

Space and Time Dependence on Transfer Process of Electrophotography

Yuji Furuya
Research & Development Lab. Hitachi Koki Co., Ltd. Hitachinaka-City, Ibaraki-Pref. 312-8502 Japan

Abstract

Electrostatic transfer process of 4-layers structure (Photoreceptor, toner, paper, and transfer belt) which is specified by multiple layers of R-C parallel circuit and expressed by a set of 3 differential equations, is investigated rigorously with a method of eigen value and eigen vectors. Initial conditions of solutions are determined by static voltage difference of each layer, which are given by electrostatic Poisson's equations, and then space dependence and time one of solutions are unified by this analysis. Time evolution of each voltage, transfer efficiency and Paschen's discharge are discussed.

Introduction

Electrostatic transfer of toner image has been widely investigated in electrophotography [1-3], because the process plays a significant role on transfer efficiency, blurring of the toner image due to Paschen's discharge (*PD*), paper transport and registration of multicolor images. Although a basic model, which is represented by multiple layers of R-C parallel circuit, is simple, complicated coefficients of a set of differential equations have presumably prevented to obtain rigorous solutions.

This study shows that the coefficients are simply described by using dielectric thickness (D_j) and time constant (τ_j) instead of C_j and R_j, respectively. The solutions are obtained easily by a method of eigenvalue and eigenvectors. It is shown that PD occurs in non- image white area, dominantly.

Static Solutions; Poisson's eqs.

A present model is shown in Fig. 1. Each layer has its electrical resistance (R_j) and capacitance (C_j). Thickness of each layer is d_j, and dielectric thickness d_j/ε_j is denoted by D_j, where ε_j is permittivity. Initial voltage on photoreceptor ($j=1$) is V_o and the voltage of non-image white area equals to V_o. Voltage of toner image area on photoreceptor is denoted by V_a where is neighboring of the non-image area. V_o and V_a are related to surface charge density σ with $\sigma_o D_1 = V_o$ and $\sigma_a D_1 = V_a$, respectively. Toner image layer ($j=2$) is specified by charge density ρ, but the non-image area of photoreceptor is occupied by air layer (D_2'). Paper ($j=3$) and transfer belt ($j=4$) are no charged layers at initial state. Transfer voltage V_t is applied on these 4-layers states.

We apply one-dimensional continuous model and electrostatic Poisson's equations are solved (; $d^2\phi_j/dx^2 = 0$ or $-\rho/\varepsilon_2$). With use of boundary condition of electrical flux density, continuous condition of the voltage at each interface, the voltage distributions $\phi_j(x)$ are given by next equations, where coordinates of the each interface are $x_j = \Sigma d_j$, (j =1~4, e.g. $x_3=d_1+d_2+d_3$).

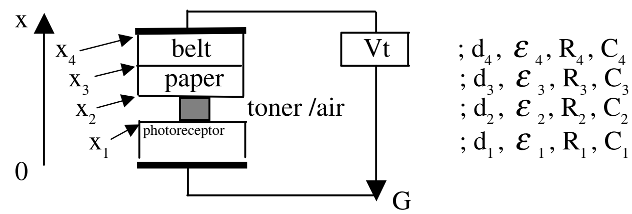

Figure 1. One dimensional continuous model for transfer process.

$$\phi_1(x) = (H/\Sigma D + \sigma a + \rho d_2)\, x/\varepsilon_1,$$
$$\phi_2(x) = Vt - (\rho/2\varepsilon_2)(x-x_2)^2 + H(x-x_2)/(\varepsilon_2 \Sigma D) - H(\gamma+\delta),$$
$$\phi_3(x) = Vt + H(x-x_3)/(\varepsilon_3 \Sigma D) - H\delta,$$
$$\phi_4(x) = Vt + H(x-x_4)/(\varepsilon_4 \Sigma D), \quad (1)$$

where

$$H = Vt - Va - \rho d_2(D_1 + D_2/2), \quad (2)$$

and $\Sigma D = D_1 + D_2 + D_3 + D_4$, $\alpha = D_1/\Sigma D$, $\beta = D_2/\Sigma D$, $\gamma = D_3/\Sigma D$, $\delta = D_4/\Sigma D$, and $\alpha+\beta+\gamma+\delta = 1$. (3)

The static voltage difference of each layer is specified as

$$U_j = \phi_j(x_j) - \phi_{j-1}(x_{j-1}), \text{ and } U_1 = (H\alpha + Va + \rho d_2 D_1),\ U_2 = (H\beta + \rho d_2 D_2/2),\ U_3 = H\gamma,\ U_4 = H\delta,\ \Sigma U_j = Vt, \quad (4)$$

Voltage distribution of the toner layer $\phi_2(x)$ shows a minimum at x_o and ideal transfer efficiency η_o is obtained as a region of the toner layer where electrostatic force acts for the paper layer.

$$\eta_o = (x_2 - x_0)/d_2 = -(H/\rho\, d_2\, \Sigma D)\,, \tag{5}$$

Similar relations of the voltages for the non-image area in the photoreceptor are given with substituting (0, Vo, εo, D_2', Σ D') for a set of (ρ, Va, ε2, D2, Σ D) in the above eqs. (1)~(4).

Dynamic Solutions

Transfer current (*I*) of area *S* in multiple layers of R-C parallel circuit is described by (6), where *Vj(t)* is dynamic voltage difference of each layer.

$$I = V_1/R_1 + C_1\, dV_1/dt = V_2/R_2 + C_2\, dV_2/dt = \ldots = \ldots \tag{6}$$

With using time constant $\tau j = RjS/Dj$ and dielectric thickness $Dj = S/Cj$ of each layer, (6) is expressed by (7) and (8).

$$I/S = V_1/\tau_1 D_1 + (1/D_1)dV_1/dt = V_2/\tau_2 D_2 + (1/D_2)dV_2/dt = \ldots = \ldots, \quad \text{where } \Sigma\, Vj = Vt\,, \text{ and } \Sigma\, dVj/dt = 0\,. \tag{7}$$

$$dV_j/dt = (D_j/\Sigma D)\, \Sigma\, (V_k/\tau_k) - V_j/\tau_j\,, \qquad j, k = 1\sim 4. \tag{8}$$

Current density is also denoted by (9) and dominated by the lowest layer of τ_k .

$$I/S = (1/\Sigma D)\, \Sigma\, (V_k/\tau_k)\,, \tag{9}$$

Eqs. (8) and (9) are very simple compared with the case of coefficients Rj and Cj [2)], and also applied to Vt of DC and AC states. Independent equations of (8) are 3 with (7) , and then, V_2 , V_3 and V_4 are selected. These eqs. (8) are represented by notation of matrix , where $\underline{X}$ is a matrix and *{ . . . }* is a column vector.

$$\underline{dV/dt} = \{ dV_2/dt\,, dV_3/dt\,, dV_4/dt \} = \underline{A}\{ V_2, V_3, V_4\} + (Vt/\tau_1)\{ \beta, \gamma, \delta \} \tag{10}$$

$$\underline{A} = \begin{pmatrix} (\beta y_2 - 1/\tau_2),\ \beta y_3,\ \beta y_4 \\ \gamma y_2, (\gamma y_3 - 1/\phi_3),\ \gamma y_4 \\ \delta y_2,\ \delta y_3, (\delta y_4 - 1/\tau_4) \end{pmatrix}$$

where

$$y_j = 1/\tau_j - 1/\tau_1. \tag{11}$$

Eigen value[4)] λ of (10) which corresponds to V(t)= Vexp(λ t) is obtained by *det* ($\underline{A}$ - λ $\underline{E}$)=0 , where $\underline{E}$ is unit matrix. λ is determined by (12) with using of (3).

$$\lambda^3 + \lambda^2/\tau_0 + \omega\lambda + (\alpha\tau_1 + \beta\tau_2 + \gamma\tau_3 + \delta\tau_4)/(\tau_1\tau_2\tau_3\tau_4) = 0\,, \tag{12}$$

where

$$1/\tau_0 = (1-\alpha)/\tau_1 + (1-\beta)/\tau_2 + (1-\gamma)/\tau_3 + (1-\delta)/\tau_4,$$

and

$$\omega = [\alpha\tau_1(\tau_2+\tau_3+\tau_4) + \beta\tau_2(\tau_3+\tau_4+\tau_1) + \gamma\tau_3(\tau_4+\tau_1+\tau_2) + \delta\tau_4(\tau_1+\tau_2+\tau_3)]/(\tau_1\tau_2\tau_3\tau_4) \tag{13}$$

The resistance of photoreceptor is usually very large.

Therefore, we apply an approximation of $\tau_1 \to \infty$, and then above (12) and (13) become simply, as follows.

$$\lambda^3 + \lambda^2/\tau_0 + \omega\lambda + \alpha/(\sigma_2\tau_3\tau_4) = 0\,, \tag{14}$$

$$1/\tau_0 = (1-\beta)/\tau_2 + (1-\gamma)/\tau_3 + (1-\delta)/\tau_4, \tag{15}$$

$$\omega = [\alpha(\tau_2+\tau_3+\tau_4) + \beta\tau_2 + \gamma\tau_3 + \delta\tau_4)]/(\tau_2\tau_3\tau_4) \tag{16}$$

There is a next relation among the each coefficient of λ in (12) and (14).

$$1/\tau_0 \propto \tau^{-1} \gg \omega \propto \tau^{-2} \gg \text{const. term} \propto \tau^{-3}\,, \tag{17}$$

As the constant term of (14) is very small, next approximation is applied to obtain the solutions of (14).

$$\text{Solution of } \lambda \cong 0: \quad \lambda(0) = -\alpha/\omega(\tau_2\tau_3\tau_4) \equiv -\varsigma$$

$$\text{Solution of } \lambda \gg 0: \quad \lambda^2 + \lambda/\tau_0 + \omega = 0,$$

$$\lambda(-) = -\omega\tau_0 \equiv -\mu$$

$$\lambda(+) = -(1/\tau_0 - \omega\tau_0) \equiv -\kappa\,, \tag{18}$$

Here, we used a relation of $(1-x)^{1/2} \cong (1-x/2)$. Regarding to the solutions of λ in the case of 3-layers structure of photoreceptor, toner and paper (; $\delta = 0,\ \tau_4 \to \infty$) , ς becomes 0, and κ and μ are given by a similar combination of (15), (16) and (18). Therefore, it seems that the above approximation is not irrelevance.

Eigenvectors for κ , μ and ς are denoted by $\underline{a}$, $\underline{b}$ and $\underline{c}$, respectively. The $\underline{a}$ is obtained by $\underline{A}\underline{a} = \lambda(+)\underline{a}$ of (11) and (18). Let the eigenvector $\underline{a}$ denote by $\{ a_1, a_2, a_3 \}$, the values of a_1 , a_2 and a_3 are determined by a relation of ratio as follows, $(a_1 : a_2 : a_3) = (1 : h : j)$. Similarly, $\underline{A}\underline{b} = \lambda(-)b$, and we describe (b1 : b2 : b3)= (1 : k : m) for b , and (c1 : c2 : c3)= (1 : n : p) for c . A set of (h , k , n), and (j , m , p) are expressed by column vectors.

$$\{ h, k, n \} = (\gamma\tau_3/\beta\tau_2)\{ (1-\kappa\tau_2)/(1-\kappa\tau_3),\ (1-\mu\tau_2)/(1-\mu\tau_3),\ (1-\varsigma\tau_2)/(1-\varsigma\tau_3) \}, \tag{19}$$

$$\{ j, m, p \} = (\delta\tau_4/\beta\mu_2)\{ (1-\kappa\tau_2)/(1-\kappa\tau_4),\ (1-\mu\tau_2)/(1-\mu\tau_4),\ (1-\varsigma\tau_2)/(1-\varsigma\tau_4) \}, \tag{20}$$

From these calculations, Vj (t) is given by next equation on the approximation of $\tau_1 \to \infty$.

$$\underline{V(t)} = \{ V_1(t), V_2(t), V_3(t), V_4(t) \} = B_2 \exp(-\kappa t)\{ -1-h-j, 1, h, j \} + B_3 \exp(-\mu t)\{ -1-k-m, 1, k, m \} + B_4 \exp(-\varsigma t)\{ -1-n-p, 1, n, p \} + Vt\{ 1,0,0,0 \}, \tag{21}$$

Unknown coefficients B_2 , B_3 and B_4 are determined by conservative law of the interface charge at the instant (t=0) when Vt is applied. We suppose that at the time $t= 0-$, 4-layers structure is formed , however Vt= 0 , and Vt $\neq 0$ (constant) at $t= 0+$. As the charge of each layer is Qj(0-) = CjUj(0-) at t=0-, and Qj(0+)= CjVj(0+) at t=0+, respectively. The conservative law of the interface charge is described by next equations.

$$U_k(0-)/D_k - U_{k+1}(0-)/D_{k+1} = V_k(0+)/D_k - V_{k+1}(0+)/D_{k+1}\ , \quad k=1\sim3, \qquad (22)$$

From solving (22), next relations are obtained.

$$V_j(0+) = U_j(0+)\ ,\ j = 1\sim4\ , \qquad (23)$$

Let a matrix denote $\underline{x} = (\underline{a}, \underline{b}, \underline{c})$, and then , $\underline{x}$ and the reciprocal matrix $\underline{x}^{-1}$ are described as follows.

$$\underline{x} = \begin{pmatrix} 1, 1, 1 \\ h, k, n \\ j, m, p \end{pmatrix}, \quad \text{and } \underline{x}^{-1} = 1/\theta \begin{pmatrix} kp - mn, & m-p, & n-k \\ jn - hp, & p-j, & h-n \\ hm - kj, & j-m, & k-h \end{pmatrix}$$

$$\theta = h(m-p) + k(p-j) + n(j-m), \qquad (24)$$

Next relation is obtained by (21), (23) and (24).

$$\underline{Uj(0+)} = \{U_2, U_3, U_4\} = \underline{x}\{B_2, B_3, B_4\}$$
$$\therefore \{B_2, B_3, B_4\} = \underline{x}^{-1}\{U_2, U_3, U_4\}, \qquad (25)$$

Time evolution of Vj (t) in (21) is given uniquely with (25). For the voltage distributions ϕ j (x, t), next equations are required, where Δ Vj(t) = Vj(t) - Uj(0+), and z is an influence factor for toner charge density due to the application of Vt.

$$\phi_1(x, t) = \phi_1(x) + \Delta V_1 x/d_1,$$
$$\phi_2(x, t) = \phi_2(x) - z\tau V_2(x-x_2)^2/d_2^2 + (1-z)\Delta V_2(x-x_2)/d_2 + \Delta V_1 + \Delta V_2,$$
$$\phi_3(x, t) = \phi_3(x) + \Delta V_3(x - x_3)/d_3 - \Delta V_4,$$
$$\phi_4(x, t) = \phi_4(x) + \Delta V_4(x - x_4)/d_4,\ \Sigma\ \Delta Vj = 0, \qquad (26)$$

Equations of (26) show unification of space and time dependence on the formulation of electrostatic transfer process.

We consider an abrupt change of *Vt off state* from Vt = constant voltage to Vt = 0 at a time t=to , where t_o is called as transfer time , in a similar way to (22). Result is shown in (27) .

$$\underline{V'(t)} = \{V'_1(t), V'_2(t), V'_3(t), V'_4(t)\}$$
$$= \{V_1(t) - Vt, V_2(t), V_3(t), V_4(t)\}$$
$$- G\exp(-\kappa t')\{-1-h-j, 1, h, j\}$$
$$- L\exp(-\mu t')\{-1-k-m, 1, k, m\}$$
$$- M\exp(-\varsigma t')\{-1-n-p, 1, n, p\}, \qquad (27)$$

where

$$\{G, L, M\} = Vt\,\underline{x}^{-1}\{\beta, \gamma, \delta\}, \text{ and } t' = t - t_0.$$

According to (26), the ideal transfer efficiency (5) at the time t_o is also modified as follows.

$$\eta(t_0) = [\eta o\,\rho\, d_2D_2 - (1-z)\Delta V_2(t_0)] / [\rho d_2D_2 + 2z\Delta V_2(t_0)] \quad (28)$$

Numerical Evaluation and Discussion

Numerical values used here are shown in Table 1.

Vt =1000 V is applied , and Vo= -650V, Va= -100V, and ρd2 (D1+ D2 /2)= -300V are used.

Table 1. Numerical values for Transfer Process

	photoreceptor	toner	paper	belt
d j (μm)	17	13.2	80	630
ε j (ε_o)	3	1.7	2.5	11
τ j (msec)	27×10^3	830	9.3	390

$\varepsilon_o = 8.854 \times 10^{-12}$ (C/Vm)

In the application of above values, (κ, μ, ζ) are (0.075, 1.53×10^{-3} , 1.57×10^{-4})$msec^{-1}$ on the toner image area, which correspond to relative humidity 50%. For the non-image area, τ'_2 (air) → ∞ and (κ', μ') are (0.082, 6.31×10^{-4}) $msec^{-1}$.

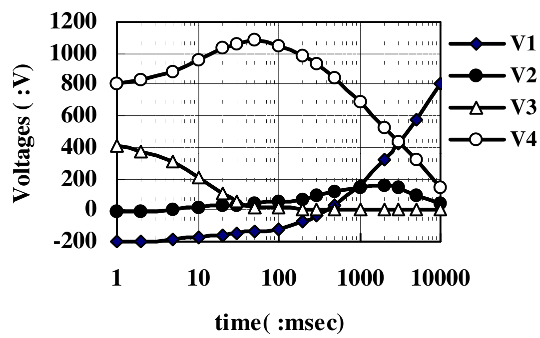

Figure 2. Time dependence of Layer Voltage on Toner image area. V1: Photoreceptor, V2: Toner layer, V3: Paper, V4: Transfer Belt. Vt =1000V is applied.

Vj(t) of the toner image area is shown in Fig. 2. It is found out that V2(t) and V4(t) show maxima, and V3(t) of the paper layer decreases and quickly reaches to 0 V. In a condition of high humidity, the maxima of V2(t), V4(t), and the decreasing of V3(t) occur in shorter time. Therefore, transfer efficiency (28) becomes quickly higher value in high humidity, however the value also depends on the transfer time to.

Electric field *E2* of the toner layer is evaluated from (26). E2(x) changes linearly with x from E2(x_1)= -2.2 V/μ m to E2(x_2) = -20.7V/μ m at z =0 and Δ V_2(max)= 168V in Fig.2. The value of E2(x_1) is comparable with a detachment field of toner from substrate [5]. Electric field E* of air bulb in continuous medium [6] is enhanced within a range of E*≦3E2/2. Electric field of PD in a small air gap [7] is about 75 V/μ m. Therefore, PD in fine air bulbs in the toner layer may occur in $E_2 \geq 50$ V/μm, and PD seems to be difficult in the toner image area.

In Fig. 3, layer voltages for non-image white area are shown. $V'_2(t)$ of the air layer increases with time slowly and exceeds about 400V at 220msec. It is predicted that PD occurs in the condition of $V(air) \geq 312+6.2d$ (μ m) volt [7]. At present case, as air gap d equals to the thickness of the toner layer, an abrupt change of $V'_2(t)$ due to PD may take place at this instance.

Accordingly, it is considered in a scheme of continuous model that PD on the transfer process occurs in the non-image white area rather than the toner image area, and edge parts of the toner image area are distorted and blurred dominantly[8] as a result of PD of the non-image area.

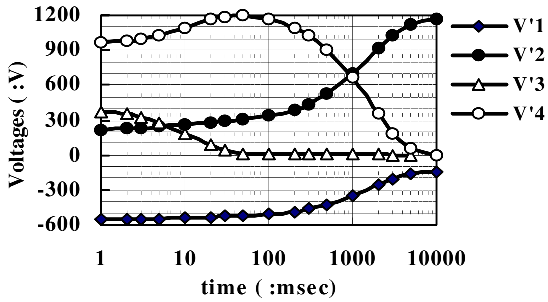

Figure 3. Time dependence of Layer Voltage on Non-image white area. V'1: Photoreceptor, V'2: Air layer, V'3: Paper, V'4: Transfer Belt. Vt =1000V is applied.

Fig. 4 and 5 show *Vt off state* at to≥300msec from Vt=1000V to Vt=0 V. In this case, the voltages of paper layer show large overshoot at to. Each voltage in the toner image area decreases to 0 volt with time in Fig. 4. However, residual voltages in the photoreceptor and the air layer are shown in the non-image white area of Fig.5.

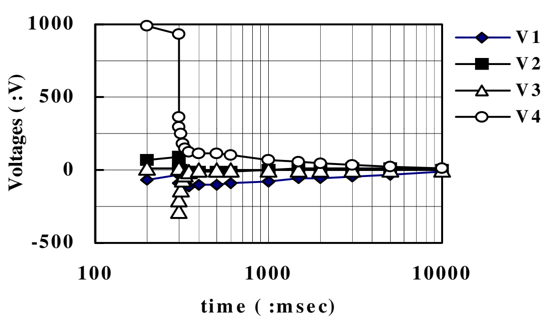

Figure 4. Time dependence of Layer Voltage for Vt Off State on Toner image area. V1: Photoreceptor, V2: Toner layer, V3: Paper, V4: Transfer Belt. Vt= 0 V in t ≥300msec.

In a separating of the paper from the photoreceptor after the transfer process, if we suppose the charge Q is constant in the air layer, PD occurs at the condition of V'= $(Q/\varepsilon_o S)d'=312+6.2d'$. It is considered that PD in the separating is also related to the non-image white area, even if Vt= 0 V, because of the residual voltages.

Usually, transfer efficiency is measured by large solid image area. The efficiency gives rise to a peak and decreases with increasing Vt.[1] This is not explained by (28), but may be affected by the overshoot of paper voltage on the Vt off state, especially in high Vt. It is also considered that the phenomenon is different from the blurring of small image area in low Vt and should be resolved on the basis of micro structure[9] and adhesive force[5] of the toner layer.

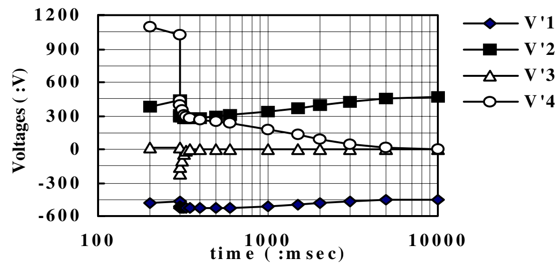

Figure 5. Time dependence of Layer Voltage for Vt Off State on Non-image white area. V'1: Photoreceptor, V'2: Air layer, V'3: Paper, V'4: Transfer Belt. Vt= 0 V in t ≥300msec.

Conclusion

Voltage changes for position and time on the transfer process have been solved rigorously with one-dimensional continuous model by using Poisson's equations and a set of 4-differential ones. Each voltage is given by a combination of time-dependent 3 exponential terms and as a function of dielectric thickness (D_j), time constant (τ_j), transfer voltage (Vt) and transfer time (t_o).

It is pointed out that Paschen's discharge of the air layer in the non-image white area easily occurs and the discharge results in the blurring of the edge parts of the toner image area neighboring to the non-image area. Transfer efficiency is also formulated, but the explanation of decreasing of the efficiency in high Vt is difficult. Therefore, the analysis based on the micro structure of the toner layer is required for the efficiency.

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Biography

Yuji Furuya received his BS. M.S. and Ph. D. in Physics from Hokkaido University Japan in 1968, 1970 and 1977, respectively. Since 1974, he has worked R & D division of printer in Hitachi Koki Co. Ltd. His work has primarily focused on high resolution and high speed digital printing. He is a member of the Physical Society of Japan and the Imaging Society of Japan.
E-mail: furuya@hitachi-koki.co.jp