Statics of Magnetic Bead Chain in Magnetic Field

Nobuyuki Nakayama, Hiroyuki Kawamoto and Makoto Yamaguchi
Department of Mechanical Engineering, Waseda University
Tokyo, Japan

Abstract

Experimental, numerical, and theoretical investigations have been carried out on statics of a magnetic bead chain in the magnetic filed. Chains formed on a solenoid coil were observed and chain lengths and slant angles were measured by a digital microscope. It was deduced that (1) the chain length depends on both the weight of magnetic particles and the magnetic flux density; (2) chains have inclinations in the inclined magnetic field and that inclinations of the chains are enlarged by the gravitational force. These static configurations of chains are approximately determined to minimize the total potential energy that consisted of the gravitational and magnetic potential energy. These characteristics were qualitatively confirmed by the numerical calculation with the two-dimensional Distinct Element Method. The investigation is expected to be utilized for the improvement of the two-component magnetic brush development system in electrophotography.

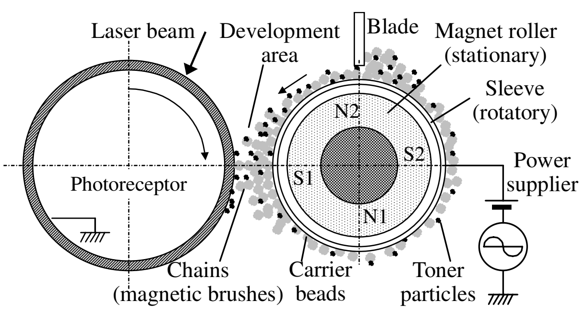


Figure 1. Magnetic brush development system.

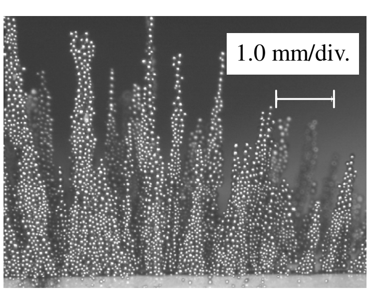


Figure 2. Chains of magnetic particles in magnetic field.

Introduction

A magnetic brush development system in electrophotography process is shown in Figure 1. Magnetic carrier beads with electrostatically attached toner particles are introduced into the vicinity of a rotatory sleeve with a stationary magnetic roller inside it[1]. Magnetized carrier beads in the magnetic field form chain clusters on the sleeve as shown in Figure 2. Tips of chains touch the photoreceptor surface at the development area and toner particles on chains move to electrostatic latent images on the photoreceptor to form real images. Carrier chains play important roles in this development system in order to realize high quality imaging. Sufficient length and moderate stiffness are required to obtain satisfactory image density and to prevent image defects. Therefore, it is necessary to clarify the relationship between kinetic characteristics of formed chains and design parameters, such as magnetic flux density and physical properties of carriers.

In these points of view, the experimental and theoretical studies[2,3] had been reported on the magnetic interactions between magnetic particles. Numerical studies[4-6] have been also conducted using the Distinct Element Method (DEM)[7] to clarify detailed behaviors of particles in the development system. However, the mechanism of the chain forming process and the effect of design parameters had not been investigated well. In this study, parametric experiments using a solenoid coil as a source of magnetic field had been performed and a numerical simulation based on the DEM was conducted to clarify the statics of chains. Then, the results were discussed in the point of potential energy minimization.

Experiment

Experimental Method

An experimental setup is illustrated in Figure 3. Spherical magnetic particles (Toda Kogyo Corp.) shown in Figure 4 were provided in the area with 10 mm in diameter at the center of the end plate on the solenoid coil. Magnetic particles made by the polymerization method are 88.1 μm in average diameter, 3550 kg/m^3 in volume density and 4.34 in relative magnetic permeability. Chains of magnetic particles formed in the magnetic field were observed and recorded by a digital microscope (Keyence Corp., VH-7000). Lengths and slant angles of chains were measured from stored still images.

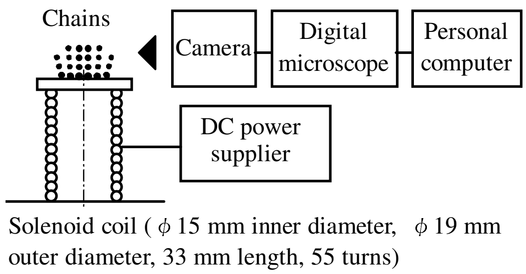


Figure 3. Experimental setup.

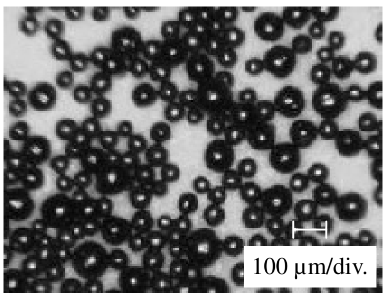


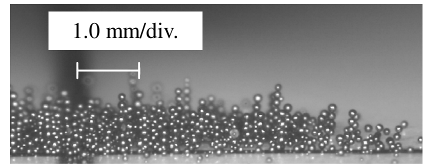
Figure 4. Photograph of magnetic particles.

Axial magnetic flux density B' along the center axis of the coil is approximated by $B'(z) = B_0(1 - cz)$, where B_0 and c (= 66.87 1/m) are constants and z (m) is the axial coordinate. Maximum magnetic flux density B_0 is proportional to the coil current with a proportional constant 0.006156 T/A.

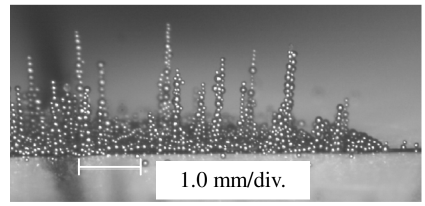
Experimental Results

Figure 5 shows the examples of observed chain profiles. In Figure 5 (a) with coil current 1 A and particle weight 0.13 kg/m^2, chain formation was not clearly observed. However, a lot of thin and long chains were formed in (b) with coil current 3 A. Although the magnetic field intensity was common, chains became thicker and longer in (c) when the weight was increased from 0.13 to 0.25 kg/m^2.

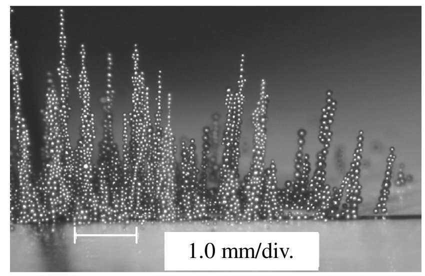
Measured chain lengths were plotted in Figure 6 as a function of the radial coordinate on the coil. While the length of each chain varies widely, the second order regression curves show clearly the dependency of the length on the coil current and the radial coordinate. In Figure 7, chain lengths at the center of the coil were plotted as a function of the particle weight. Chain lengths increase with the increase in the particle weight and the coil current.



(a) Coil current 1 A, particle weight 0.13 kg/m^2



(b) Coil current 3 A, particle weight 0.13 kg/m^2



(c) Coil current 3 A, particle weight 0.25 kg/m^2

Figure 5. Observed chain profiles.

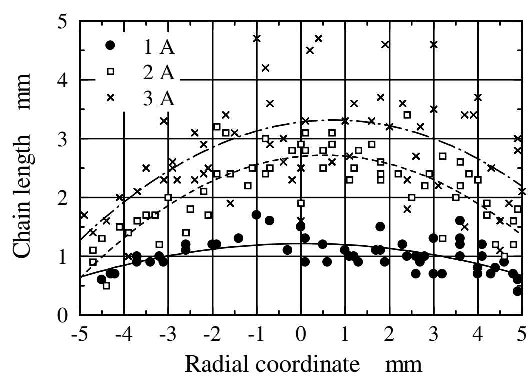


Figure 6. Measured chain length at various coil current. (0.38 kg/m^2 weight)

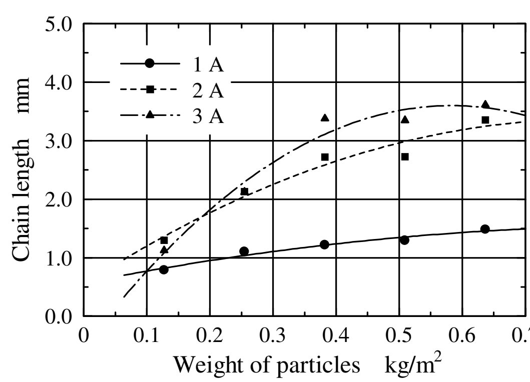


Figure 7. Relation between chain length and weight of particles.

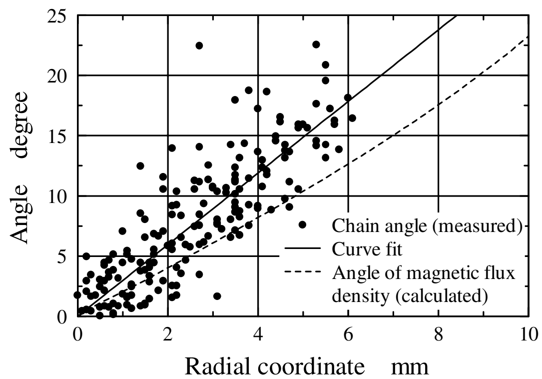


Figure 8. Relation between chain angle and radial coordinate.

Slant angles of chains are plotted in Figure 8 as a function of the radial coordinate with the calculated angle of the magnetic flux density. The angles increase linearly with the increase in the radial coordinate. Angles of chains are larger than that of the magnetic flux density. It is supposed that the gravitational field enlarged the slant angle.

Numerical Simulation

Simulation Method

Numerical simulation was performed on chain forming process based on the two-dimensional DEM[7]. In the calculation the momentum equations are solved for each particle with three degrees of freedom including rotation. In this study, mechanical interaction force, magnetic force, air drag, and gravitational force are included in the equations. The mechanical interaction force in normal direction at the contact point was estimated from Hertzian contact theory. The interaction force in tangential direction was assumed proportional to the normal force with a proportional constant 0.25. The effect of rolling friction, van der Waals force and electrostatic force were neglected.

The magnetic force $\boldsymbol{F}_j$ and rotational moment $\boldsymbol{M}_j$ of the j-th particle with the magnetic moment $\boldsymbol{m}_j$ are given by the following expressions under the assumption that each particle behaves as a magnetic dipole placed at the center of the magnetized particle[2].

$$\begin{aligned} \boldsymbol{F}_j &= (\boldsymbol{m}_j \cdot \nabla)\boldsymbol{B}_j, \\ \boldsymbol{M}_j &= \boldsymbol{m}_j \times \boldsymbol{B}_j. \end{aligned} \tag{1}$$

The magnetic moment $\boldsymbol{m}_j$ and magnetic flux density $\boldsymbol{B}_j$ at the position of the j-th particle are

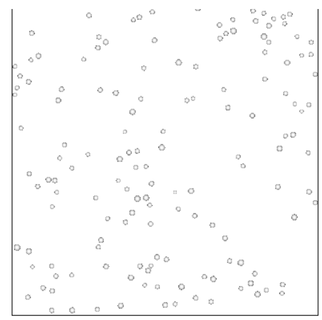
$$\boldsymbol{B}_j = \boldsymbol{B}_j' + \sum_{\substack{k=1 \\ j \neq k}}^{N} \boldsymbol{B}_{kj}, \tag{2}$$

$$\boldsymbol{m}_j = \frac{4\pi}{\mu_0} \frac{\mu - 1}{\mu + 2} \frac{{a_j}^3}{8} \boldsymbol{B}_j, \tag{3}$$

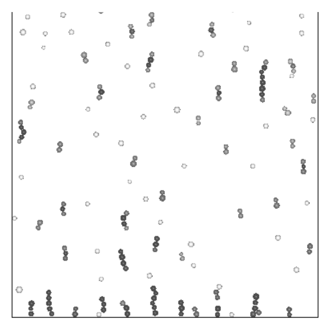
where N is the number of the particles, μ_0 is the permeability of free space, μ is the relative permeability of particles, a_j is the diameter of the j-th particle. The first term in the right hand side of Equation (2) is the applied magnetic field by the coil and the second term is the field at the j-th particle due to dipoles of the other particles. $\boldsymbol{B}_{kj}$ is given by

$$\boldsymbol{B}_{kj} = \frac{\mu_0}{4\pi} \left(\frac{3\boldsymbol{m}_k \cdot \boldsymbol{r}_{kj}}{\left|\boldsymbol{r}_{kj}\right|^5} \boldsymbol{r}_{kj} - \frac{\boldsymbol{m}_k}{\left|\boldsymbol{r}_{kj}\right|^3} \right), \tag{4}$$

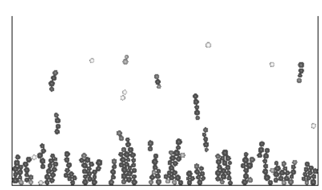
where $\boldsymbol{r}_{kj}$ is the position vector from the k-th to the j-th particle. The magnetic force is determined by solving Equations (2), (3) and (4) simultaneously and by substituting the results to Equation (1).



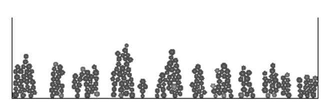
(a) t = 0 ms



(b) t = 10 ms



(c) t = 30 ms



(d) t = 50 ms

Figure 9. Simulated chain forming process. Image densities of particles designate magnitude of applied forces.

Simulated Results

Simulated chain forming process is shown in Figure 9. Initially, 225 particles were placed at random positions in the area with 5 mm in width and 8 mm in height as is shown in Figure 9 (a). The numbers of particles were estimated from the weight of magnetic particles used in the experiment. The number of 225 particles in this simulation is equivalent to 0.64 kg/m^2 weight in the experiment. Young's modulus of the particles was assumed to be 10 GPa and that of the fixed boundaries was assumed to be 100 GPa. Poisson's ratio and friction coefficient were assumed to be 0.3 and 0.2, respectively. The motion of each particle was calculated every 100 ns after applying the gravitational field and the magnetic field equivalent to coil current 3 A that corresponds to the maximum magnetic flux density 0.018468 T.

In the simulated chain forming process shown in Figure 9, discretely placed particles form clusters connected in lines first. The clusters fall on the floor and grow up to treelike chains. It is also observed that the forces applied to particles increase with the growth of chain length. It is clearly recognized that the calculated profile was similar to that in the experimental observations shown in Figure 5.

Calculated dependency of maximum chain lengths on the weight of particles and the coil current is plotted in Figure 10. Calculated results qualitatively agreed with the measured results shown in Figure 7. That is, the chain becomes longer with the larger number of magnetic particles in the field of the higher magnetic flux density. However, the simulated results give smaller values of the chain length than the experimental results. One of possible reasons is that particles are apt to fall off chains in the simulation because of the neglected rolling friction. In addition to consideration of the rolling friction, three-dimensional DEM calculation is also necessary to improve the quantitative accuracy.

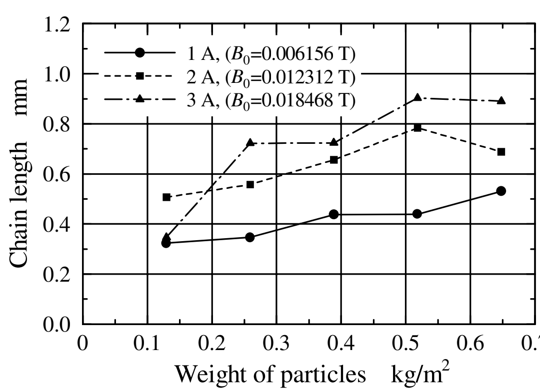


Figure 10. Calculated relations between chain length and weight of particles.

Theoretical Discussion

Chain Length

We assumed the hypothesis that the chain length is determined to minimize its total potential energy. The total potential energy is given by the sum of magnetic energy U_m expressed by Equation (5) and gravitational energy.

$$U_m = -\frac{1}{2}\sum_{j=1}^{N} \boldsymbol{m}_j \cdot \boldsymbol{B}_j' \quad (5)$$

On the assumption that particles are connected along a straight line in vertical direction, as illustrated in Figure 11 (a), the relation between the chain length and the magnetic energy was calculated by solving Equations (2), (3) and (4). The gravitational energy is simply determined by the chain configuration. Calculated results are shown in Figure 12. In the figure, both the total energy and the average energy are plotted for each coil current. The most stable chain length to minimize the energy exists for each coil current and the length increases with the increase in coil current. It is also shown that the most stable length to minimize the total energy is different from that to minimize the average energy.

The calculated relations between the most stable chain length and coil current are plotted in Figure 13 with experimental results and numerically simulated results. A solid line describes the relation between the coil current and the chain length, l_t, to minimize the total energy, and the broken line describes the relation between the coil current and the chain length, l_a, to minimize average energy. Experimental values are close to l_a with smaller number of particles, on the other hand, the values are close to l_t with larger number of particles. The theoretically estimated length l_t agrees with the experimental results for over 0.38 kg/m^2 weight particles. It is shown that the chain length is determined to minimize the total potential energy if sufficient particles to form chains are provided. However, if the number of particles is limited, the length is determined to minimize the average potential energy.

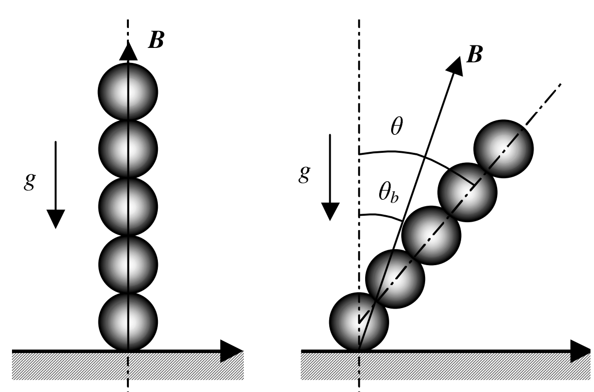
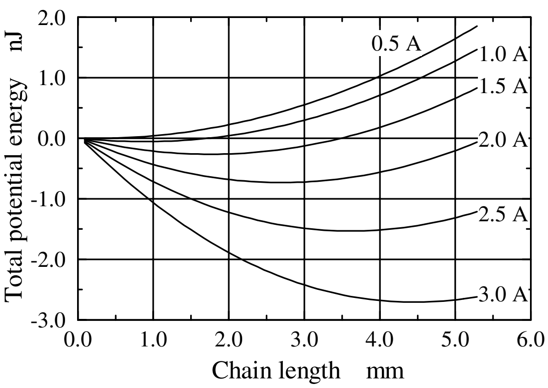
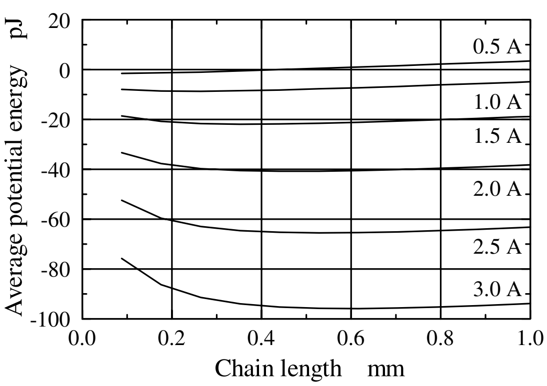


Figure 11. Schematic diagram of straight and inclined chain.



(a) Total potential energy



(b) Average potential energy

Figure 12. Calculated relations between potential energy and chain length.

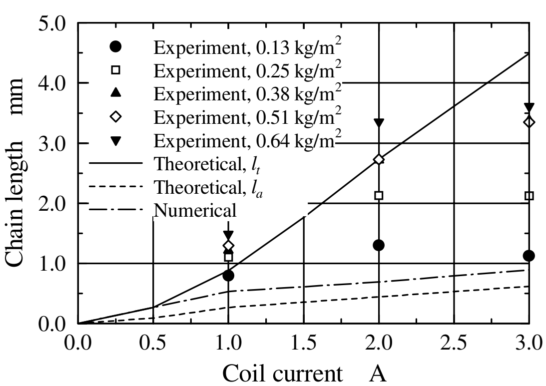


Figure 13. Comparison of calculated chain length with experimental results.

Slant Angles of Chains

It is supposed that the slant angle of the chain is also determined to minimize the potential energy. The estimated total potential energy of the inclined chain, as is described in Figure 11 (b), are plotted in Figure 14 as a function of the chain angle, θ, for different angles of magnetic flux density, θ_b. Nine particles are considered in this energy estimation which minimize the total potential energy under the condition of coil current 1 A. It is shown that the most stable angle of chain exists for each angle of magnetic flux density if the angle is less than 20 degree.

The most stable chain angle is plotted in Figure 15 compared with the experimental result. The symbols express the relation between regression curve of the measured chain angle and calculated angle of magnetic flux density described in Figure 8. A solid line expresses the theoretical results obtained from Figure 14. The theoretical result is similar to the experimental. It is confirmed that the gravitational field magnifies the chain angle. However the theoretically estimated chain angle is larger than the experimental result. A non-conservative theory must be established to quantitatively evaluate this phenomenon.

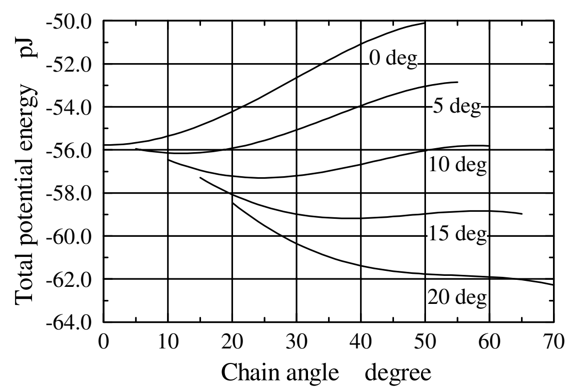


Figure 14. Effect of magnetic flux angle on the relation between potential energy and chain angle.

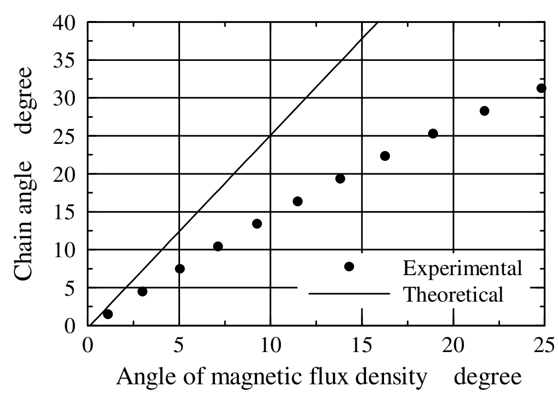


Figure 15. Comparison of calculated and experimental results of chain angle.

Conclusion

Lengths and slant angles of magnetic bead chains formed on the solenoid coil were measured to clarify the statics of chains and the chain forming mechanism. Motions of particles were numerically simulated using DEM and calculated results were compared with experimental results. Then, the stable conditions of chains were discussed in points of potential energy minimization. The followings were clarified from the investigation: (1) The chain lengths increase with the increase in magnetic flux density and the weight of particles. (2) Slant angles of inclined chains formed in the inclined magnetic field were enlarged by the gravitational force. (3) The chain forming process and chain profiles can be simulated qualitatively well by the two-dimensional Distinct Element Method. (4) These static condition of chains is approximately determined to minimize the total potential energy that consists of the magnetic and gravitational energy. The present investigation is expected to be utilized for the improvement of the two-component magnetic blush development system.

Acknowledgement

The authors would like to express their thanks to the Ministry of Education, Science and Culture of Japan for the financial support, to Toda Kogyo Corp. for supplying magnetic particles, and to Janjomsuke Wiphut (Bank of Thailand), N. Sukou, S. Yamada, and A. Sasakawa (Waseda Univ.) for their help of carrying out experiments.

References

1. E. M. Williams, *The Physics and Technology of Xerographic Processes*, Krieger Publishing, FL (1993).
2. R. S. Paranjpe and H. G. Elrod, Stability of chains of permeable spherical beads in an applied magnetic field, *J. Appl. Phys.*, **60**, 1 (1986), 418-422.
3. R. S. Paranjpe and H. G. Elrod, A magnetomechanical model of the magnetic brush, *J. Appl. Phys.*, **63**, 6 (1988), 2136-2140.
4. Y. Shibata, et al., Granular modeling coupled with the magnetic field, *Proceedings of Symposium on Computational Methods in Structural Engineering and Related Fields*, **17**, (1993), 115.
5. S. Serizawa, Numerical simulation of carrier chain formation in magnetic brush development in electrophotography, *Japan Society of Mechanical Engineering D&D 99*, No. 99-7 (I) (1999), A288.
6. T. Kubota, et al., Numerical analysis for behavior of development in magnetic brush system of electrophotography by particles method, *Japan Hardcopy '99 Fall Meeting*, (1999), 26.
7. P. A. Cundall and O. D. L. Strack, A discrete numerical model for granular assemblies, *Géotechnique*, **29**, 1 (1979), 47-65.

Biography

Nakayama, Nobuyuki holds a BS degree in Physics from Tohoku Univ. (1983). In 1983, he joined Fuji Xerox, and has been engaged in the research of electrophotography as a Researcher. From 2000, he has also been a student of Department of Mechanical Engineering, Waseda Univ. He is working on numerical simulation and measurement of powder dynamics in electrophotography. He is a member of the Imaging Society of Japan and the Japan Society of Mechanical Engineering.