# **Linear Pixel Shuffling (II): An Experimental Analysis of Tone and Spatial Characteristics**

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## **Abstract**

The tone and spatial characteristics of halftone patterns can be altered significantly by the printing process. In the current study, different types of halftone patterns were examined both for their printed tone reproduction characteristics (dot gain) and for their printed spatial characteristics (noise power spectra). Linear Pixel Shuffling introduces unique spatial properties and provides a variety of controls over both the printed noise power spectrum and the degree on clustering in the halftone image.

## **Introduction**

The linear pixel shuffling (LPS) algorithm described in Part  $(I)$ <sup>1</sup> offers unique opportunities for controlling the spatial properties of digital halftones. The current report describes the noise power spectra of LPS halftones in comparison with other well known halftones. The printing process introduces both physical and optical dot gain, and the impact of the noise power spectrum of halftones on dot gain is examined. An experimental metric is developed that relates the noise power spectrum of halftones to the magnitude of their optical dot gain. The results show that the LPS halftone technique occupies a unique spatial domain between clustered dot halftones and higher frequency halftones such as Floyd-Steinberg and other blue noise systems.

## **Noise Power Spectra**

Figure 1 illustrates two LPS halftone patterns at a nominal dot area fraction of  $F_0$ =0.5. Image (A) is an LPS halftone used as a simple halftone mask, as described in Part  $(I)^1$ . Image (B) is the LPS technique used to order the address of pixels, followed by diffusion of the truncation error. Instead of propagating the error only in a forward direction, the LPS technique allows diffusion of error in all directions, as described previously.<sup>2</sup>

Figure 1 illustrates two patterns as they exist in computer files. In the computer they are assigned an addressability of 300 pixels per inch. The noise power spectra of these two virtual images are shown in Figures 2 and 3. The noise power is displayed as both the radial average spectrum and as a pictorial 2-D spectrum in which gray level is approximately Log(power). For comparison, lines at 1.18 cy/mm and 5.9 cy/mm have been added to Figure 2 to illustrate the limiting frequencies one can achieve at 300 dpi addressability and Fn=0.5 with clustered dot and a Bayer dispersed dot halftones. The noise power spectrum of a common Floyd-Steinberg at Fn=0.5 is shown as a dotted line in Figure 3.

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*Figure 1: (A) LPS hafltone mask and (B) LPS with error diffusion, both at Fn = 0.5.* 







*Figure 3: Radial average and 2-D noise power spectrums of the LPS halftone with error diffusion, at*  $F_n = 0.5$ .

Error diffusion algorithms are well known to change the distribution of noise power in a halftone.<sup>3</sup> The Floyd-Steinberg systems are higher in frequency (blue shifted) relative to a white noise system but are lower in noise relative to the limiting Bayer dispersed dot. Similarly, the LPS mask of Figure 2 is blue shifted relative to white noise but lower in frequency relative to Floyd-Steinberg. Addition of error diffusion to the LPS halftone slightly red shifts the pattern relative to the LPS mask, as is evident both by inspection of Figures  $1(A)$  and  $(B)$  and by examination of the noise power spectra. From these and many other examples, the authors have observed the noise power of LPS systems to be higher in frequency than clustered dots, but lower in frequency than Floyd-Steinberg, which is lower in frequency than the Bayer clustered dot. This same ordering is manifested in tone reproduction characteristics, as shown below.

#### **Relating Noise Power to Tone Reproduction**

Microdensitometry allows a direct measure of dot area fraction, F, and the average reflectance of the paper between the dots,  $R_n$ . Figure 4 illustrates this with data from LPS halftone masks.



*Figure 4:*  $R_{p}$  versus *F* for microdensitometry measurements of two *independently printed LPS halftone masks. The line is modeled with m = 0.7.* 

The variation of  $R_p$  with F is a manifestation of optical dot gain and can be modeled as described previously  $4$  and as shown in Equation (1)

$$
R_p = R_g \left[ P_{00} \cdot \left( 1 - \sqrt{R_{ink} / R_g} \right) + \sqrt{R_{ink} / R_g} \right] \tag{1}
$$

where  $P_{00}$  is the mean probability that light will emerge from the paper between the halftone dots after having entered the paper between the dots. The reflectance values of  $R_{g}$  and  $R_{ink}$ are of the unprinted paper and of the ink printed at  $\bar{F}=1$ .

In order to relate  $P_{00}$  to dot fraction, F, it is convenient to define pseudo-pixel regions in the printed image in terms of printer addressability. In the current study we have regions (pseudo pixels) that are 1/300 inch square. Ideally, there are two kinds of regions: those between ink and those under ink. As shown previously<sup>4</sup> the value of  $P_{00}$  can be modeled in terms of another probability,  $P_a$ , for light exiting a particular region of the paper after having entered the paper at that same region. The value of  $P_a$  is a constant independent of dot area fraction, F, and dependant only on the convolution of the paper PSF with the size of the region.

The amount of light that scatters out of the selected region is  $(1-P_a)$ . Some of this escaping light,  $P_b$ , emerges from the paper in another region between the dots. The value of  $P_b$  does depend on F, and the total probability,  $P_{\text{on}}$ , of light exiting the paper somewhere in the image, but between ink dots, is the sum of  $P_a$  and  $P_b(1-P_a)$ .

$$
P_{oo} = P_a + P_b (1 - P_a)
$$
 (2)

The value of  $P<sub>b</sub>$  has previously been shown to be modeled well by equation  $(3)$ .<sup>4</sup>

$$
P_{b} = (I - F)^{m} \tag{3}
$$

If the pseudo pixel regions with and without ink were randomly distributed around the selected region, then the value of  $P_b$  would be directly proportional to 1 - F, so m = 1. One would expect so called stochastic halftones, such as Floyd-Steinberg, to behave in this way. However, if the regions were highly clumped such that the nearest neighbors are more likely to be of the same type, then  $P_b$  would be larger than  $(1 - F)$ , and this requires  $0 < m < 1$ . A similar argument suggests the Bayer dispersed dot halftone should have a value of  $m > 1$ . In order to test this idea, values of  $R_p$ versus F were measured for a clustered dot, a Floyd-Steinberg, and a Bayer dispersed dot halftone. The results are shown in Figure 5. The data in Figures 4 and 5 were modeled by solid lines calculated by choosing a value for m and then applying equations  $(3)$ ,  $(2)$ , and  $(1)$  solved for  $R<sub>n</sub>$ . The values of m were adjusted to fit the data, and the resulting values of m are shown in the Figures.



*Figure 5: Rp versus F for (x) a 30 LPI clustered dot halftone, (o) A Floyd-Steinberg halftone, and (+) a Bayer Dispersed dot halftone. All were printed at 300 dpi printer addressablity. The corresponding solid lines were fit with m= 0.5, m=1.0, and m=1.1.* 

## **Conclusion**

It is evident from these results that the four halftone types in this study are in the order Clustered, LPS, Floyd-Steinberg, Bayer Dispersed with regard both to their spatial frequency for noise power and their characteristic value of m. The value of m can be interpreted as an index of the relative degree of clustering of addressable units in the printing process. While the physical spreading of ink relative to the size of the pseudo-pixel regions would also manifest itself in larger values of m, this effect is constant for a given printing system. Thus, it is clear that the degree of clumping associated with these halftone patterns, independent of dot gain, is related to the noise power spectra of the patterns. This means a pattern with a higher spatial frequency is expected to have less clustering. This is consistent with intuition, but the value of m provides a practical and measurable index for the degree of clustering characteristic of different halftone patterns. As discussed in Part  $(I)$ , some amount of clustering is desirable as printing engines achieves higher and higher addressability. This is because

the control of a single, isolated addressable unit of colorant becomes more difficult in higher addressable print engines. Thus a significant potential advantage of the LPS system is that while it is blue shifted relative to the clustered dot halftone it provides a greater degree of clustering and thus of control compared to Floyd-Steinberg systems.

## **References**

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#### **Acknowledgment**

This research was supported by a grant from Hewlett-Packard.

## **Biography**

Jonathan Arney is an associate professor at the Center for Imaging Science at Rochester Institute of Technology. His research interests include physical and optical characteristics of colorants on substrates, with particular emphasis in halftone systems. He serves as faculty advisor for the Student Chapter of IS&T and as a council member for the Rochester Chapter of IS&T. Jonathan received the IS&T Journal Award (Science) in 1990, the IS&T Service Award in 1995, and in 1996 he received the Raymond C. Bowman Award for contributions to education in the fields of Photographic and Imaging Science.