

# Modeling Ink-penetration for Ink-jet Printing

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## Abstract

A model describing the effects of ink penetration for water based ink jet printing is proposed. Dependence of the ink penetration on ink percentage and ink spreading have been taken into account in this model. Halftone tints printed with the primary inks have been measured spectrally to verify the model. The agreement between measurements and the model is accurate enough to show important characteristics of ink penetration such as color shifts and color gamut reduction.

## 1. Introduction

Ink penetration has long been known as an important factor that influences the quality of tone reproduction. The effects of the penetration are two-fold: optical and chromatic [1]. First the reflectivity of substrate paper under the ink dots is changed due to the presence of the ink in the substrate. Second the pure (i.e. remaining) ink layer on the ink-paper interface becomes thinner (or even completely vanishes) due to the ink penetration and therefore it is more transparent to the incident light. Because transmittance (or correspondingly the absorption) of the ink is strongly wavelength dependent, such ink redistribution as ink penetration will certainly influence the spectral transmittance of the printed ink and therefore leads to color shift for the printed image comparing to the case of no ink penetration. However the description and understanding as stated above were only available qualitatively. Quantitative description and prediction for the ink penetration has been a challenge for modeling and simulation of the tone reproduction. The difficulties lie, above all, in dealing with the ink penetrated paper which show strong scattering and absorption characteristics. Moreover the optical properties of the ink penetrated paper is non-isotropic (especially for the half tone cases) and depends strongly on the ink distribution which is very difficult to obtain if not impossible and even more difficult to model and to simulate.

Only few studies concerning simulations for the ink penetration have been reported [1, 2, 3, 4, 5, 6, 7]. These

studies have mainly been made for images with full ink coverage. Arney and Alber [7] has made an attempt to modeling halftone images by applying a tonal value independent model. In their model it is assumed that the depth of the ink penetration is independent of the ink percentage and equal to that of full ink coverage. This assumption is questionable. Intuitively, the depth of the ink penetration under a well separated ink dot (small ink coverage) should be smaller than that under overlapping ink dots (large ink percentage) where the ink spreading on the surface is restricted due to the overlap. The larger the ink percentage the more restricted the ink spreading and therefore the more the ink penetration down into the substrate paper. Therefore a model in which the depth of the ink penetration correlates with the ink coverage is needed for a successful simulation.

Very recently we presented a proposal in representing the effect of the ink penetration. It was shown, for a full tone plate, that the effect of the ink penetration is optically equal to an induction to an extra ink layer lying on the paper's surface while the substrate remains clean [8, 9]. In this report we have further established correlations between the spectral transmittance of the induced ink layer and the depth of the ink penetration. In addition a framework describing both ink penetration and optical gain, for any halftone images, is presented. This approach has been applied to simulate images printed by a inkjet and comparison with measurements have been made. Chromatic variations due to the ink penetration have also been discussed.

## 2. Method and Model for the Ink Penetration

As shown in our previous work [10], the effect of the ink penetration (Fig. 1a) can optically be described as introducing an extra ink layer with transmittance,  $\gamma$ , while the whole paper remains pure without penetrating ink (Fig. 1b). Consequently the reflectance of a halftone image

reads [8, 9],

$$R = R_{MD} - \Delta R \quad (1)$$

where

$$R_{MD} = \sum_{j=0}^{N-1} (T_j \gamma_j)^2 R_0 \sigma_j \quad (2)$$

and

$$\Delta R = \sum_{j=0}^{N-1} \sum_{i < j} (T_i \gamma_i - T_j \gamma_j)^2 P_{ji} \sigma_j \quad (3)$$

are terms corresponding to Murray-Davis assumption and Yule-Nielsen effect, respectively.  $T_j$  and  $\gamma_j$  are the spectral transmittance values for the remaining- and induced ink layers of region  $\Sigma_j$ . In Eqs. 2 and 3,  $P_{ji}$  describes the conditional probability for a photon exiting the substrate from the region  $\Sigma_j$  if it enters the substrate at the region  $\Sigma_i$ , and  $\sigma_i$  and  $\sigma_j$  are the areas of the corresponding regions. In addition,  $R_0$  is the reflectance of a bunch of clean paper.

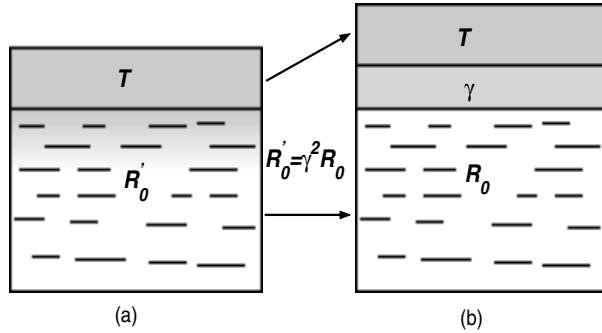


Figure 1: Equivalent descriptions of ink penetration (for full tone images). The ink penetrated substrate (reflectance  $R'_0$ ) is equivalent to an induced ink layer (transmittance  $\gamma$ ) and the pure substrate (reflectance  $R_0$ ), where  $R'_0 = \gamma^2 R_0$ .

It is worth noticing that the spectral reflectance values,  $T$  and  $\gamma$ , also depend on the ink percentage because of their dependence of the thickness of the remaining ink layer (for  $T$ ) and of the depth of the ink penetration ( $\gamma$ ). Therefore the key issue in applying the model to a half tone image is to obtain  $\gamma$  for an ink dot (or a cluster of micro dots) of any ink percentage. If the ink percentage commanded by an inkjet is  $a$ , it becomes eventually  $a'$  due to ink spreading after printing. For an ink droplet with a certain volume, a reasonable assumption is that the thickness of the ink layer is inversely proportional to the ink spreading on the surface. As the depth of the ink penetration for a full tone plate,  $Z_1$ , is experimentally obtainable [5, 6], one may write the depth of the ink penetration,  $Z_a$ , for the halftone

plate, in terms of  $Z_1$  and as a function of the ink percentage, i.e.

$$Z_a = f\left(\frac{a}{a'}\right) Z_1 \quad (4)$$

Evidently,  $f(a/a') = 1$  when  $a = 1$ .

The general expression of  $\gamma_a$  (for any ink percentage,  $a$ ) has been derived [11] and can be computed as the following. For clarity of the discussions, the ink percentage is denoted as subscript hereafter. A superscript prime is added to quantities representing the ink penetrated paper only. Therefore  $R'_a$  and  $R'_1$  represent reflectance values for the ink penetrated paper of ink coverage  $a$  and 100% (with a bunch of pure paper as background,  $R_0$ ). The only exception of the notation is for  $R'_\infty$  which is the reflectance of the ink penetrated paper in infinitive thickness. Applying Kubelka-Munk theory we have derived expression for computing  $R'_a$  [11],

$$R'_a = \frac{1 - \mathcal{X} R'_\infty}{R'_\infty - \mathcal{X}} \quad (5)$$

where

$$\mathcal{X} = \frac{1 - R_0 R'_\infty}{R'_\infty - R_0} \left[ \frac{(R'_\infty - R_0)(1 - R'_1 R'_\infty)}{(1 - R_0 R'_\infty)(R'_\infty - R'_1)} \right]^{f\left(\frac{a}{a'}\right) - 1} \quad (6)$$

The transmittance corresponding to the induced ink layer,  $\gamma_a$ , can then be computed by

$$\gamma_a = \sqrt{\frac{R'_a}{R_0}} \quad (7)$$

Evidently the  $\gamma_a$  depends strongly on the function of the ink penetration,  $f(a/a')$ , which in turn depends on the paper-ink interaction.

Simulation of spectral reflectance of halftone images printed with primary inks and on uncoated copy paper have been carried out. The macro-dot is assumed to be round in shape. The ink spreading is simulated by replacing the radius of the commanded dot  $r$  with  $r(1 + \Delta r)$  ( $\Delta r$  is a constant for each primary color and depends on the properties of the ink-paper interaction). Another key issue for the simulation is the form of the function describing the ink penetration,  $f(a/a')$ . To get the exact form of this function is a difficult task. It depends on at least two sorts of factors. Evidently the function depends on the ink-paper interaction. On the other hand it depends also on the construction of the printer, for example, how the macro-dot is built and how the ink droplet is controlled.

In order to get information about the inkjet, halftone patches ( $a = 10, 30, 50, 70, 100\%$ ) were printed on inkjet transparencies. Contrary to building a layer of ink penetration, an ink layer on the surface of the transparency is created. We assume that  $f(a/a')$  has a similar form for both

cases, even though parameters in the function differs from one case to another. The simulations for the spectral reflectance values (of print on the transparency) have shown excellent agreement with the experimental data with a simple exponential form [11]

$$f\left(\frac{a}{a'}\right) = \left(\frac{a}{a'}\right)^n \quad (8)$$

In the expression  $n$  is a constant for each primary color. The  $n$  ( $\Delta r$ ) parameters used for transparencies are, 2(0.2), 2.5(0.25) and 3(0.3), for cyan, magenta and yellow, respectively. We therefore apply the same form of function but different  $n$  and  $\Delta r$  parameters in the simulations for the ink penetration.

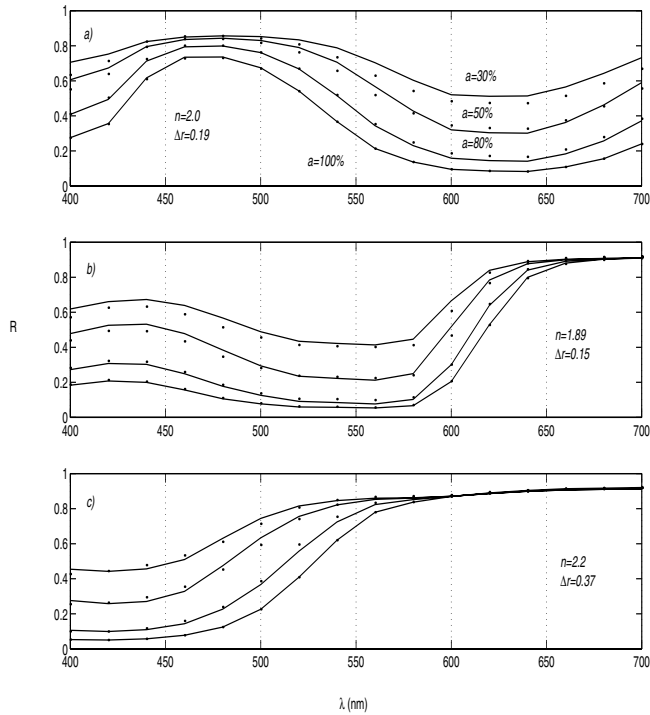


Figure 2: Simulated (solid lines) and measured (dot lines) reflectance values for half-tone plates, a) cyan, b) magenta and c) yellow. Ink percentage,  $a = 30, 50, 80, 100\%$ , downwards.

### 3. Results and Discussions

Figure 2 shows the simulated spectral reflectance values (solid lines) for the primary colors, cyan, magenta and yellow. The parameters,  $n$  and  $\Delta r$ , describing the ink penetration and the ink spreading, have been denoted in the figure. The spectral reflectance of the uncoated copy paper ( $R_0$ ) and that of the full tone patches ( $R_1$ ) were used as input data for the simulations of the half-tone images. Experimentally half-tone patches were prepared by using inkjet and printed on ordinary (uncoated) copy papers ( $80g/m^2$ ). The commanded ink percentages for the

patches were,  $a = 30, 40, 50, 80, 90, 95\%$  and  $100\%$ , respectively. The spectral reflectance of the half-tone patches were dried and then measured by using Elrepho 2000.

Figure 2 shows also the experimental data. The agreement between the simulated curves (solid lines) and the experimental ones (dot lines) are reasonably good for all of the primary colors. The agreement is especially good for the patches with big ink percentage (say  $a \geq 50\%$ ). The reason behind this fact may have to do with the so called edge effect. In the other words, the depth of the ink penetration under the edge of the printed ink dot is generally smaller than that in the center. The edge effect is particularly serious when the ink dot is small. However when the ink dot becomes bigger, this effect becomes less important and more uniform in depth. Then the agreement between the simulations and experimental measurements becomes better.

In order to study chromatic effects of the ink penetration. The simulated- and measured spectral reflectance values were used to compute the color coordinates ( $L^*a^*b^*$  color system) of the patches. The coordinates were then projected onto a 2D plane ( $a^*$  vs.  $b^*$ ) for the convenience of the discussion. In calculating the color coordinates  $10^\circ$  observer under CIE illuminant  $D_{65}$  was assumed. To demonstrate the effect of the ink penetration on color rendering,  $a^*$  vs  $b^*$  curves with- and without ink penetration have been shown in the figure. Evidently the ink penetration results in a remarkable color shift. For example, cyan patches appear less greenish when there exists ink penetration, and the magenta patches appear less reddish. Similarly, the yellow patches become less yellowish on one hand due to the ink penetration. On the other hand, it tends to be a bit greenish for the light- and medium tone patches, and reddish when tonal values are close to 1. In addition quantity  $\sqrt{a^{*2} + b^{*2}}$  in the case of having ink penetration is much smaller than that having not which means that saturation of the color for the printed images is dramatic reduced due to the ink penetration. In a 3D point of view (say  $L^*a^*b^*$  color system), this effect results in the so called color gamut reduction [12].

As one can expect from the good agreement between the simulated- and measured spectral reflectance values, the simulated  $a^*$  vs.  $b^*$  curves based the tonal value dependent model (dependent model), match very well those of the experiments (see Fig. 3). It is very interesting to notice that the experimental curves with- and without the ink penetration, in the light tone region (around  $a^* = 0$ ,  $b^* = 0$ ), lie very close to each other. This phenomenon implies that there is little ink penetration when the ink coverage is small and it increases when ink coverage increases.

As comparison to the previous method, simulations based on tonal value independent model (independent model) for the ink penetration have also been carried out

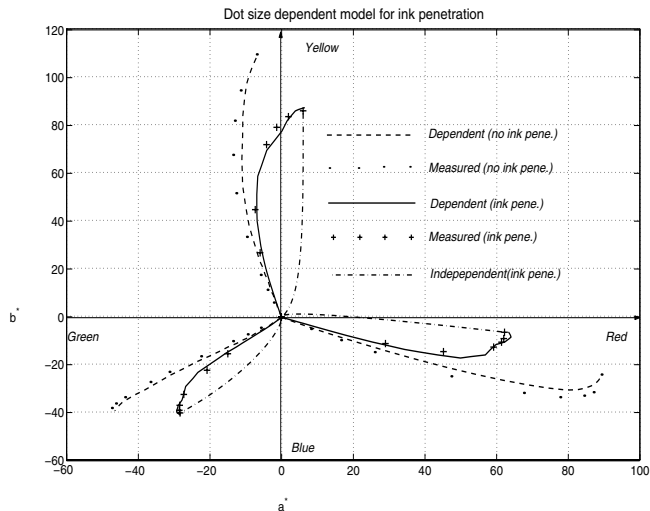


Figure 3: Color variation upon the ink penetration, a comparison between simulations and experiments. Ink penetration: ink percentage dependent model (solid line); ink percentage independent model (dot dash line); experiment (+). No ink penetration: simulation (dash line); experiment (dot line)

and presented in Fig. 3 (dot and dash lines). Clearly the independent model for the ink penetration failed to reproduce the main feature of the experimental curves.

#### 4. Conclusion and Summary

We have presented a model to describe the effects of the ink penetration. It is shown that optically the ink penetration can be described by introducing an induced ink layer which lies on the substrate surface while the whole substrate remains pure. The induced ink layer plays exactly the same role as the remaining layer (or an ordinary ink layer). The induced ink layer together with the remaining ink layer determines the optical and chromatic characteristics of the printed images. The advantages of the model are: 1) the penetrating ink layer which is strong both in absorption and scattering is simplified as a layer with strong absorption (the induced ink layer) and a layer with strong scattering (the clean substrate). 2) Difficulties in dealing with a non-uniform substrate (inked substrate differs from non-inked one) are avoided. 3) Algorithms [13, 14] that were developed for non-ink penetration systems become directly applicable to the case of having the ink penetration.

Comparing to the previous work, dependence of the ink penetration on the ink percentage has been considered. Simulations showed that the tonal value dependence of the ink penetration is of critical importance for reproducing the experimental features. The simulations agree fairly well with the experiments in terms of the spectral reflectance and chromatic appearance. The simulations for

halftone images with- and without the ink penetration have further shown the significant impacts from the ink penetration to the color rendering of halftone images, i.e. color shift and color gamut reduction.

#### 5. Acknowledgement

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