Modeling Electrophotographic Developer Flow with a Viscous Fluid Flow Model

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Abstract

In laser printers, the developing station forms toner images on the photoconductor drum surface. The primary concern for formation of stable toner images is obtaining smooth developer flow. Then, observation of the developer flow at a free surface in an actual developing station has been done. For better flow optimization, developer flow details such as local flow velocity vectors and streamlines, not only at the free surface but also in the developer flow, should be clarified. A simulation method for optimizing the flow was examined.

In the simulation method, a large calculation domain in the developing station is required. Therefore, viscous fluid analysis to minimize the calculation load is employed. Viscosity should be defined as fitting the developer particle flow. To do this, viscosity measurements were carried out. It was found that the developer flow should be non-Newtonian fluid, in which viscosity is proportional to the reciprocal of its shear rate. This means that shear stress in the developer flow should be constant.

This property was used to simulate the developer particle flow behavior successfully, even in three-dimensional models. Flow details, on both large and small scales in the developing station, could be observed.

Introduction

In laser printers using an electrophotographic process, the developing station forms toner images on the photoconductor drum surface. Toner needs to be stably supplied to their developing zone. In a two component developing system, the powder known as developer is used for toner supply. New toner is mixed with developer in the mixing zone of the developing station. Then, it is carried through the conveyance zone to the developing zone where it is supplied to the photoconductor drum.

Conventional observations of the developer flow on the free surface were made during development of actual developing stations. However, better flow optimization, requires details of developer flow such as local flow velocity vectors and streamlines, not only on the free surface but also in the developer flow. Therefore, a simulation method to optimize the flow should be examined.

A direct calculation method of the movement of powder particles using DEM analysis is widely need to simulate powder flow.[1,2] However, since DEM analysis needs a huge calculation time, it is unsuitable for calculations, which need a large-scale domain such as to optimize the developer flow in the developing station. Viscous fluid analysis, which has a comparatively small calculation load, is a suitable alternative to calculate the large-scale domain.

An approach applicable to simple flow like the flow on a slope is desirable for the fluid equation;[3,4] however, no fluid equation has been reported which includes all flow phenomena.[5] Modeling for only a target flow is possible from an industrial standpoint.

We examine modeling using the fluid equation for developer flow in the developing station. In order to make the best use of fluid analysis, viscosity should be defined as fitting the developer particle flow. We examine how to define the effective viscosity of the developer particle flow, and we build the fluid analysis model for the developer flow. Then, we apply this model to the mixing zone in an actual developing station, and compare with measured and model simulation results.

Analysis Model

Developer flow in the model is calculated by the fluid analysis program, which uses the Navier-Stokes equation for incompressible fluid, equation (1). V is fluid velocity for the normal direction; F and p are the external force and pressure on the fluid; ρ and μ are the fluid density and fluid viscosity, which are assumed as constant.

$$\frac{\partial \vec{V}}{\partial t} + \left(\vec{V} \bullet \vec{\nabla}\right)\vec{V} = \vec{F} - \frac{1}{\rho}\vec{\nabla}p + \frac{\mu}{\rho}\vec{\nabla}^2\vec{V} \qquad (1)$$

The developer is made up of powder particles; when dealing with their flow as a viscous fluid, effective viscosity, μ, and density, ρ, should be used for the developer flow. Density is defined as apparent density in general.[6] The effective viscosity needs to be defined for the powder flow.

In addition, in the fluid analysis, the behavior of fluid on the wall surface needs to be investigated, and the boundary conditions on the wall surface need to be defined. The effective viscosity and the boundary conditions on wall surface are determined by measuring spindle torque while agitating the developer using a predetermined spindle with a viscometer (model RVDV III, Brookfield Co.) for general viscous fluid.

Figure 1 shows the three types of spindle forms used for measurements. They are the cylinder spindle, type #4LV, the disk spindle, type #6RV/H, and the T bar spindle, type E. In the cylinder spindle and the disk spindle, friction with the wall surface of the spindle moves the developer. These spindles can be used to investigate the developer behavior on the wall surface, and to define the boundary conditions on the wall surface. Developer is moved by pushing the bar for the T bar spindle so it can be used to define the effective viscosity.

Slip Boundary

The flow behavior on the wall surface is seen by using cylinder and disk spindles.

Equations (2) and (3) relate the measured torque, T_M (dyne-cm), and spindle rotation speed, N (rpm), for the cylinder spindle, type #4LV. R_c (cm), R_b (cm), L (cm) are the diameter of the cylinder, the diameter of the spindle, and the effective length of spindle as shown in Figure 1. μ (poise), and $\dot{\gamma}(s^{-1})$ are the viscosity and the shear rate on the spindle cylinder side, respectively.

$$T_M = 2\pi\mu R_b^2 L\dot{\gamma} \tag{2}$$

$$\dot{\gamma} = \frac{\pi R_c^2}{15\left(R_c^2 - R_b^2\right)} N \tag{3}$$

As shown in equations (2) and (3), in Newtonian fluid, the torque measured by the viscometer is proportional to the spindle rotation. Therefore, the relationship of the torque and spindle rotation speed is investigated. In addition, as shown in equation (1), pressure affects flow. So, the relationship between pressure and torque is investigated by changing the depth to which the spindle is inserted into the developer.

Figure 2 plots torque vs. spindle rotation for changing spindle insertion depth into the developer. Figures 2 (a) and (b) are the cylinder spindle, type #4LV, and the disk spindle, type #6RV/H. Torque becomes roughly constant with the spindle rotation for both spindles and all spindle insertion depths.

Shear rate, $\dot{\gamma}$, is constant for the spindle rotation speed in Figure 2 (a). This means that the developer has slipped on the spindle surface. In the disk spindle, type #6RV/H, the surface geometry making contact with the developer is so complicated that shear rate, $\dot{\gamma}$, cannot be calculated geometrically. However, in the disk spindle with Newtonian fluid, the shear rate, $\dot{\gamma}$, should be proportional to the spindle rotation speed, and the torque should be proportional to shear rate, $\dot{\gamma}$. The developer slip boundary on the spindle surface can be confirmed from the results in Figure 2 (b). As a result, the slip boundary to wall surface making contact with the developer can be defined.

In addition, in Figure 2, torque changes with the spindle insertion depth. The following can be considered as causes.

1. As pressure increases, torque increases.
2. The increase of pressure changes the developer density, and the flow behavior changes.

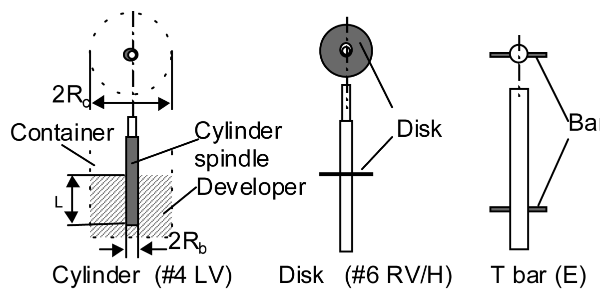

Figure 1. Spindle forms

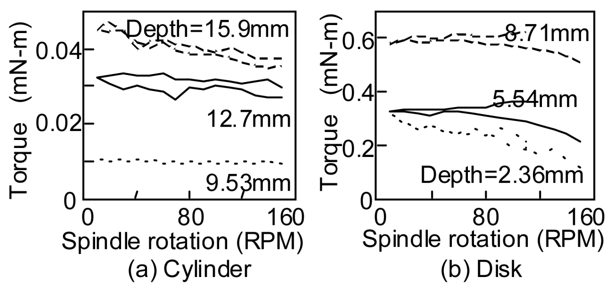

Figure 2. Torque vs. spindle rotation for insertion depth

Table 1. Influence of pressure on density

Mass (kg)	Height (m)	Mean density (kg/m^3)
0.22730	0.0690	2621.4
0.45526	0.1425	2542.3
0.25359	0.0810	2491.4
0.50688	0.1640	2459.5

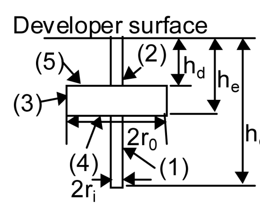

Figure 3. Contact surface of disk spindle

Table 2. Friction factor of the cylinder spindle

Depth (mm)	Torque (mN-m)	Friction Factor
9.53	0.010	0.617
12.7	0.030	1.041
15.9	0.038	0.844

Table 3. Friction factor of the disk spindle

Depth (mm)	Torque (mN-m)	Friction Factor
2.36	0.230	0.676
5.54	0.350	0.659
8.71	0.580	0.799

In order to investigate the influence of pressure on density, the amount of developer put into the cylinder container is changed, and density is measured with the developer heights. The pressure on the developer increases with the developer height. However, as shown in Table 1, the developer densities are similar to 2500 kg/m^3 even with different heights. Therefore, the developer density is regarded as constant for pressure. And the change of the torque with spindle insertion depth is attributed to the pressure influence.

Next, the friction factor required for the slip boundary is defined. Torque estimates for the disk spindle in each domain are shown in Figure 3. Domain (1) is the lower axial cylinder surface, domain (2) is the upper axial cylinder surface, domain (3) is the disk side wall, domain (4) is the disk bottom wall, and domain (5) is the disk top wall. Equations (4) - (8) show the torque for the disk spindle. T_n is the torque for domain n (=1-5). ρ, g and f are the developer density, gravity constant, and friction factor. The sum of equations (4) –(8) is the torque estimated by measurement. In addition, the torque to the cylinder spindle can be expressed with equation (4) only.

$$T_1 = \left(\pi r_i^2 \rho g\right)\left(h_c^2 - h_e^2\right) f \tag{4}$$

$$T_2 = \left(\pi r_i^2 \rho g\right)\left(h_d^2\right) f \tag{5}$$

$$T_3 = \left(\pi r_d^2 \rho g\right)\left(h_e^2 - h_d^2\right) f \tag{6}$$

$$T_4 = \left(\frac{2}{3}\pi \rho g h_e\right)\left(r_d^3 - r_i^3\right) f \tag{7}$$

$$T_5 = \left(\frac{2}{3}\pi \rho g h_d\right)\left(r_d^3 - r_i^3\right) f \tag{8}$$

The friction factor is calculated for different depths used in Figure 2.

The factors are shown in Tables 2 and 3 for the cylinder and disk spindles. In this calculation, even when the spindle depth changes, the obtained friction factor must be constant. However, in Table 2, the change in friction factor is large, while in Table 3, it is roughly constant. For the cylinder spindle, the measured torque is so low that measurement error is included in the result.

It is concluded that disk spindle measurements give the friction factor; the averaged friction factor, f= 0.7, is used for the analysis.

Developer Particle Flow

In order to determine the effective viscosity of the developer, the flow behavior in T bar spindle is examined next. Figure 4 plots torque vs. spindle rotation for varying the spindle insertion depth into the developer using the T bar, type E. The torque becomes roughly constant with the spindle rotation for all spindle insertion depths. For Newtonian fluids, as shown in equations (2) and (3), torque should be proportional to spindle rotation. Therefore, this result suggests that the developer flow has to be modeled as a non-Newtonian fluid. Figure 5 plots torque vs. the spindle insertion depth for the T bar. When the spindle insertion depth is under 12mm, the torque increases with the spindle depth. And the developer above the T bar is moved by spindle rotation. At spindle insertion depths over 12mm, the torque becomes roughly constant to the spindle depth. And no developer movement by spindle rotation can be observed from its free surface.

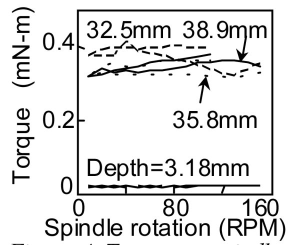

Figure 4. Torque vs. spindle rotation in T bar

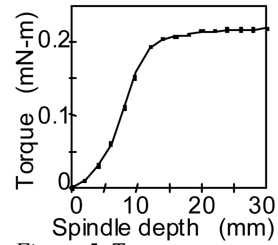

Figure 5. Torque vs. spindle depth in T bar

Table 4. Results of Newtonian model

Rotation (RPM)	Viscosity (Pa-s)	Torque (mN-m)
100	0.22	0.0268
20	0.22	0.0046
20	1.10	0.0208

Table 5. Results of non-Newtonian viscosity model

Rotation (RPM)	n	K	Torque (mN-m)
100	0	46	0.0290
20	0	46	0.0210

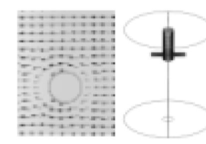

Figure 6. Analysis model

As shown in equation (2), torque can be expressed as proportional to either the viscosity or the shear rate. For the case of the spindle insertion depth under 12mm (Figure 5), the torque is roughly proportional to the spindle insertion depth. The increasing of spindle insertion depth affects the developer drag, i.e., drag by effective viscosity. Therefore, the measured torque is roughly proportional to the drag by effective viscosity, and the effective viscosity is estimated as constant using the measured torque. However, when the spindle insertion depth exceeds 12mm, since the measured torque becomes constant with the spindle insertion depth, the effective viscosity is not proportional to the torque. Therefore, the effective viscosity is estimated using the result of the spindle insertion depth under 12 mm. In addition, the friction between developer surface and T bar surface creates another shear stress. Therefore, it is concluded that torque can be expressed as a function of the viscosity and the friction factor.

As mentioned above, the developer flow is modeled as a non-Newtonian fluid, and the effective viscosity is estimated by using the measured torque. We use the power law viscosity model shown in the equation (9). Here, μ, $\dot{\gamma}$, K and n are the viscosity, shear stress, constant and the power number.

$$\mu = K\dot{\gamma}^{(n-1)} \tag{9}$$

In order to estimate the effective viscosity, the value of K and n in equation (9) is determined. In this case, K and n are determined by matching the calculated torque from the non-Newtonian viscosity model, to the measured torque. In addition, in order to verify that developer flow can be modeled by using the non-Newtonian Power Law fluid model, the calculation using this model is also performed simultaneously. A finite element model (Figure 6) is created for analysis of T bar measurement set up with spindle insertion depth of 3.18mm, where the developer movement can be observed.

The analysis is performed for T-bar rotations of 20 RPM and 100 RPM. Tables 4 and 5 show the results of the Newtonian and the non-Newtonian viscosity models. When the effective viscosity is set constant in the Newtonian model, the torque increases almost linearly to rotational speed, and it differs from the measured results shown in Figure 4. On the other hand, the result of roughly constant torque is obtained in the non-Newtonian viscosity model using K=46 and n=0.

If the result (n= 0, K= 46) obtained in Table 5 is substituted in equation (9), equation (10) is obtained in which the viscosity is proportional to the reciprocal of the shear rate.

$$\mu = \frac{46}{\dot{\gamma}} \tag{10}$$

Shear stress, τ, can be then calculated from equation (11).

$$\tau = \mu\dot{\gamma} \tag{11}$$

If equation (10) is substituted into equation (11), τ= 46. This means that shear stress, τ, in developer flow is constant.

Influence of Toner Concentration

We examined the developer properties, effective viscosity and friction factor. In addition, we defined the constant density in developer flow. Then we examined other developer properties here.

In developer flow, toner concentration is defined by the ratio of toner mass to developer mass, so, along with charge, it is a developer properties. In a standard developer, charge stabilizes in a short time. So, it can be regarded as constant. On the other hand, toner concentration varies with time and space. In the development of developing station, it is important to know the change of toner concentration with time and space. Here, we examine the toner concentration.

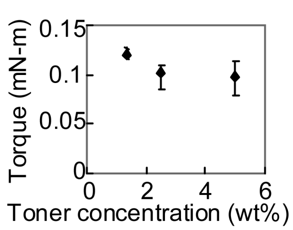

Figure 7. Torque vs. toner concentration

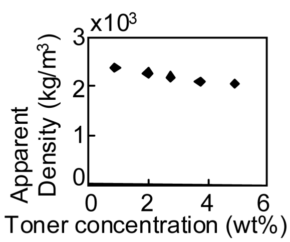

Figure 8. Developer density vs. toner concentration

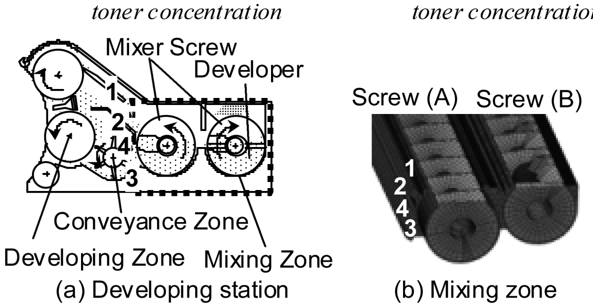

Figure 9. Mixing zone in developing station

Figure 7 shows the measurement results for spindle torque using the viscometer at three toner concentrations. We used the T bar, type E, with the spindle insertion depth of 6.35 mm which is the center of the region with developer movement. Figure 7 shows torque decreases slightly with toner concentration. This change originates from the change of density. Figure 8 shows developer density vs. toner concentration. The density decreases slightly with toner concentration. In addition, for commonly used toner concentrations of 2 wt% to 5 wt%, change of torque and density are under 5%. Here, the measured torque using T bar spindle is related to viscosity. So, viscosity and density can be regarded as constant for toner concentration.

As mentioned above, the developer flow can be analyzed with the non-Newtonian viscosity model, which can be expressed with equation (10), using the slip boundary with the friction factor.

Model and Validation

The Model Geometry for Mixing Zone in Developing Station

We applied the analysis model to the mixing zone in the developing station (Figure 9). Figure 9 (a) shows the whole developing station. Figure 9 (b) shows the composition of mixed zone and a general view of the finite element model used for analysis. This mixing zone arranges two spiral screws, which are left and right, screw (A) and screw (B). These screws rotate and circulate the developer in the depth direction of the developing station to mix the supplied toner and the developer. In this developing station, developer goes in and out between the developing zone and the mixing zone. In order to reflect this in the analysis model, the developer mass flow, as boundary conditions, is given at the holes 1, 2, 3, and 4 on the left side in Figure 9 (b).

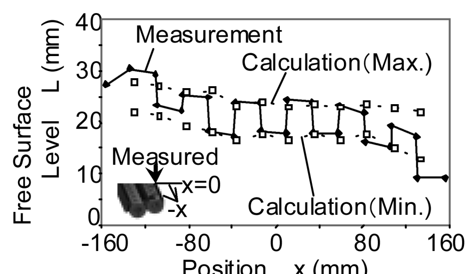

Figure 10. Comparison of measured and calculated free surface level

Table 6. Comparison of measured and calculated torque

Screw	Calculated (kg-cm)	Measured (kg-cm)
(A)	0.687	0.649
(B)	0.611	0.692

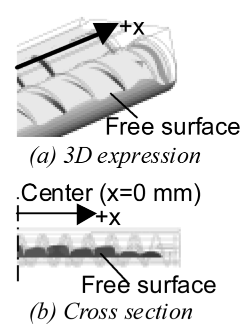

Figure 11. Example of calculated free surface level

Revision of Slip Condition Based on Observation in the Developing Station

In order to estimate the initial velocity in developer flow, we put pieces of polystyrene into the developer and measured their movement. We found that developer is moved in the axial direction by the screw spiral for a longer than the time estimated from the velocity of the developer observed on the free surface. This result means that the slip rate on the screw wall differs between the free surface and bottom.

Therefore, as shown in equations (12) and (13), functions are added to the model. Here, U_w, U_{ns}, and U_{slip} mean the developer velocity on the screw wall, the developer velocity on the screw wall with no slip condition, and the developer velocity on the screw wall with full slip condition. The c, F, g, and n mean the slip rate, the friction factor on the screw wall, the gravity vector, and the vector of the direction in which the developer moves.

$$U_w = cU_{ns} + (1+c)U_{slip} \tag{12}$$

$$c = -\frac{1}{2}F\left(\frac{\vec{g}}{|g|}\cdot\vec{n} - 1\right) \tag{13}$$

In equations (12) and (13), the full slip condition is set for the lower end of the screw, and no slip condition is set for the upper end.

Verification by Experiment

In order to validate the result obtained by calculation, we measured the developer free surface level in the mixing zone and the torque given to the screw using an actual developing station. The measured and calculated results are shown in Figure 10. Here, height from the bottom of the mixing zone is defined as free surface level. The free surface level is measured for the place shown in the insert. The free surface level in the mixing zone is changed by the screw rotation. The measured data are shown as the result on the stopped screw at the fixed position. On the other hand, the calculated data are shown as the maximum and minimum results on the rotating screw. The difference between measured and calculated values is less than 10%; agreement is good.

Table 6 shows measured and calculated torques. Their difference is about 5%.

Example Calculated Results

Figure 11 shows a 3-dimensional calculation example using the model. Figure 11 (a) shows the developer free surface in 3 dimensions, and Figure 11 (b) shows the cross section on the developer free surface. For the right side in each figure, the free surface level is low. This result fully reproduces the measurement result shown in Figure 10. Thus, since the measurement result is reproducible, the cause is known. And, since the cause is known, we can cope with problems for developer flow.

Conclusion

We examined a calculation technique for developer flow in the developing station by using viscous fluid analysis. We found that the developer can be dealt with as non-Newtonian fluid, which has the relation, $\mu = K/\dot{\gamma}$ (K is a constant; effective viscosity, μ; and shear rate, $\dot{\gamma}$). In addition, the slip boundary with the friction factor is given as boundary conditions. In this model, we regarded that density and charge as constant.

Three-dimensional analysis in the mixing zone was carried out using this model. Measured and calculated results were compared for the free surface and the screw torque. The calculation was in good agreement with measurement, with less than 10% errors.

The model allowed detailed flow for large and small regions in the developing station to be obtained.

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Biography

Yasuo Takuma was born in Osaka prefecture, Japan in 1959. He received his BE and ME degrees in 1983 and 1985, respectively. He began researching imaging technology for laser printers, in Hitachi, Ltd. from 1985 and in Hitachi Koki Co., Ltd. from 1995. He is a member of ISJ and JSAP.