

Yule-Nielsen Effect and Ink-penetration in Multi-chromatic Tone Reproduction

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Abstract

A framework describing influences of ink penetration and Yule-Nielsen effect on the reflectance and tristimulus values of a halftone sample has been proposed. General expressions of the reflectance values and CIEXYZ tristimulus values have been derived. Simulations for images printed with two inks have been carried out by applying Gaussian type of point spread function (PSF). Dependence of Yule-Nielsen effect on the optical properties of substrate, inks, the dot geometry, ink penetration etc., have been discussed.

1. Introduction

Yule-Nielsen effect[1] which is also called optical dot gain refers to the fact that a photon which enters the ink dot can exit from the non-inked paper or vice versa due to light scattering in the substrate. This effect makes Murray-Davis equation an inaccurate approach to halftone images. Yule-Nielsen effect depends on the optical properties of the materials (paper, ink) and geometrical distribution of ink dots (resolution, size and shape). To account for the effect is practically interesting in graphic arts and has long been a research topic in theoretical, simulation and experimental perspectives [1-7]. However, so far the studies have mainly been focused on monochromatic images and little has been done for the multichromatic ones.

In addition to Yule-Nielsen effect, ink penetration is another important factor that influences the quality of tone reproduction. Ink penetration has two folds of optical effects [7]. First the reflectivity of the substrate under the ink dots becomes smaller than that of the clean one due to the presence of penetrating ink segment. Second the pure ink layer becomes thinner therefore more transparent to the light. It has been a common practice in modeling and simulations that the substrate is assumed to be clean and optically isotropic [2, 4]. However when there exists ink penetration, these assumptions are broken down and the substrate becomes a non-isotropic base of the image. Moreover the optical characteristic of ink penetrated paper depends on the distribution of the penetrating ink which is very difficult to obtain if not impossible. Therefore suitable approximations that simplify the problem is needed.

2. Model and Methodology

2.1. An Approximation for Ink Penetration

The point reflectance of a print at the point (x, y) is given by [4]

$$R(x,y) = T(x,y)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T(x',y')p(x-x',y-y')dx'dy' \tag{1}$$

$p(x-x', y-y')$ is usually called point spread function (PSF) which is the probability per unit area that a photon enters the surface of the paper at point (x', y') and then exits the paper at a point (x, y). It has been shown that for a full tone print (the substrate is covered by a solid ink layer), the effect of ink penetration can equally be described as there is an extra *ink* being inserted under the remaining ink layer (see Fig. 1) while the substrate as whole remains clean (not being ink penetrated at all)[8]. We apply this approach as an approximation to a halftone image and express the transmission function $T(x, y)$ as,

$$T(x,y) = 1 - (1 - T_1\gamma)D(x,y) \tag{2}$$

where function $D(x, y)$ is 1 if there is ink at (x, y) and 0 if there is no ink at (x, y). T_1 is the transmittance of the remaining ink layer and $\gamma = R'_g/R_g$ is the transmittance of the introduced *ink* layer, where R_g and R'_g are the reflectance values of the clean and inked substrate, respectively. The computation of γ has been reported earlier [9].

The advantage of this approximation is that whether the substrate is clean or being ink penetrated simulation goes in exactly the same way. Therefore models that were developed for cases without ink penetration can directly be applied. For example isotropic assumption of the PSF, $p(x, y) = p(\sqrt{x^2 + y^2})$ is valid, which simplifies the calculation a great deal. Finally, it is worth to note that this approach is strict only for a full tone print and is an approximation for the halftone image because it neglects the spatial variation of penetrating ink inside the paper which is seldom the same as the full tone one.

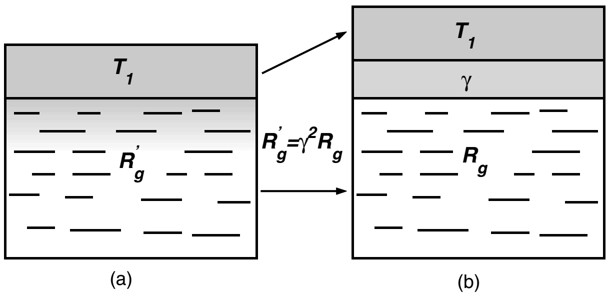


Figure 1: Equivalent descriptions of ink penetration. (a) remaining ink layer (transmittance T_1) and ink penetrated substrate (reflectance R_g'); (b) the ink penetrated substrate is replaced by an extra ink layer (transmittance γ) and a clean substrate (reflectance R_g), where $R_g' = \gamma^2 R_g$.

2.2. Yule-Nielsen Effect in Multichromatic Tone Reproduction

Fig. 2 is a side view of a halftone image with 2 inks. As shown in the figure, the substrate can be subdivided into 4 distinct regions and they are denoted as Σ_j (j=0,1,2,3). We define P_{ij} (i,j=0,..,3) as probabilities for a photon that exits the substrate from region Σ_j if it enters the substrate from Σ_i and ca be expressed as[8],

$$P_{ij} = \frac{1}{\sigma_i} \int_{\Sigma_i} \int_{\Sigma_j} p(x_i - x_j, y_i - y_j) dx_i dx_j dy_i y_j \quad (3)$$

where σ_i is the area of the region Σ_i. If the transmittance

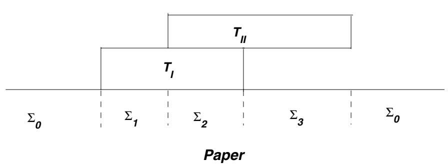


Figure 2: A side view of a multichromatic halftone image

of the ink layer and ink penetration of Σ_j (area σ_j) are T_j and γ_j, respectively, the overall reflectance of the image is given by [8]

$$R = \sum_{j=0}^{3} T_j^2 \gamma_j^2 R_g \sigma_j - \sum_{j=0}^{3} \sum_{i<j}^{3} (T_i \gamma_i - T_j \gamma_j)^2 P_{ji} \sigma_j \quad (4)$$

Eq. (4) can directly be generalized to any multichromatic image. Provided the image consists of N distinct ink areas, we have

$$R = R_{MD} - \Delta R \quad (5)$$

where

$$R_{MD} = \sum_{j=0}^{N-1} T_j^2 \gamma_j^2 R_g \sigma_j \quad (6)$$

Table 1: Transmittance of distinct ink regions

Regions	Σ_0	$\Sigma_1(\Sigma_3)$	Σ_2
Transmittance	1	$T_I(T_{II})$	$T_I T_{II}$

is the reflectance of the halftone image under Murray-Davis assumption, and

$$\Delta R = \sum_{j=0}^{N-1} \sum_{i<j}^{N-1} (T_i \gamma_i - T_j \gamma_j)^2 P_{ji} \sigma_j \quad (7)$$

is the term corresponding to Yule-Nielsen effect. From these one can draw such a conclusion that Yule-Nielsen effect is a general phenomenon in tone reproduction. Because ΔR is a non-negative quantity, Eq. (5) means that the real reflectance of the halftone image, R, is smaller than that predicted by Murray-Davis assumption.

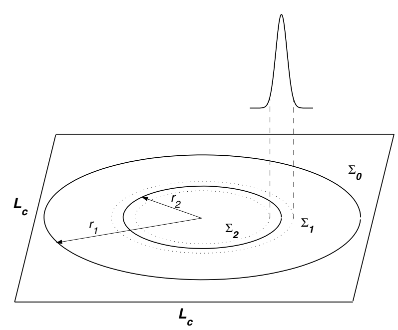


Figure 3: A systematic diagram for point spread function and dot geometry

3. Simulations and Examples

As applications of the present model, simulations for images consisting of two inks have been carried out. Gaussian type of point spread function has been applied in the simulation, which is of the form

$$p(x_i - x_j, y_i - y_j) = \kappa e^{-[(x_i - x_j)^2 + (y_i - y_j)^2]/\delta^2} \quad (8)$$

where κ is a factor of normalization. The optical properties of the substrate is characterized by a parameter δ. For generality, δ is defined relatively to the length of a halftone cell, L_C. For simplicity, we assume that the inks are printed one on top of the other (coaxial, or dot on dot) and round in shape. As shown in Fig. 3, the printed image consists of four distinct ink regions, the bare paper (Σ_0), two primary colors ($\Sigma_1(\Sigma_3)$) and one secondary color (Σ_2). The transmittance values corresponding to these areas are listed in Tab. 1. Fig. 4 is the reflectance computed under the Murray-Davis assumption (R_{MD}) which varies with respect to the ink dot areas, a and b, in a bi-linear

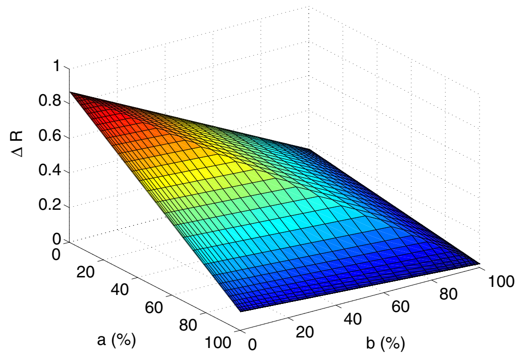


Figure 4: Computed R_{MD}, two inks, dot on dot, $R_g = 0.87$, $T_I = 0.35$, $T_{II} = 0.45$.

fashion. Therefore R_{MD} has a simple roof like structure with maximum along the line $a = b$.

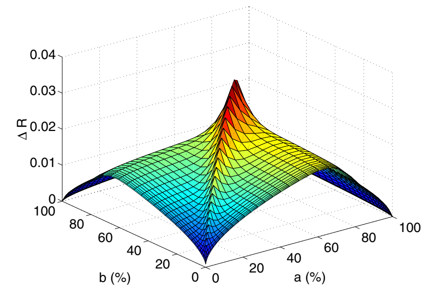
Fig. 5a)-d) correspond to the Yule-Nielsen effect with respect to different δ values. The following facts are observed,

1. ΔR has local maxima when the ink dots have identical sizes ($a = b$, i.e. they are completely overlapped with each other).

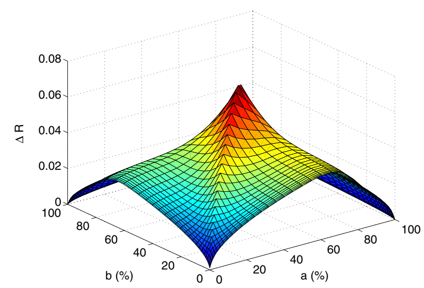
2. The local maxima become less prominent when the Gaussian parameter, δ, gets bigger or equivalently the PSF becomes broader and flatter.

3. The magnitude of ΔR becomes bigger when δ is bigger.

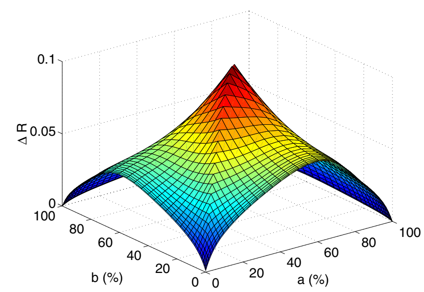
The appearance of the local maxima is a hybrid effect of the transmittance difference between adjacent regions and the effective extension of PSF. For simplicity of explanation, we assume that the ink 1 dot (Σ_1) has a fixed radius, r_1 (or area a) (see Fig. 3). Now we examine the process when the radius of the ink 2 dot (Σ_2), r_2, increases from $r_2 = 0$ to $r_2 = r_1$. Considering a photon enters the substrate at a point (x, y) in Σ_2 (secondary color region), the PSF that describes the probability of finding the photon in a point (x', y') becomes very small when $\sqrt[2]{(x-x')^2 + (y-y')^2} \geq 2\delta$. Therefore only when the photon hits a point inside the region marked by dot line circles, there is remarkable probability to find the photon in the adjacent area (Σ_1, primary color region). Consequently if $r_2 \ll r_1$, there is little chance for the photon to exit from the noninked area (Σ_0). However this possibility becomes much larger when $r_2 \approx r_1$ (or $a \approx b$). Considering the fact that ΔR is proportional to $(T_i - T_j)^2$ (see Eq. (7), assuming $\gamma_i = \gamma_j = 1$ for simplicity), ΔR has the biggest value between $T_0 = 1$ (the bare paper) and $T_2 = T_I T_{II}$ (the secondary color). Therefore ΔR increases when r_2 is approaching r_1. However after $r_2 = r_1$, if r_2 further increases ($r_2 \geq r_1$), ΔR falls again. Thus



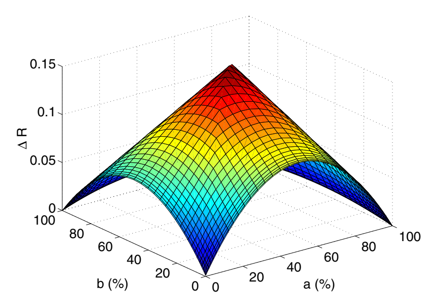
(a) $\delta = 4\% L_c$



(b) $\delta = 8\% L_c$



(c) $\delta = 14\% L_c$



(d) $\delta = \infty$

Figure 5: Computed ΔR with respect to the dot areas. Two inks, dot on dot, L_c length (width) of a halftone cell. $R_g = 0.87$, $T_I = 0.35$, $T_{II} = 0.45$.

ΔR reaches its maximum when $a = b$. This explanation is consistent with the fact No. 2. When δ gets bigger, the PSF becomes broader and flater. Correspondingly the area marked by the dot line circles gets wider and therefore the local maximum of ΔR becomes broader and (relatively) less prominent, even though its absolute quantity increases. Because there is bigger probability for the photon to enter the paper from one region and to exit from the other (or even others) when δ is big, Yule-Nielsen effect becomes stronger (i.e. bigger ΔR). An extreme case of it is that $\delta \to \infty$(see Fig. 5d)). In this case the PSF becomes constant over the whole image. Therefore the photon has equal probability to be found any where of the paper, no matter where the photon enters the paper and it is said the photon being "completely scattered"[4]. Then the local maxima disappear and only the global maximum remains at $a = b = 50\%$.

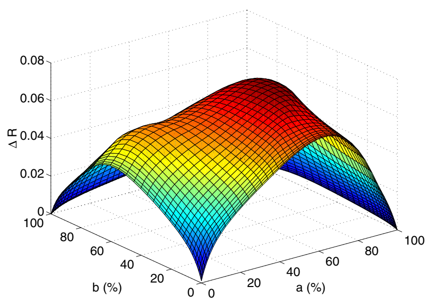


Figure 6: Computed ΔR for two inks, dot off dot (separation of the dot centers = $20\%L_c$), $\delta = 14\%L_c$, $R_g = 0.87$, $T_I = 0.35$, $T_{II} = 0.45$.

In the case that the centers of the dots are not identical, the shape of the regions (Σ_i) vary with respect to the distance between the centers and the sizes of the ink dots. Therefore ΔR has a more complicated shape as shown in Fig. 6.

4. Yule-Nielsen Effect on Color Reproduction

According to definition, for example, CIE X-stimulus value can be computed as

$$X = \int R(\lambda)S(\lambda)\overline{x}(\lambda)d\lambda \qquad (9)$$

where $S(\lambda)$ is the energy distribution of the illumination and $\overline{x}(\lambda)$ is the tristimulus function. To stress the dependence of reflectance value on the wavelength of the light, R is denoted as $R(\lambda)$. Substituting the $R(\lambda)$ by using Eq. (5), one can further express the X-stimulus as

$$X = X_{MD} - \Delta X \qquad (10)$$

where X_{MD} is the tristimulus value computed according to the Murray-Davis assumption and ΔX is due to Yule-Nielsen effect. They are expressed as,

$$X_{MD} = \int R_{MD}(\lambda)S(\lambda)\overline{x}(\lambda)d\lambda \qquad (11)$$

$$\Delta X = \int \Delta R(\lambda)S(\lambda)\overline{x}(\lambda)d\lambda \qquad (12)$$

If the X-stimulus value of the substrate is X_0, due to the non-negativity of ΔX, there is,

$$X_0 - X \geq X_0 - X_{MD} \qquad (13)$$

Therefore the present model predicts more saturated color than does by applying the Murray-Davis assumption. This is actually the chromatic consequence of Yule-Nielsen effect.

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