# Basic Characteristic of Ball Motion in Twisting Ball Display 

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#### Abstract

Twisting ball display is promising as a candidate for Digital Paper but ball behavior must be clarified. We carry out basic experiments into ball behavior by using enlarged scale model balls. Threshold electric fields for bi-stable motion and response time when switching ball direction are measured under various conditions. A theoretical study on the moments of ball rotation is introduced. Driving moment of coulomb force is shown to dependent on electric field strength, difference in surface electric charge densities on the hemispheres, and ball diameter. An equation that yields ball rotation is described that is based on the coulomb force.


## Introduction

The amount of digital information accessed continues to increase; the rapid spread of the Internet is clearly one of the major causes. People can choose softcopy or hardcopy to access digital information. Generally speaking, hardcopy offers ease of reading and simpler handling; softcopy offers the advantages of ease of digital processing and reuse. Digital Paper ${ }^{1}$ appears to be the ideal medium that combines the advantages of both hardcopy and softcopy. This study focuses on the twisting ball display method, which is a promising candidate for realizing Digital Paper. Ball motion is studies experimentally ${ }^{2-4}$ and theoretically to help this method's development.


Figure 1. Basic structure of twisting ball display.

## Principle of Twisting Ball Display

The principle of twisting ball display ${ }^{5,6}$ is shown in Figure 1. It consists of balls with black and white hemispheres that lie in a dielectric-liquid. Individual balls are held in cavities formed in a transparent dielectric polymer sheet. The balls can be rotated by setting electric fields across the sheet. Images are formed by setting the appropriate electric field pattern.

## Experimental Method

Ball behavior was observed using enlarged model balls that are 100 times larger than those anticipated to be used in practical display sheets. The experimental apparatus is shown in Figure 2. A white nylon ball (specific gravity 1.14) was colored using a black dielectric paint. It was placed in glass cell that held two liquids with different specific gravity. The balls floated on the horizontal boundary plane between the two liquids.


Figure 2. Experimental apparatus (side view).

Balls with diameters of $3.2 \mathrm{~mm}, 4.0 \mathrm{~mm}, 4.8 \mathrm{~mm}$ and 5.6 mm were used. The dielectric liquids were hydrocarbon: Exon Isopar-G (Specific gravity 0.75) and hydrofluoride: 3M PF-5052 (Specific gravity 1.70).

Table 1. Observed behavior of rotation.

| Voltage <br> $(\mathrm{kV})$ | diameter |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\phi 3.2 \mathrm{~mm}$ | $\phi 4.0 \mathrm{~mm}$ | $\phi 4.8 \mathrm{~mm}$ | $\phi 5.6 \mathrm{~mm}$ |
| 0.5 | none | none | none | none |
| 1 | slight movement | none | none | slight movement |
| 1.5 | unstable rotation | slight movement | unstable rotation | unstable rotation |
| 2 | bi-stable rotation | unstable rotation | unstable rotation | unstable rotation |
| 2.5 | bi-stable rotation | bi-stable rotation | bi-stable rotation | unstable rotation |
| 3 | bi-stable rotation | bi-stable rotation | bi-stable rotation | bi-stable rotation |
| 3.5 | over rotation | bi-stable rotation | bi-stable rotation | bi-stable rotation |
| 4 | over rotation | bi-stable rotation | bi-stable rotation | bi-stable rotation |



Figure 3. Minimum driving voltage for bi-stable rotation.
D.C. voltage was set between the pair of glass electrodes. Ball motion was recorded using a video camera and analyzed. Response time was defined as the time taken for a ball to rotate by 180 degrees. Measurements were repeated 10 times for each condition and the mean response time was calculated. Applied voltage ranged up to maximum 4.0 kV .

## Experimental Results

Observed ball behavior is shown in table 1. We found that voltages in excess of 3 kV drove the $3.2 \varphi$ ball into over rotation.

Minimum voltages necessary for bi-stable rotation are plotted in Figure 3. The threshold voltage tends to increase with ball diameter. Response time is plotted in Figure 4 for $3.2,4.0$, and $4.8 \varphi$ balls. The response time decreases as the voltage increases and saturates at a common value independent of ball size.

## Theoretical Considerations of Ball Rotation

Coulomb driving force was analyzed theoretically. Driving moment was derived from the coulomb force given by the surface charges on the balls; the surface charge density on the hemispheres ( $\boldsymbol{A}$ and $\boldsymbol{B}$ ) have different values ( $\sigma_{A}$ and $\sigma_{B}$ in Figure 5). These surface electric charges are derived


Figure 4. Response time of colored ball.
from the electrical double layer on the surface region of balls in the dielectric liquid. The driving moment is zero when the hemisphere boundary is perpendicular to the electric field $\boldsymbol{E}$ : the coulomb force created by the surface electric charges are cancelled in this case, see Figure 6. If the boundary is $\theta$ degrees off the perpendicular (see Figure 7), the projected area of the portions of the sphere within the angle of $\pm \theta$ contribute to an effective rotation moment.


Figure 5. Electrostatic charge distribution on a ball.


Figure 6. Case 1: Boundary is orthogonal to electric field.


Figure 7. Case 2: Boundary is not orthogonal to electric field.


Figure 8. Unit discs within ball: disc parallels electric field.


Figure 9. Rotation moment on unit disk.

We determine the driving moment for a unit disk taken from the sphere with unit thickness parallel to the driving electric field $\boldsymbol{E}$. The position of the disk is set by angle $\alpha$ as shown in Figure 8. The radius $\boldsymbol{r}$ of the disk is given by ball's radius $\boldsymbol{R}$ and angle $\alpha$ as follows:

$$
\begin{equation*}
r=R \sin \alpha \tag{1}
\end{equation*}
$$

Figure 9 shows a sliced unit disk parallel to the driving electric field. Moments $\Delta \boldsymbol{M}_{A}$ and $\Delta \boldsymbol{M}_{\boldsymbol{B}}$ are defined as unit moments for a unit angle at angle position $\psi$ and ( $\psi+\pi$ ), which correspond to hemisphere $\boldsymbol{A}$ and $\boldsymbol{B}$ respectively. Total moment is described as:

$$
\begin{equation*}
\Delta M=\Delta M_{A}-\Delta M_{B}=\boldsymbol{q}_{A} E \boldsymbol{E}-\boldsymbol{q}_{B} E \boldsymbol{E} \tag{2}
\end{equation*}
$$

In equation (2), charge values $\boldsymbol{q}_{A}, \boldsymbol{q}_{B}$ are defined as unit surface electric charges for the unit angle $\Delta \psi$ of the unit disk in hemispheres $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively.

Length h is determined as:

$$
\begin{equation*}
\boldsymbol{h}=\boldsymbol{r} \sin \psi \tag{3}
\end{equation*}
$$

When we define surface charge density for hemisphere $\boldsymbol{A}$ and $\boldsymbol{B}$ as $\sigma_{A}$, and $\sigma_{B}$ respectively, charge value $\boldsymbol{q}_{A}, \boldsymbol{q}_{B}$ are written using radius $\boldsymbol{r}$ as follows:

$$
\begin{equation*}
\boldsymbol{q}_{A}=\sigma_{A} \boldsymbol{r}, \boldsymbol{q}_{B}=\sigma_{B} \boldsymbol{r} \tag{4}
\end{equation*}
$$

Unit moment $\Delta \boldsymbol{M}$ is written as follows:

$$
\begin{equation*}
\Delta \boldsymbol{M}=\boldsymbol{E r}^{2}\left(\sigma_{A}-\sigma_{B}\right) \sin \psi \tag{5}
\end{equation*}
$$

The term $\left(\sigma_{A}-\sigma_{B}\right)$ represents the difference in surface electric charge densities of hemisphere $\boldsymbol{A}$ and $\boldsymbol{B}$. When we write $\sigma_{A}-\sigma_{B}=\sigma$, moment $\Delta \boldsymbol{M}$ is finally described as:

$$
\begin{equation*}
\Delta M=\boldsymbol{E r}^{2} \sigma \sin \psi \tag{6}
\end{equation*}
$$

The effective moment on the disk is the moment considered only within the following limited range of angle $\psi$ :

$$
\frac{\pi}{2}-\theta \leq \psi \leq \frac{\pi}{2}+0
$$

The total driving moment on unit disk $\boldsymbol{M}_{\text {disc }}$ is written as:

$$
\begin{equation*}
M_{d i s c}=\int_{\frac{\pi}{2}-\theta}^{\frac{\pi}{2}+\theta} E r^{2} \sigma \sin \Psi d \Psi \tag{7}
\end{equation*}
$$

The driving moment for the whole sphere is given by

$$
\begin{align*}
M & =\int_{0}^{\pi} M_{d i s c} d \alpha \\
& =\int_{0}^{\pi} \int_{\frac{\pi}{2}-\theta}^{\frac{\pi}{2}+\theta} E \sigma(R \sin x)^{2} \sin \psi d \psi d x  \tag{8}\\
& =\pi R^{2} \sigma E \sin \theta
\end{align*}
$$

Finally, the driving moment given by equation (8) is plotted as a sign curve in Figure 10. It is noted that moment $\boldsymbol{M}$ is proportional to $\sigma$ : the difference of surface charge densities for hemisphere $\boldsymbol{A}$ and $\boldsymbol{B}$, electric field $\boldsymbol{E}$, and $\boldsymbol{R}^{2}$ : the square of the ball's radius.


Figure 10. Rotation moment on a ball.


Figure 11. Moment pairs on unit disc.

When we consider viscous drag moment $\boldsymbol{M}_{\text {drag }}$, rotation will occur in the following condition:

$$
\begin{equation*}
\mathbf{M}-\mathbf{M}_{\text {drag }}>\mathbf{0} \tag{9}
\end{equation*}
$$

The motion equation for a ball is written using angular speed $\omega$ and inertia moment I as

$$
\begin{equation*}
M-M_{d r a g}=I \frac{d \omega}{d t} \tag{10}
\end{equation*}
$$

where, I is determined using the specific gravity of the sphere $\rho$ as,

$$
\begin{equation*}
I=\frac{8}{15} \rho R^{5} \tag{11}
\end{equation*}
$$

$\boldsymbol{M}_{\text {drag }}$ is written, using viscous resistance rate $\eta$ of the liquid and a constant value $\boldsymbol{k}$, as:

$$
\begin{equation*}
\boldsymbol{M}_{\text {dragg }}=k \boldsymbol{R}^{2} \eta \omega \tag{12}
\end{equation*}
$$

Equation (10) indicates that minimum electric field for ball rotation depends ball diameter as seen in Figure 3. The saturation in response time (see Figure 4) also can be explained by equation (10).

## Summary

(1) The behavior of ball rotation due to imposed electric field was clarified experimentally using enlarged scale model balls.
(2) Driving moment was analyzed theoretically and found to be proportional to the difference in surface electric charge density on the hemispheres, driving electric field, and the square of ball diameter. A motion equation for ball rotation was developed using this driving moment.
(3) The ball behavior observed in the experiments can be explained by the results of the theoretical analysis.

Further study should be done on the correspondence between the experimental results and theoretical analyses with close examination at practical dimensions.

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## Biography

Tomohiro Tanikawa was born in 1977. He received his B.S. degree in 1999 from Tokai University. He is expected to receive his M.S. degree from Graduate School of Tokai University in 2001. He is now engaged in a study of twisting ball display at Tokai University.

