Prediction of the Reflection Spectra of Three Ink Colour Prints

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Abstract

We have developed a colour prediction model and an ink-spreading model. The present study aims at confirming the validity of both models for the case of ink-jet prints using cyan, magenta and yellow inks.

Our colour prediction model, augmented by the ink-jet spreading model, predicts accurately the reflection spectra of halftoned samples printed on an HP printer and on an Epson printer. For each printer, the reflection spectra of 125 samples uniformly distributed in the CMY colour cube were computed. The average prediction error between measured and predicted spectra is about $\Delta E = 2.5$ in CIELAB. The model requires the estimation of a set of parameters which are deduced from a small set of measured samples. Such a model simplifies the calibration of ink-jet printers, as well as their recalibrations when ink or paper is changed.

Keywords: physical modelling, colour prediction, halftones, ink-jet printing, dot gain, ink spreading, calibration.

Introduction

In previous publications,[1,2] we have developed a colour prediction model which unifies in a common mathematical framework the phenomena of surface reflection, multiple internal reflections in the ink layers, light scattering in the paper and ink spreading. Results such as the Clapper-Yule equation, the Murray-Davis relation and the Williams-Clapper equation are particular cases of this model. The prediction model only requires a small set of measured input data for predicting the spectra of printed patches.

Ink-spreading, a particular kind of dot gain, induces significant colour deviations in ink-jet printing. Therefore, the estimation of the area covered by each ink combination is crucial in colour prediction models. In a previous study,[3] we modelled ink spreading by considering a small set of ink drop configurations which are sufficient to deduce the ink spreading in all other cases. The present study aims at confirming the validity of the colour prediction model and of the ink spreading model for the case of 3 ink layers.

The colour prediction model

This section gives only an overview of the main parts of our colour prediction model for halftone prints. The full model has been presented in detail in a previous publication.[2] Let us denote by i the light fluxes oriented downwards, and by j the light fluxes oriented upwards. The interaction of the light with the halftone print can then be modelled by three matrices, as briefly explained below (see Figure 1).

In the present study, we consider high quality ink-jet paper consisting of an ink-absorbing layer in optical contact with the substrate of reflectance R_g which is a diffuse white reflector. This reflector is supposed to be Lambertian and is never in contact with the inks. Since the coating has a refractive index n different from that of air, multiple internal reflections occur at the interface between the air and the coating. This phenomenon significantly increases the optical density of the ink-absorbing layer. In order to take this into account the interface is modelled by a matrix called the *Saunderson correction matrix* M_{SC}.[2]

In halftone prints, the inks are not applied uniformely on the paper. There are regions covered with ink or a superposition of several inks and regions without ink. Our model assumes that each region which is covered by a uniform ink combination behaves according to the Kubelka-Munk model. The absorption and the scattering of light in an infinitely thin layer of the coating is modelled by a *Kubelka-Munk matrix* M_{KM}.[2] The interaction of the light fluxes in the coating is then modelled by the exponential of the product of M_{KM} and the thickness X of the coating.

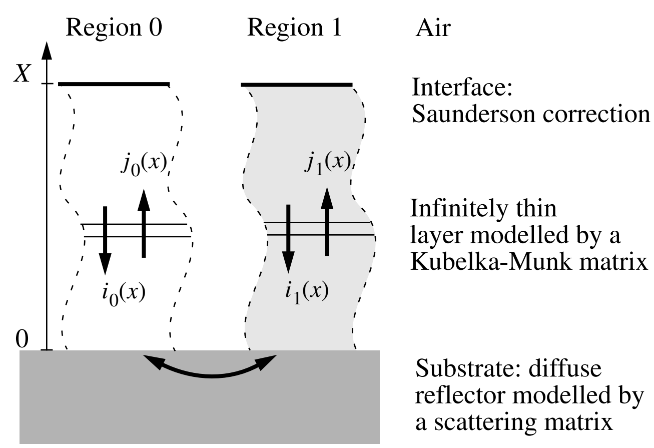

Figure 1. Model of the printed surface. On top of the substrate, each region is considered to be a uniform layer which behaves according to the Kubelka-Munk model. The exchange of photons between different regions takes place in the substrate.

Finally, the light scattering in the substrate, which is an exchange of photons between regions covered by different ink combinations, is modelled by a *scattering matrix* M_S.[2] Each coefficient of this matrix corresponds to the probability of a photon to be scattered from one region to another. Note that this phenomenon is related to the Yule-Nielsen effect.

In the particular case of a printed surface made of a non-inked area (region 0) and an inked area (region 1), the light fluxes above the air-coating interface are denoted i_0, i_1, j_0 and j_1. They are related to the light fluxes at the top of the substrate by the following matrix equation:

$$\begin{bmatrix} i_0 \\ i_1 \\ j_0 \\ j_1 \end{bmatrix} = M_{SC} \cdot \exp(M_{KM}X) \cdot M_S \cdot \begin{bmatrix} i_0(0) \\ i_1(0) \\ R_g i_0(0) \\ R_g i_1(0) \end{bmatrix} \tag{1}$$

Thanks to Equation (1) the emerging light fluxes j_0 and j_1 can be expressed as functions of the incident light fluxes i_0 and i_1. Let a_0 be the fraction of area occupied by region 0, and let $a_1 = 1 - a_0$ be the fraction of area occupied by region 1. The reflectance R of the printed sample is then given by:

$$R = \frac{\begin{bmatrix} a_0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} j_0 \\ j_1 \end{bmatrix}}{\begin{bmatrix} a_0 & a_1 \end{bmatrix} \cdot \begin{bmatrix} i_0 \\ i_1 \end{bmatrix}} = \frac{(1 - a_1)j_0 + a_1 j_1}{(1 - a_1)i_0 + a_1 i_1} \tag{2}$$

Modelling ink spreading

The model presented in the previous section requires an estimation of the fraction of area occupied by each ink combination, and an estimation of the scattering probabilities (coefficients of the M_S matrix). Since these parameters depend on the microscopic structure of the halftone print (shape of the ink drop impact and haftoning algorithm), they must be computed by simulating the printing process.

Unfortunately, the shape of the ink drop impact is not constant. For instance the superposition of ink drops causes a significant dot gain, i.e. a change of the covered area. When ink drops are printed one over another or just partially overlap, an ink spreading process takes place. This phenomenon is a complex interaction between the inks and the printed surface. It is strongly related to physical properties like wettability and solvent absorption. Therefore the inks behave differently on every surface.

In a previous work, we introduced a simplified model of ink spreading where the shape of the impact is approximated by a parametric curve $r(\theta)$ in polar coordinates.[3] According to our observations made under the microscope, the shape of the ink drop impact depends on the state of the surface ("wet" or "dry") and the configuration of the neighbouring impacts. Most ink-jet printers use a hexagonal grid when printing in colour. Hence, each ink drop impact has six neighbours. Therefore, the circumference of the impact model is parametrised by six radius vectors having a common origin at the impact centre. Each vector is oriented to the direction of the midpoint between two neighbouring impacts (see Figure 2). Furthermore, we assume that a neighbour influences only locally the shape of the impact. Hence, the length r_i of the i-th radius vector depends only on the state of its two closest neighbours and on the state of the surface at the impact centre. In order to get realistic impact shapes, the circumference between two vectors is approximated by a parametric curve which is a polynomial of degree three. In other words, $r(\theta)$ is a piecewise polynomial of degree three.

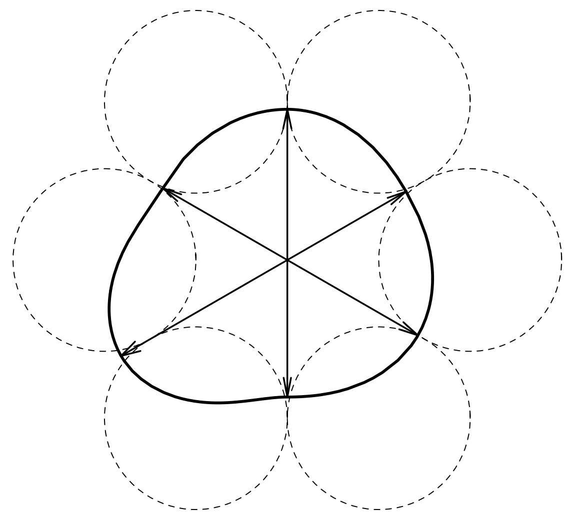

Figure 2. Six radius vectors define the circumference of the impact. The dashed circles indicate the locations of neighbouring impacts.

The density profile of an isolated ink impact was measured under the microscope and approximated by a parabolic function. We assume that the dye density D at the location defined by the polar coordinates (ρ, θ) $(0 \le \rho \le r(\theta))$ is given by:

$$D(\rho, \theta) = D_M \cdot \left[1 - \left(\frac{\rho}{r(\theta)}\right)^2\right] \tag{3}$$

where D_M is the density at the centre of the impact. Note that the amount K of dye remains constant during the spreading process, only the spatial distribution is changed. Therefore, the maximal density D_M at the centre of the impact must be computed for each ink drop configuration so that:

$$K = \iint D(\rho, \theta)\rho d\rho d\theta \tag{4}$$

Note that D_M decreases when the impact is enlarged.

The function $D(\rho, \theta)$ is a numerical model of a single ink drop impact. This function is parametrised by six radius vectors, and each vector depends on the state of only two neighbours and the state of the surface on which the ink drop is printed. Using a combinatorial approach based on Polya's counting theory,[4,5] we have shown that a reduced number N

of non-equivalent ink drop configurations are sufficent to determine the ink spreading in all other cases:

$$N = \frac{k \cdot (k+1) \cdot (k+2)}{2} \qquad (5)$$

where k is the number of inks.

In the particular case of a three-ink print, a set of $N = 30$ non-equivalent ink drop configurations must be considered. These configurations are listed in Figure 4. The mathematical method involved in this computation is described in reference 3.

Now we have all elements allowing to simulate the printing process. The printed surface is modelled by using high resolution grids, one grid for each ink. The value of a grid point corresponds to the local amount of a given ink (see Figure 3). The numerical model of a single ink drop impact $D(\rho, \theta)$ is used as a stamp. Wherever the halftoning algorithm puts an ink drop on the surface of the printed medium, the numerical model is stamped at the same location on the high resolution grid (see Figure 3). Stamp overlapping is additive.

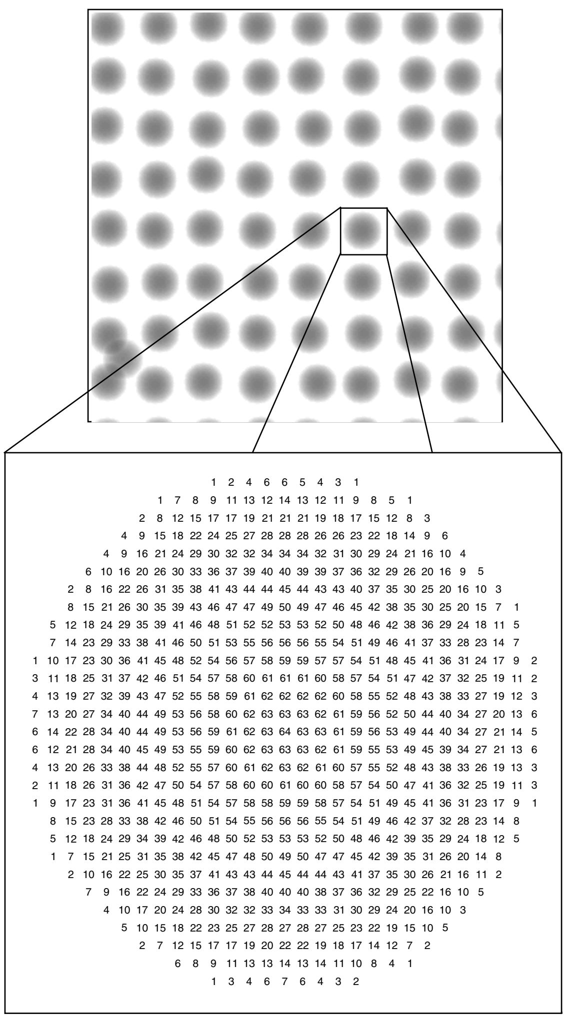

Figure 3. High resolution grid modelling the printed surface. The value of a grid point corresponds to the local amount of dye. The density profile of an isolated ink impact is parabolic.

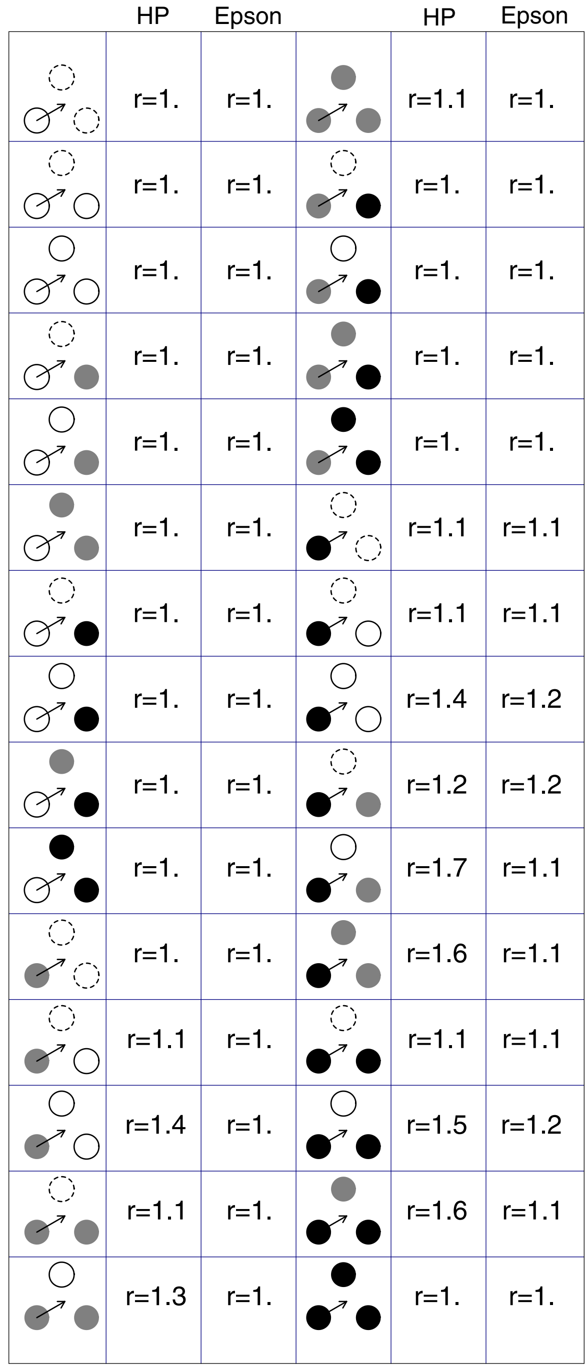

Figure 4. List of the 30 non-equivalent configurations in a three-ink print. The dashed circles indicate the locations of neighbours which are not covered with ink. The white disks correspond to one ink drop, the grey disks correspond to two ink drops, and the black disks correspond to three ink drops. The vectors indicate the orientation of the spreading. The magnitude of the spreading of HP inks printed on HP paper and the magnitude of the spreading of Epson inks printed on Epson paper are respectively given in the columns entitled HP and Epson, where $r = r_i / r_0$.

Prediction results and discussion

We printed two series of 125 colour samples uniformely distributed in the CMY colour space. Each series is a set of 5×25 samples printed on five sheets of paper. The area coverage of the yellow ink is constant for all samples printed on the same sheet, and varies from sheet to sheet. The first series was printed on Epson "PhotoQuality Glossy Paper" using an Epson Stylus Color printer.[6] The second series was printed on HP "Photo Paper" using an HP DeskJet 550C printer.[7] All samples were produced with a clustered dither algorithm with 33 tone levels.

For both series, the radius r_0 of an isolated circular impact was measured accurately under the microscope, and for each ink drop configuration, the corresponding radius vector length r_i was estimated visually under the microscope. In the HP series, the superposed cyan, magenta and yellow ink drop impacts have almost the same size, whereas in the Epson series, the radius of the yellow drop impact is 20% larger than the radius of the cyan drop impact, which is, in turn, 30% larger than the radius of the magenta drop impact. All ink spreading coefficients $r = r_i/r_0$ are listed in Figure 4. Note that the ink spreading coefficients for the Epson series are given for the magenta drop impact.

Table 1: Prediction results in CIELAB for the Epson series.

Area coverage of the yellow ink:	Mean ΔE	$\sqrt{\frac{\sum \Delta E^2}{n}}$	Maximal ΔE
0%	1.67	1.97	3.58
25%	1.81	1.96	3.11
50%	1.95	2.14	4.13
75%	2.91	3.02	5.59
100%	2.85	3.11	5.75

Table 2: Prediction results in CIELAB for the HP series.

Area coverage of the yellow ink:	Mean ΔE	$\sqrt{\frac{\sum \Delta E^2}{n}}$	Maximal ΔE
0%	2.07	2.34	4.70
25%	2.57	2.85	4.85
50%	2.59	2.90	5.48
75%	3.27	3.49	6.85
100%	3.92	4.28	6.89

Using our prediction model, we computed the spectra of the 250 samples, and compared them with the measured spectra. The average colorimetric deviations between measured and predicted spectra of the Epson series and of the HP series are given in Table 1 and Table 2 respectively. The results obtained for the Epson series are a little better than the results for the HP series. This could be due to the stronger spreading of the HP inks.

Conclusion

We confirmed the validity of the colour prediction model and of the ink spreading model for the case of three inks. The spreading process was modelled by enlarging the drop impact according to the configuration of its neighbours and the state of the surface. The number of cases which must be analysed is reduced to a small set by using Polya's counting theory. In a three-ink-printing process, only 30 cases must be considered to deduce the ink spreading in all cases.

The printing process was simulated by stamping impacts of different shapes on high resolution grids. Each impact shape is determined by 6 radii. The values of each radius are estimated by an observation of the cases given by Polya's counting theory. This allowed us to compute the relative areas occupied by the various ink combinations in a three-ink-process. We predicted accurately the spectra of 250 samples produced by two different printers. The average prediction error is about $\Delta E = 2.5$ and the maximal error is less than $\Delta E = 7$ in CIELAB.

Such a model simplifies the calibration of ink-jet printers, as well as their recalibration when ink or paper is changed. The ink spreading model can also improve advanced Neugebauer-based colour prediction methods since they require an accurate estimation of the area covered by each ink combination.

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