Digital Halftoning Based on a Repulsive Potential Model

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Abstract

This paper presents a new digital halftoning system based on a repulsive potential model, in which black dots correspond to charged particles. Since the charged particles have the charge of the same polarity, they repel each other. By changing the coefficient of the repulsion based on the gray level of the corresponding area of an original continuous-tone image, the distance among black dots is controlled, and halftoning is realized. Because of the repulsion among the black dots, such forces that make the distance between black dots almost uniform come about. Therefore, high quality halftone images with fine texture are produced.

Introduction

Halftoning[1,2,3,4] is a process of converting a continuous-tone image to a visually similar black and white dot pattern. Halftone images have been used in conventional printing, electronic printing, facsimile, and in other applications. Since halftoning has high practical value, many algorithms have been studied and proposed. Halftoning system can be classified into following three types.

Type 1: fixed dot position and variable dot size
Type 2: fixed dot position and fixed dot size (on or off)
Type 3: variable dot position and fixed dot size

A very classical optical screen system, which has been used in printing industry since 1890's, can be classified into Type 1. Clustered dot ordered dither can be regarded as a digitally simulated version of the classical screen system. In 1970's, digital processing technology spread fairly widely. Since then a variety of popular dithering systems, for example ordered dither, error diffusion, and various colored noise masking, have been developed and put to practical use. In these dithering algorithms, output devices such as printers and displays have of fixed-sized, fixed-positioned arranged on a square grid. Therefore, these dithering system can be classified into Type 2.

The halftoning system presented in this paper falls into Type 3. In this system, the size of each black dot is the same, but the distances between black dots are not represented by integers but by real numbers. In other words, the position of each dot is not digital, but like an analog.

Repulsive Potential

The idea of our system came from Physics. Coulomb's law[5] is a very basic law in Static Electricity. According to the law, two charges of the same polarity repel each other, and two charges of the different polarity attract each other. In case the charges are not moving, the Coulomb's force is inverse proportional to the square of the distance between the charges, and proportional to the product of charges q_1 and q_2 as follows,

$$F = k\frac{q_1 q_2}{r^2} \tag{1}$$

where k is a value related to the permittivity ε. In our system, point charges correspond to black dots of output devices. Since the size of black dots is supposed to be identical, the magnitude of each charge is identical as well. Namely, $q_1 = q_2 = q$. Therefore, Equation (1) can be rewritten to Equation (2). It is also possible to extend our algorithm so that it may permit variable-sized black dots, but it is left for further study.

$$F = k\frac{q^2}{r^2} \tag{2}$$

In this case, the force is always repulsive. Therefore, we used the word "repulsive potential model". Moreover, we used Equation (3) instead of Equation (2) so that more general n values may be allowed.

$$F = k\frac{q^2}{r^n} \tag{3}$$

In Coulomb's law, $n = 2$ as mentioned above. However, according to our experiments, the quality of halftone image is not very good in cases $n = 2$. It is probably because the force does not become weak enough even if the distance is large, which means that a dot pattern of a portion in an image is considerably affected by other distant portions in the same image. An ideal solution of this problem might be to use Yukawa's potential, shown in Equation (4), instead of Coulomb's law.

$$F = \frac{\partial}{\partial r}\left(-\frac{e^{-\frac{r}{\lambda}}}{r} \right) \tag{4}$$

But a problem is that Equation (4) is too complex because an exponential function is included. Therefore, we adopted Equation (3).

Next, relation between gray level g ($0 \le g \le 1$) and distance r ($r \le r < \infty$) must be examined, where r_0 is the minimum distance between dots, and corresponds to the darkest dot pattern.

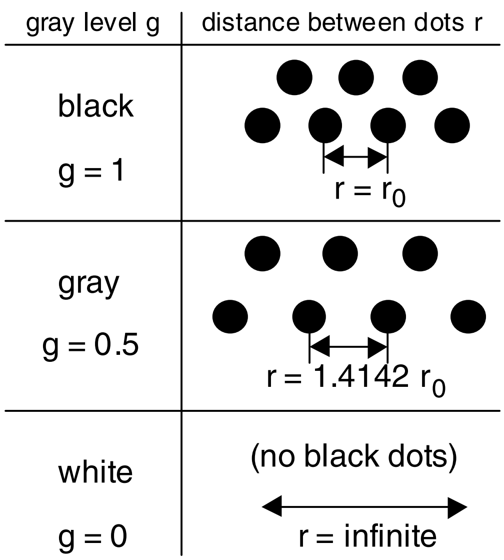

Figure 1. Relation between g and r.

From Figure 1, the relation can be formulated like this.

$$g = \left(\frac{r_0}{r} \right)^2 \tag{5}$$

From Equations (3) and (5), we can lead a relation between g and k. Suppose that a charge experiences the forces from the left and right charges as shown in Figure 2. In this Figure, the left half and the right half have different k values, k_1 and k_2 respectively. Therefore, the density of the point charges, in other word the distances between point charges r_1 and r_2 are different each other. Since point charges are placed very regularly, the composition of forces from all the left point charges is proportional to a force from a nearest single point charge on the left. In a similar way, composed forces from all the right point charges are proportional to a force from a nearest single point charge in the right. It is obvious that the left half and the right half are geometrically similar each other. Therefore, as for the balance of forces, Figure 2 can be simplified into Figure 3.

In this case the two forces F1 and F2, must be balanced in order that the central charge may stand still.

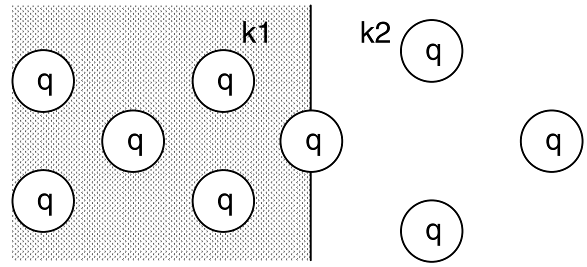

Figure 2. Balance of Coulomb's forces

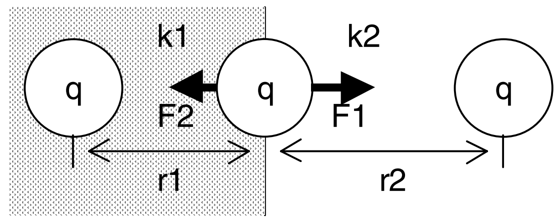

Figure 3. Balance of Coulomb's forces

$$\frac{k_1 q^2}{r_1^{\,n}} = \frac{k_2 q^2}{r_2^{\,n}} \tag{6}$$

Therefore, by dividing both sides by q^2,

$$\frac{k_1}{r_1^{\,n}} = \frac{k_2}{r_2^{\,n}} \tag{7}$$

On the other hand, from Equation (5),

$$\frac{1}{r_1^{\,2}} = \frac{g_1}{r_0^{\,2}} \qquad \frac{1}{r_2^{\,2}} = \frac{g_2}{r_0^{\,2}} \tag{8}$$

From Equations (7) and (8), variables r_1 and r_2 are erased.

$$k_1 g_1^{\frac{n}{2}} = k_2 g_2^{\frac{n}{2}} \tag{9}$$

A possible solution of Equation (9) is to determine k_1 and k_2 as follows.

$$k_1 = \frac{1}{g_1^{\frac{n}{2}}} \qquad k_2 = \frac{1}{g_2^{\frac{n}{2}}} \tag{10}$$

Because g is the gray level of each pixel, k is decided based on the equation. In this case, the denominator turns to be zero if g is zero. It seems to be a practical solution to add a very small number to the denominator to avoid the problem.

The example above is the simplest case, and only two gray levels are included in an image. However, the concluded Equation (10) can be applied to more realistic images which have several gray levels.

Algorithm

The proposed algorithm is shown in Figure 4.

(1)Initial Image

The total number of black dots should be calculated first, based on the average gray level of a given image. For example, suppose that the gray level is 0.5, and the number of pixels of a side is 128, then the total number of the black dot is $128 \times 128 \times 0.5 = 8192$.

These dots are scattered at random over the image, and used as the initial dot pattern Please be careful that x and y coordinates are expressed by real numbers, not integers.

(2) Calculation of *k* Values

For each pixel, *k* is calculated based on the original image by Equation (10). Hereafter, *k* values are not changed.

(3) Calculation of Forces and Movement of Black Dots

For each dot in the image, the force caused by the neighboring dots is calculated by Equation (3), and the position of the dot is updated by Equation (11),

$$\begin{aligned} x' &= x + \alpha \times f_x \\ y' &= y + \alpha \times f_y \end{aligned} \qquad (11)$$

where (x,y) is the old position, and (x',y') is the new position of a specific dot. fx and fy are the x and y component of the force. α is a coefficient. The simplest way is that α is a constant coefficient. However, by decreasing the value of α as the number of iteration increases, better convergence can be achieved. This process is iterated several times for all the dots. After 50-100 times iteration, the arrangement of the dots usually converges.

Experiments

First we chose the value of *n* as two, but the quality of halftone images is not very good. Probably this is because the force travels to too distant places. We changed the value of n to eight, and the picture quality was improved.

Figures 5, 6 and 7 shows gray level representation characteristics. The original gray level image is a test image produced artificially. In Figure 6, upper half of the original image has the gray level 0.5, and the lower half has the gray level 0.25, where 1.0 and 0.0 means perfect black and perfect white respectively.

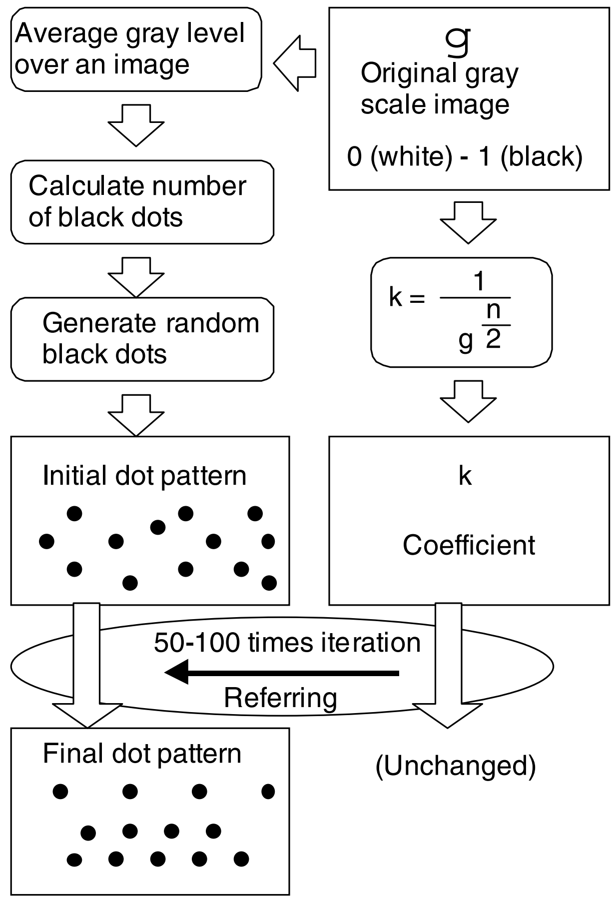

Figure 4. Flow diagram of the proposed system

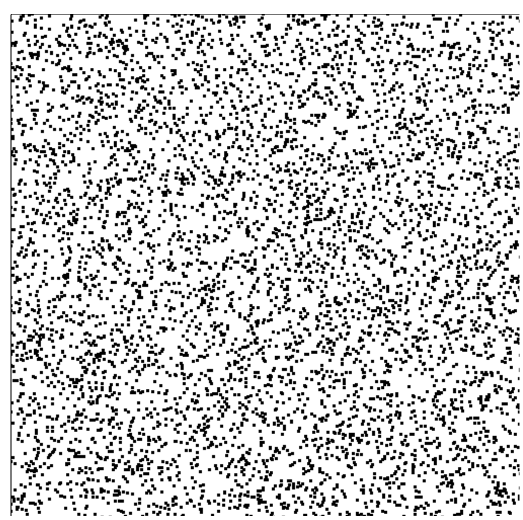

Figure 5. Initial dot pattern

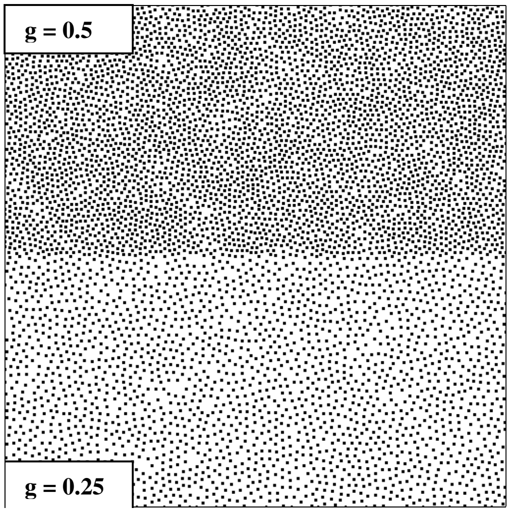

Figure 6. Final dot pattern (n = 8, 50 times iteration)

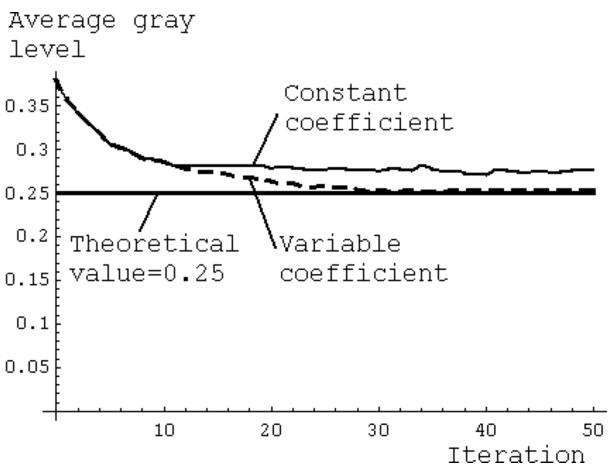

Figure 7. Iteration and convergence

Figure 8. Original image (64 x 64 pixels)

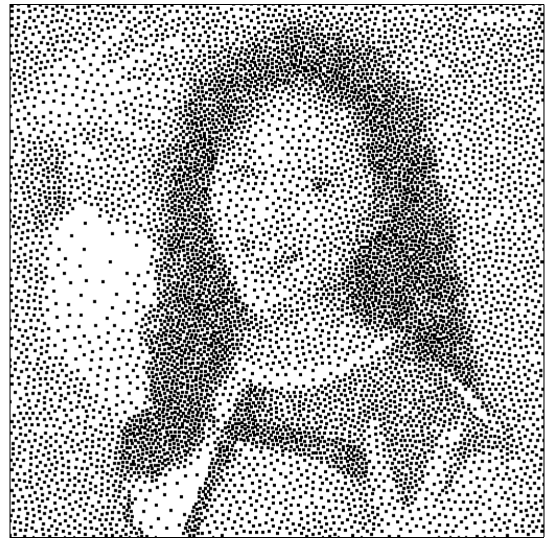

Figure 9. Final dot pattern (n=8, 50 times iteration)

Figure 7 shows a manner of convergence for the test image. The horizontal axis is the iteration times and the vertical axis is the average gray level of the lower half of the generated halftone image. As the number of iteration increases, the average gray level converges to a theoretical value. Therefore, the precision of gray level representation is pretty high.

Figure 8 is a gray scale image. It originated from ISO/JIS SCID (Standard color image data) "portrait", but it has been converted to a low-resolution monochrome image. Figure 9 is the final dot pattern of the image.

Conclusion

A new type of digital halftoning technique has been described that are based on repulsive potential model, the origin of which is Coulomb's law in Static Electricity. In the proposed system, each dot is regarded as a point charge. All the black dots are the same size, and the position of each dot varies like an analog. By experiments, it has been shown that high quality halftone images with fine texture are produced.

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Biography

Kazuhisa Yanaka is a professor of Kanagawa Institute of Technology, Japan. He gained BE, ME and Dr.Eng. degrees from the University of Tokyo, in 1977, 1979, and 1982 respectively. He joined Electrical Communication Laboratories of NTT in 1982 and developed videotex terminals, teleconferencing systems, and image coding algorithms. He moved to Kanagawa Institute of Technology in 1997.

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