Orthogonal Illuminant Model and Its Application

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Abstract

This paper discusses about the most different illuminant (termed the orthogonal illuminant) compared with a reference illuminant. This is the first report on the definition of the orthogonal illuminant and on the solution of the optimization problem. As an application, the orthogonal illuminant is applied to derive the optimized spectral reflectance for color constancy. No reports have appeared about the optimization for color constancy because of the lack of such models as the orthogonal illuminant. In experiments, the solution of the orthogonal illuminant and the solution of the optimized spectral reflectance for color constancy are indicated and discussed.

1.Introduction

This paper discusses about the most different illuminant (termed the orthogonal illuminant) compared with a reference illuminant based on metameric color-mismatch volume.

Metameric colors (metamers) are color stimuli with the same tristimulus values but different spectral radiation power distributions. One of the most important applications of a set of metamers generated with respect to a given illuminant and observer is to the determination of the magnitude of the color mismatches that will occur when the illuminant or the observer is changes. There have been studies[1)] of the boundaries of mismatches of metamers by N. Ohta and G. Wyszecki, but no reports have appeared about which illuminant yields the largest magnitude of color-mismatch volume. The illuminant which yields the largest color-mismatch volume is termed the orthogonal illuminant. The color-mismatch volume corresponds to the degree of difference.

This is the first report defining of the orthogonal illuminant and the solution to the optimization problem. The main subjects of this paper are the definition of the orthogonal illuminant and the derivation of the orthogonal illuminant by solving the optimization problem whose cost function is the volume of the color mismatch. Mismatch coordinates are known to form a closed solid in a color space. Linear programming is employed to calculate the volume of the solid. It is difficult to derive a solution that maximizes the volume of the solid by analytical methods. Hence, a search method for optimization called simulated annealing[2)] is employed.

In experiments, the orthogonal illuminant is derived for an illuminant of the completely flat spectrum: the ideal white illuminant. The orthogonal illuminant can be applied to derive the optimized spectral reflectance for color constancy, and its experimental results are also provided.

2.Orthogonal Illuminant Model and Application

2.1.Orthgonal Illuminant Model

Two objects with different spectral reflectance functions $\rho(\lambda)$ and $\rho'(\lambda)$ give rise to metamer stimuli when illuminated by $s(\lambda)$ if their corresponding tristimulus values X,Y,Z and X',Y',Z' are equal as follows:

$$
\begin{aligned}
&\sum_{\lambda} s(\lambda)\rho(\lambda)\bar{x}(\lambda) / \sum_{\lambda} s(\lambda)\bar{y}(\lambda) \\
&\quad = \sum_{\lambda} s(\lambda)\rho'(\lambda)\bar{x}(\lambda) / \sum_{\lambda} s(\lambda)\bar{y}(\lambda)\,, \\
&\sum_{\lambda} s(\lambda)\rho(\lambda)\bar{y}(\lambda) / \sum_{\lambda} s(\lambda)\bar{y}(\lambda) \\
&\quad = \sum_{\lambda} s(\lambda)\rho'(\lambda)\bar{y}(\lambda) / \sum_{\lambda} s(\lambda)\bar{y}(\lambda)\,, \\
&\sum_{\lambda} s(\lambda)\rho(\lambda)\bar{z}(\lambda) / \sum_{\lambda} s(\lambda)\bar{y}(\lambda) \\
&\quad = \sum_{\lambda} s(\lambda)\rho'(\lambda)\bar{z}(\lambda) / \sum_{\lambda} s(\lambda)\bar{y}(\lambda)\,.
\end{aligned}
\tag{1}
$$

where λ: wavelength.

Under the first illuminant (reference illuminant), the metameric match is described as follows, where $\left(X^{(1)},Y^{(1)},Z^{(1)}\right)$ is the coordinate of the metameric match for different values of $\rho(\lambda)$,

$$
\begin{aligned}
X^{(1)} &= \sum_\lambda s^{(1)}(\lambda)\rho(\lambda)\bar{x}(\lambda) / \sum_\lambda s^{(1)}(\lambda)\bar{y}(\lambda) \\
&= \sum_\lambda s^{(1)}(\lambda)\rho'(\lambda)\bar{x}(\lambda) / \sum_\lambda s^{(1)}\bar{y}(\lambda), \\
Y^{(1)} &= \sum_\lambda s^{(1)}(\lambda)\rho(\lambda)\bar{y}(\lambda) / \sum_\lambda s^{(1)}(\lambda)\bar{y}(\lambda) \\
&= \sum_\lambda s^{(1)}(\lambda)\rho'(\lambda)\bar{y}(\lambda) / \sum_\lambda s^{(1)}\bar{y}(\lambda), \\
Z^{(1)} &= \sum_\lambda s^{(1)}(\lambda)\rho(\lambda)\bar{z}(\lambda) / \sum_\lambda s^{(1)}(\lambda)\bar{y}(\lambda) \\
&= \sum_\lambda s^{(1)}(\lambda)\rho'(\lambda)\bar{z}(\lambda) / \sum_\lambda s^{(1)}\bar{y}(\lambda),
\end{aligned}
\qquad (2)
$$

where

$$\rho(\lambda) \neq \rho'(\lambda).$$

When the illuminant is changed from the first one $s^{(1)}(\lambda)$ to the second $s^{(2)}(\lambda)$, the corresponding tristimulus values are given by,

$$
\begin{aligned}
X^{(2)} &= \sum_\lambda s^{(2)}(\lambda)\rho(\lambda)\bar{x}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda), \\
Y^{(2)} &= \sum_\lambda s^{(2)}(\lambda)\rho(\lambda)\bar{y}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda), \\
Z^{(2)} &= \sum_\lambda s^{(2)}(\lambda)\rho(\lambda)\bar{z}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda),
\end{aligned}
\qquad (3)
$$

$$
\begin{aligned}
X^{(2)'} &= \sum_\lambda s^{(2)}(\lambda)\rho'(\lambda)\bar{x}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda), \\
Y^{(2)'} &= \sum_\lambda s^{(2)}(\lambda)\rho'(\lambda)\bar{y}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda), \\
Z^{(2)'} &= \sum_\lambda s^{(2)}(\lambda)\rho'(\lambda)\bar{z}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda).
\end{aligned}
\qquad (3)'
$$

Here the metamerism is broken down and spread out,

$$\left(X^{(2)}, Y^{(2)}, Z^{(2)}\right) \neq \left(X^{(2)'}, Y^{(2)'} Z^{(2)'}\right).$$

It is known that the mismatch coordinates form a closed solid in a color space. We define the illuminant which makes the magnitude of the volume of the solid the largest against a reference illuminant as the orthogonal illuminant. Linear programming is employed to calculate the volume of the solid. It is difficult to derive the solution maximizing the volume of the solid by analytical methods. Hence, a search method for optimization called simulated annealing is employed.

The closed solid can be derived using the linear programming method in which eq.(2) and $0 \le \rho(\lambda) \le 1$ are the constraints, and eq.(3) is the objective function. The term $\left(X^{(1)}, Y^{(1)}, Z^{(1)}\right)$ is a metameric color which is fixed, and for various values of $\rho(\lambda)$, $\left(X^{(2)}, Y^{(2)}, Z^{(2)}\right)$ assumes mismatch values by changing the illuminant from $S^{(1)}(\lambda)$ to $S^{(2)}(\lambda)$.

[Linear programming formulation for the problem]

Constraints

$$0 \le \rho(\lambda) \le 1. \qquad (4.a)$$

$$
\begin{aligned}
X^{(1)} &= \sum_\lambda s^{(1)}(\lambda)\rho(\lambda)\bar{x}(\lambda) / \sum_\lambda s^{(1)}(\lambda)\bar{y}(\lambda), \\
Y^{(1)} &= \sum_\lambda s^{(1)}(\lambda)\rho(\lambda)\bar{y}(\lambda) / \sum_\lambda s^{(1)}(\lambda)\bar{y}(\lambda), \\
Z^{(1)} &= \sum_\lambda s^{(1)}(\lambda)\rho(\lambda)\bar{z}(\lambda) / \sum_\lambda s^{(1)}(\lambda)\bar{y}(\lambda).
\end{aligned}
\qquad (4.b)
$$

Objective function

$$
\begin{aligned}
X^{(2)} &= \sum_\lambda s^{(2)}(\lambda)\rho(\lambda)\bar{x}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda), \\
Y^{(2)} &= \sum_\lambda s^{(2)}(\lambda)\rho(\lambda)\bar{y}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda), \\
Z^{(2)} &= \sum_\lambda s^{(2)}(\lambda)\rho(\lambda)\bar{z}(\lambda) / \sum_\lambda s^{(2)}(\lambda)\bar{y}(\lambda).
\end{aligned}
\qquad (5)
$$

The volume of the closed solid corresponds to a degree of difference between the first illuminant and the second illuminant, and we optimize the second illuminant maximizing the volume.

The optimized illuminant is the orthogonal illuminant.

2.2.Application

As one of applications, the orthogonal illuminant is applied to derive the optimized spectral reflectance for color constancy. The smaller the distance between the tristimulus coordinates generated by using an orthogonal illuminant pair, the more constancy kept.

3.Solution

We optimize the second illuminant maximizing the volume using simulated annealing[2]. In simulated annealing, $s^{(2)}(\lambda_i)$, $(i = 1,2,\cdots,n)$ quantized in the spectrum range are n dimensional parameters to be optimized, where n indicates the number of spectrum values. In simulated annealing process, reconfiguration of parameters $S^{(2)}(\lambda_i)$, $(i = 1,2,\cdots,n)$ is performed and for each reconfiguration acceptance or nonacceptance is determined. In simulated annealing, the reconfiguration and determination of acceptance are repeated, and the final state of the reconfiguration is the optimized solution. The following function ΔV is defined for the judgement of acceptance or nonacceptance of a reconfiguration.

$$
\begin{aligned}
\Delta V = &(V \text{ value after reconfiguration}) \\
&-(V \text{ value before reconfiguration}),
\end{aligned}
\qquad (6)
$$

where V indicates the volume of a mismatch color solid calculated using linear programming described previously.

If ΔV increases, the reconfiguration is accepted. If ΔV decreases, the reconfiguration is accepted based on the probability of $p_a = \exp(-\Delta V / T)$ and rejected based on the probability of $p_r = 1 - \exp(-\Delta V / T)$, where T indicates the temperature of the annealing process. The larger the value of T, in other words, the higher the temperature, the more easily the reconfiguration is accepted; the smaller the value of T, in other words, the lower the temperature, the more difficult is the acceptance of reconfiguration. These are simulations of the annealing process. The reconfiguration of parameters is performed in descending the temperature using ΔV and the probability distribution as a reference. Local minima can be avoided using a probability distribution, and with decreasing temperature, the global optimum can be attained. The T value is reduced in decrements of ΔT down to 0. The reduction of ΔT is performed when the variation of the cost function V is the noise : the equilibrium state. In the repetition of the reconfigurations as temperature decreases, when the temperature reaches 0, the reconfigured state of $S^{(2)}(\lambda_i)$, $(i = 1,2,\cdots,n)$ is the final optimized solution. We can obtain the solution of the orthogonal illuminant by this procedure.

Simulated annealing is also applied to the application described in section 2.2. In the application, reconfiguration of $\rho(\lambda_i)$, $(i = 1,2,\cdots,n)$ are performed to minimize the distance between tristimurus coordinates of an orthogonal illuminant pair.

4.Experiments

Experiments deriving the orthogonal illuminants were performed. In the experiments, an illuminant of the completely flat spectrum was employed as the reference illuminant. The metameric match was on the x, y color coordinate of the illuminant. In these experiments, the wavelength range 400nm-700nm was divided into eight sections. In the simulated annealing process, the cost function was the mismatch volume. The initial temperature was $T = 1$ and $\Delta T = 1/10^4$. The reconfiguration based on $S^{(2)}(\lambda_i)$, $(i = 1,2,\cdots,n)$ was performed using random numbers. The random numbers determined that which section i should be reconfigured and if the modified value $\Delta S^{(2)}(\lambda_i)$ should be positive or negative. The step size of the modification was $|\Delta S^{(2)}(\lambda_i)| = 1/10^2$. The reconfiguration is performed keeping the Y value of the illuminant $S^{(2)}$ equals to the Y value of the illuminant $S^{(1)}$ (=100.0).

Figures 1 shows the experimental results. Figure 1(a) shows the reference illuminant and figure 1(b) shows its orthogonal illuminant. Against the completely flat spectrum, the pulse spectrum in the shortest wavelength division is dominant in the orthogonal illuminant which is reasonable in the mean of the orthogonal.

The orthogonal illuminants were applied to derive the optimized spectral reflectance for color constancy. The smaller the distance between the tristimulus coordinates generated by an orthogonal illuminant pair, the more constancy kept. Simulated annealing was also employed to derive the optimal solution. The conditions for simulated annealing were the same with the first experiments. In the case the parameters $\rho(\lambda_i)$, $(i = 1,2,\cdots,n)$ were reconstructed in the process, and $|\Delta\rho(\lambda_i)| = 1/10^4$. The cost function of the process was the distance between two coordinates using $S^{(1)}(\lambda_i)$, $S^{(2)}(\lambda_i)$, $(i = 1,2,\cdots,n)$.

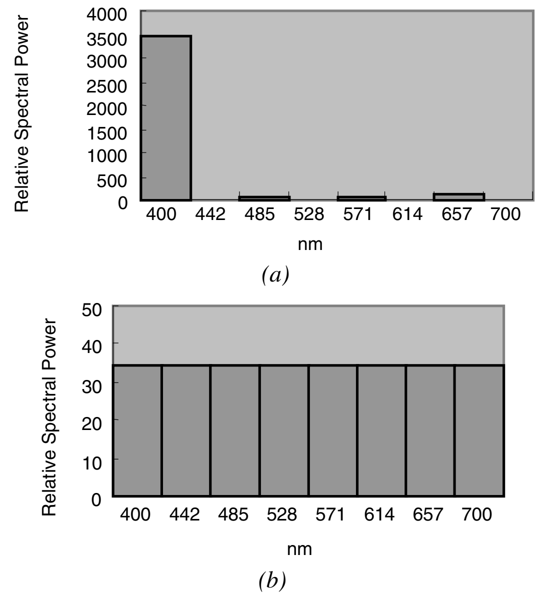

Figure 1 The reference illuminant and the orthogonal illuminant.

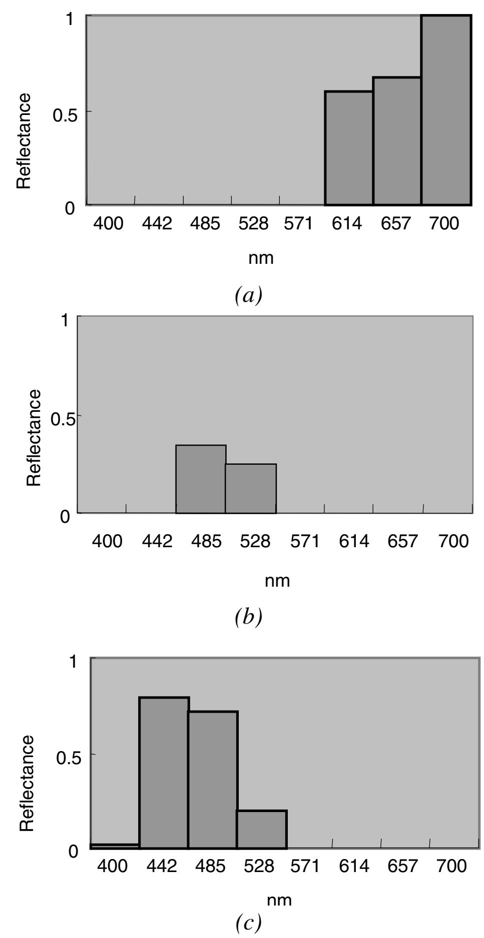

Figure 2 Optimum spectral reflectances

Figure 2 shows the optimum spectral reflectance for the illuminants of Figure 1. The result of Figure 2(a) is for Y=20, x=0.64, y=0.34, Figure 2(b) is for Y=20, x=0.22, y=0.62, and Figure 2(c) is for Y=20, x=0.16, y=0.14.

These are the first analytical results under the analytical illuminant model.

5.Conclusions

In this paper, the orthogonal illuminant has been discussed. First the definition of the orthogonal illuminant has been described, and second the solution of the illuminant has been described. As an application, the orthogonal illuminant has been applied to derive the optimized spectral reflectance for color constancy. No reports have appeared about the optimization for color constancy because of the lack of such models as the orthogonal illuminant.

Experiments have been performed to derive the orthogonal illuminant. A illuminant of the completely flat spectrum has been employed as the ideal reference illuminant. The orthogonal illuminant had a dominant pulse spectrum which is reasonable as the orthogonal.

The orthogonal illuminant was applied to derive the optimized spectral reflectance for color constancy. The results are the first analytical results.

Hereafter, we will also apply the orthogonal illuminant to other applications[3)].

References

1. N. Ohta and G. Wyszecki, Theoretical Chromaticity-Mismatch Limits of Metamers Viewed Under Different Illuminants, J. Opt. Soc. Am., 65, p.327 (1975).
2. S. Kirkpatrick, Optimization by simulated annealing, J. Statist. Phys., 34, p.975 (1984).
3. N. Matsushiro and N. Ohta, Orthogonal Illuminant Model and Its Application to Counting Metamers, IS&T/SID 8th Color Imaging Conference, accepted, Nov., (2000).

Biography

Nobuhito Matsushiro received his Ph.D degree from the University of Electro. Communications, Tokyo, Japan in 1996. He is a visiting scientist in Rochester Institute of Technology, NY, USA.

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