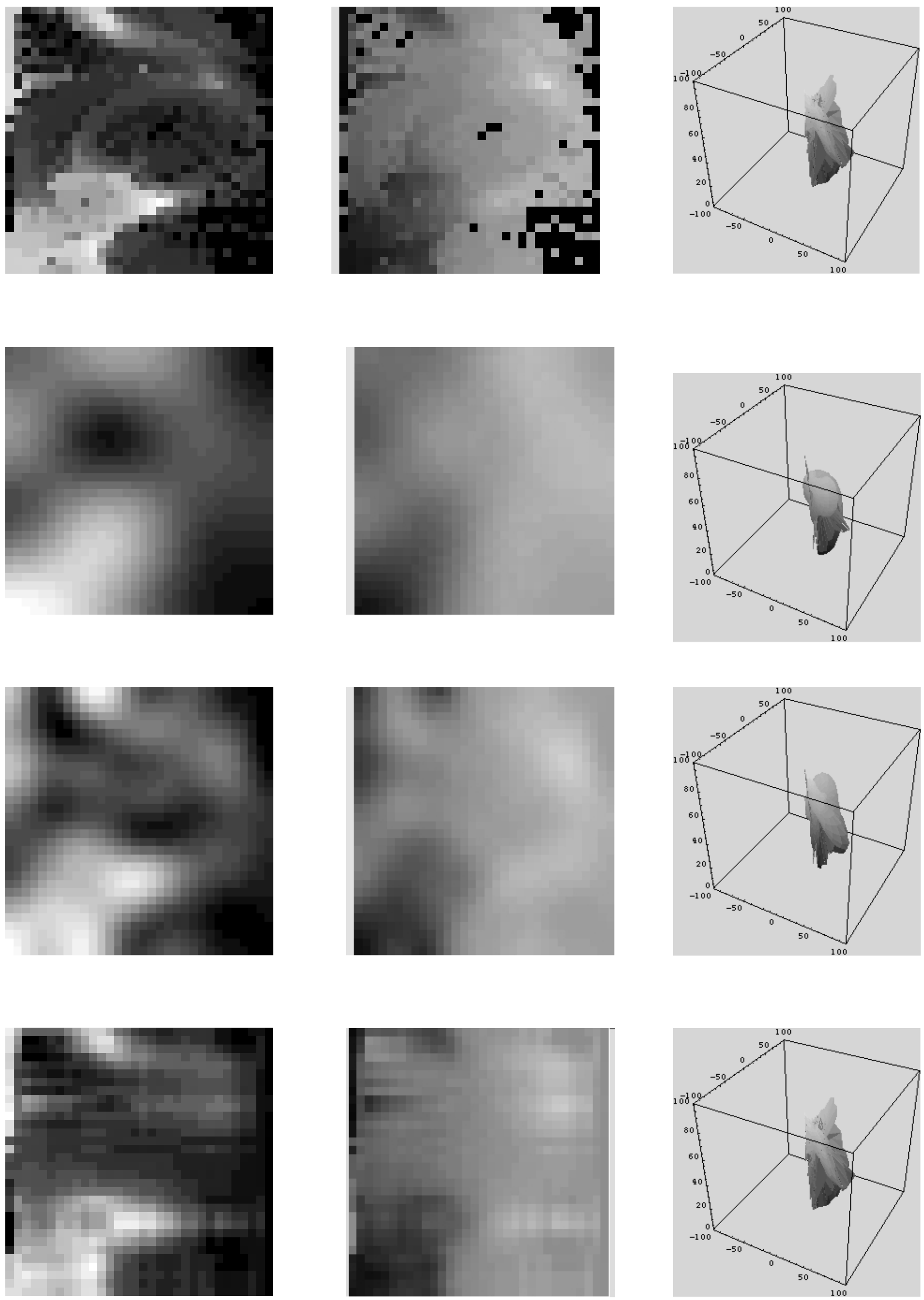
Discussion and Conclusion

The image gamut shell shape was compactly described with small number of singular values by SVD. However, the eigenvectors of matrix ***U*** and ***V*** depend on the image contents. Although the transform matrix ***A*** of DCT is fixed independent of image, the eigenvectors ***U*** and ***V*** in SVD should be computed and transmitted by every image. Because the ***r****-*mage is highly correlated in polar angle space, the basis function such as wavelet may be available to the higher rate of compression. The future works are going to be continued searching the more elegant gamut shell description independent of images

References

1. K. E. Spaulding et al, Proc. SPIE, 2414,61-68(1995)
2. L. W. MacDonald and M. R. Luo, Color Imaging, John Wiley &Sons (1999)
3. N. Katoh et al, J. E. I., 8(4),p365-379 (1999)
4. Chen Hung-Shing and H. Kotera, Proc. IS&T's NIP16, p.783-786 (2000)
5. R. Saito, and H. Kotera, Proc. 8th CIC., p.330-334 (2000)
6. Chen Hung-Shing, M. Omamiuda and H. Kotera, J. IS&T,45, 2, pp.141-151 (2001)

1.



Original 32x32 segments

DCT reduced 4 x 4 coefficients

DCT reduced 8 x 8 coefficients

SVD reduced 4 x 4 coefficients

(a) r-image *(b) Color map of gamut surface* *(c) Gamut shell*

Figure 5. Gamut shell shapes reproduced from r-image compressed by DCT and SVD