

# Extraction of Image Gamut Surface and Calculation of its Volume

Ryoichi Saito and Hiroaki Kotera

Department of Information and Image Sciences, Chiba University  
Chiba, Japan

## Abstract

Communicating color gamut is very important in color research and application. In this paper, we propose a new method for calculating the gamut volume by rearranging the color values. The entire color space is firstly divided by hue angle  $\_Hab$  and secondly by lightness angle  $\_Langle$  where the steps ( $\_Hab$ ,  $\_Langle$ ) are decided in non-uniform to include the constant color samples in each segmented sector. The most out-side points from the center are extracted, and the gamut surface is constructed by each ordered points. While, triangles constructed by each ordered points are decided to be outward or inward. The inward side is assigned to a minus volume, and finally the entire gamut volume is calculated by summing up the individual volume of tetrahedron with selected color values. The proposed method enables to describe the three dimensional gamut surfaces of color devices and color images.

## Introduction

Knowledge of the color gamut surface is useful for many color science-related tasks such as visualization, gamut volume calculation, and gamut mapping across the different color media. Some methods have been introduced<sup>1-4</sup> but the problem of calculating image gamut has no definitive solution yet.<sup>5</sup> An entire gamut volume can be calculated by tetrahedral color values listed in known orders.<sup>6</sup> The corresponding color gamut are analyzed and compared. However, the orders of color values in a given image are unknown at first. In this paper, we propose a new method for calculating the gamut volume by rearranging the color values. The entire color space is divided by hue angle and by lightness angle. The most out-side points from the center are extracted, and the gamut surface is constructed by each ordered points. These surfaces, however, are not always directed to outward, because each segmented sector has irregular size. So the surface direction should be decided to be outward or inward, and the inward surface is assigned to a minus volume to avoid double counting for the overlapped regions, then the entire gamut volume is calculated by summing up the individual volume of tetrahedron with selected color values. This method was

evaluated by theoretical printer models and resulted in the exact matching to the theoretical volume sizes.

## Volume Calculation of Gamut

The volume of a tetrahedron can be calculated:

$$V_T = \frac{1}{6} |a \cdot (b \times c)| \quad (1)$$

where  $V_T$  is the volume of the given tetrahedron and  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$  are the vectors corresponding to the three corners of the triangle on the surface of the gamut.

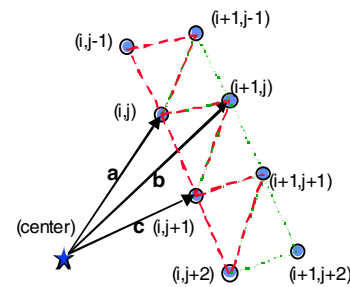


Figure 1. Calculation method of volume by tetrahedral color values listed in known orders.

The vectors are defined relative to a fixed center point inside of the gamut. The total gamut volume can be calculated by summing up the volumes of all the tetrahedrons in the gamut as follows.

$$V_G = \sum_{i,j=1}^{n-1} V_n \{(i, j), (i+1, j), (i, j+1)\} + \sum_{i,j=1}^{n-1} V_n \{(i, j+1), (i+1, j+1), (i+1, j)\} \quad (2)$$

where  $V_n$  denotes the volume of the  $n$ -th tetrahedron. Here  $n-1$  pieces of tetrahedrons are built up between  $\{(i,j), (i,j+1), (i+1,j)\}$  and  $\{(i,j+1), (i+1,j+1), (i+1,j)\}$ . In natural image, tetrahedral volume can be calculated by extracted point from random color pixels. The extraction process is described as the followings;

1) The cube root  $R_t$  of pixels  $N$  is computed by Eq. 3.

$$Rt = \sqrt[3]{N} \quad (3)$$

2) An image center  $(L^*, a^*, b^*)_{avg}$  of each pixel's color value is computed by Eq. 4.

$$(L^*, a^*, b^*)_{avg} = \frac{1}{N} \sum_{i=1}^n (L^*_i, a^*_i, b^*_i) \quad (4)$$

$i$  is pixel number.

3) The maximum lightness point ( $L^*_{max}$ ) and the minimum lightness ( $L^*_{min}$ ) are picked up.

4) Hue angles  $H_{angle}$  are calculated on  $a^*-b^*$  plane with an image center by Eq.5.

$$H_{angle} = \tan^{-1} \left( \frac{b^* - b^*_{avg}}{a^* - a^*_{avg}} \right) \quad (5)$$

5) The pixels are non-uniformly divided by  $H_{angle}$  to include the constant sample number  $Rt^2$  in each segment (See Figure 2(a)).

6) In a segmented hue angle sector,  $L^*$  angles  $L^*_{angle}$  are calculated along  $L^*$  axis by Eq.6.

$$L^*_{angle} = \tan^{-1} \left( \frac{L^* - L^*_{avg}}{[(a^* - a^*_{avg})^2 + (b^* - b^*_{avg})^2]^{1/2}} \right) \quad (6)$$

7) Again, the pixels are non-uniformly divided by  $L^*_{angle}$  with constant sample number  $Rt$  in each segment (See Figure 2(b)).

8) In each segment, the most out-side points  $(L^*, a^*, b^*)_{EXT}$  from the center are extracted.

9) In each segment, the most out-side points are extracted from each  $L^*_{angle}$  parts, then  $L^*_{max}$  and  $L^*_{min}$  are added both ends of this row.

10) For all hue angle sectors, each row is produced with process (9) and then the matrix of extracted points is completed by adding the first row to the final row.

The triangle surfaces constructed by extracted points are not always directed to outward, because each segmented sector has non-uniform size. To calculate the volume exactly, it needs to decide whether triangles are directed to outward or inward.

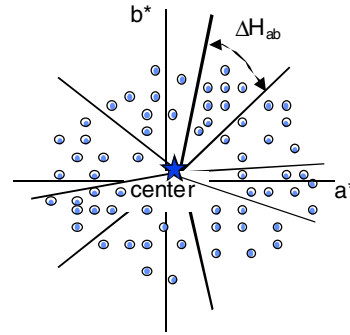
11) Using the extracted points  $(L^*, a^*, b^*)_{EXT}$ , triangle surfaces with  $\{(i,j), (i,j+1), (i+1,j)\}$  and  $\{(i,j+1), (i+1,j+1), (i+1,j)\}$  are calculated each normal vector  $nv$ .

12) The inner product A of the radiate vector by the normal vector  $nv$  for each triangle is calculated as follows. The sign of product A means whether the triangle direction is outward or inward.

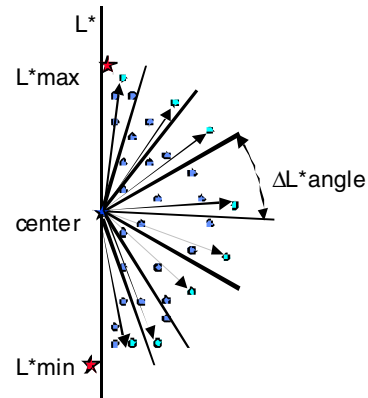
$$A = \{ (L^* a^* b^*)_{EXT} - (L^* a^* b^*)_{avg} \} \cdot nv \quad (7)$$

13) The tetrahedron constructed by an inward surface is assigned to a minus volume, and the total gamut volume is

calculated by summing up the individual volume of the tetrahedrons.



(a) Hue angle division.



(b)  $L^*$  angle division and color values selection.

Figure 2. The method of color gamut division.

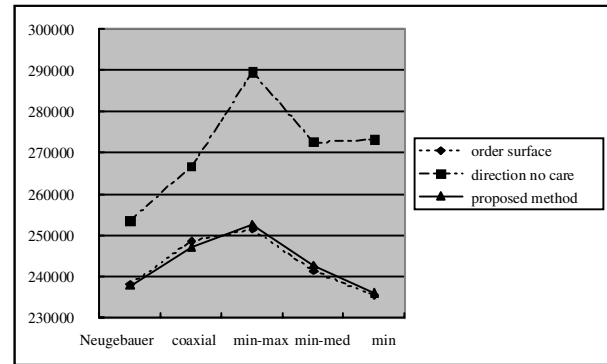


Figure 3. The comparison of calculated results in each method.

### Evaluation of Volume Calculation for Printer Models

Theoretical printer models were used to evaluate the proposed method. The c, m, y area ratios of primary color inks are changed from 0 to 1 in steps of 0.04, generating  $26^3=17576$  colors. Figure 3 shows the calculated volume of

each printer models: Neugebauer,<sup>7</sup> coaxial<sup>5</sup> (dot on dot<sup>8</sup>), three dot allocation models.<sup>5</sup> If not taking the surface direction into account, the calculated volume includes large error as compared with theoretical value. The proposed method resulted in the exact matching to the theoretical values.

### Application to Volume Calculation of Natural Scene Images

Figure 4 shows the results in natural scene image gamut extracted by proposed method; (a) original image 'bottles' with 150x180 pixels, (b) color distribution in CIELAB space, (c) wire frame image, (d) gamut surface covered with triangles. The color gamut of natural scene image has been correctly extracted and the volume is estimated as 121711.

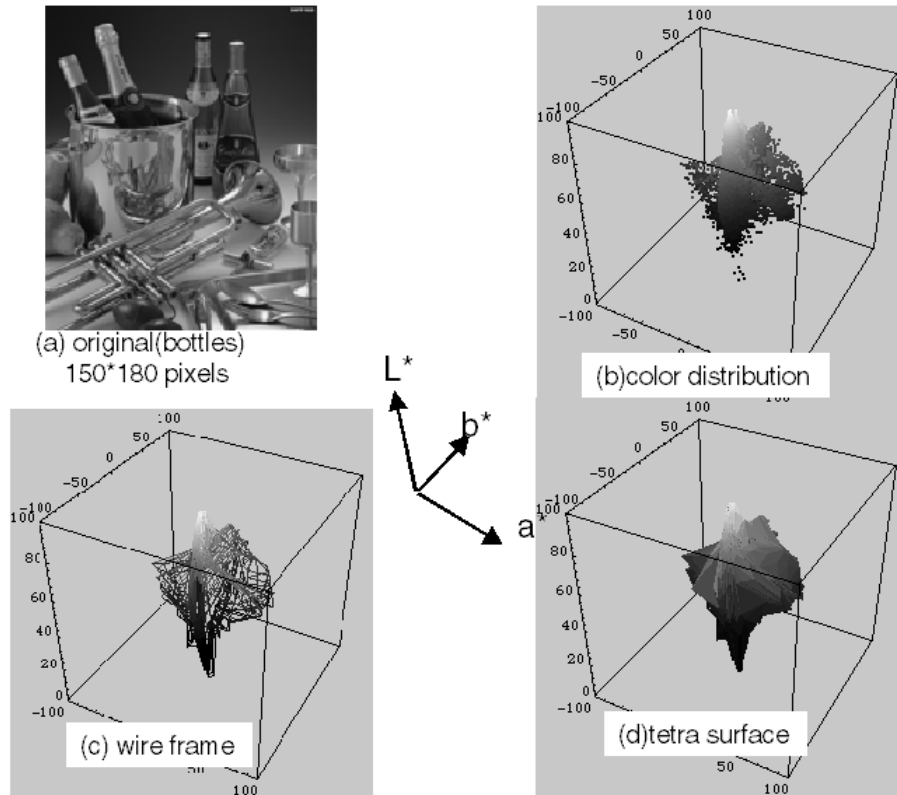


Figure 4. The original image "bottles" and comparison of its color gamut. Volume is 121711.

### Gamut Comparison of Monitor and Printer

The proposed method was applied to compare the gamut volumes between CRT monitor and ink-jet printer. In CIELAB space, color values were generated for  $L^*$  by 5 steps and for  $a^*$  and  $b^*$  by 10 steps. These LAB values were transformed from into sRGB values,<sup>9</sup> then the out of sRGB gamut data (over 1 and under 0 values) was omitted. Finally, 1675 points data were regenerated in CIELAB space and were used for the monitor gamut. On the other hand, these 1675 points data were printed out for sRGB values by ink-jet printer without color correction and these XYZ colorimetric values were used for the printer gamut.

Figure 5 shows the monitor and the printer gamut extracted by proposed method. The monitor's gamut is shown as a wire frame and the printer's gamut as a solid surface. In Blue to Magenta region and Green to Yellow region, differences of gamut between the monitor and the printer are remarkable. In medium lightness of Cyan region, both gamut boundaries are in contact with. The monitor's volume is calculated as 687373 and the printer as 293285. The monitor volume is estimated twice as large as the printer's.

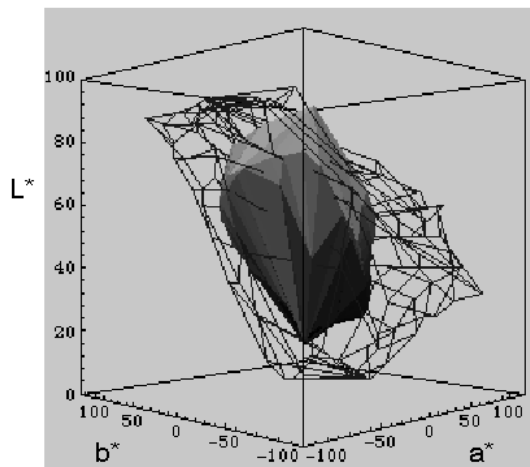


Figure 5. The comparison with monitor and printer gamut. The wire frame is monitor and the solid gamut is printer.

**Table 1. Surface area and concave ratio**

	Theoretical printer model		Natural image	
	Neugebauer	coaxial	bottles	bride
Surface area	28976.2	30662.1	30662.1	30655.9
Concave (%)	4.0	8.0	18.98	27.34

### Surface Area and Concave Ratio

The area of each extracted triangle is calculated using Heron's formula Eq. (8),

$$S = \sqrt{\frac{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}{4}} \quad (8)$$

where the variable (a,b,c) represents one of the three sides defining a triangle. The total surface area of gamut is calculated by summing up the individual area of extracted triangles. While, inward triangle areas of total surface area represent concave hull of gamut. Table 1 shows surface

area and concave ratio with each gamut. The natural scene images have larger concave ratios than the devices.

The concave ratio tells the complexity of the gamut shape.

### Discussion and Conclusion

We proposed a new method for extracting the gamut surface and calculating the gamut volume by rearranging the color values. This method can be used to get a gamut boundary surface and to calculate a volume exactly from random points of color distributions. The proposed method is available to describe the surface of image color gamut and applied to the gamut mapping.

### References

1. Thomas and Finney, *Calculus and Analytic Geometry 5 ed.*, Addison-Wesley, 1980, p.711.
2. W. D. Kappel, *IS&T's NIP13th*, p.470-p.474, Nov.2-7 (1997).
3. M. Mahy, *IS&T and SID The 4th CIC*, p.145-p.150, Nov.19-22 (1996).
4. T. Tanaka, R. Berns and M. Fairchild, *J. of EI*, **6**(2), 154 (April 1997).
5. R. Saito and H. Kotera, *J. of IS&T*, **43**(4), 345(1999).
6. N. Moroney, *Part of the IS&T/ SPIE Conf. on Color Imaging*, p.120-p.128 (1999).
7. H. E. J. Neugebauer, *Z. Wiss. Photo.*, **36**(4), 73 (1937).
8. R. Balasubramanian, *Proc SPIE*, **2413**, 356-364, 1995.
9. sRGB (<http://www.w3.org/Graphics/Color/sRGB.html>).

### Biography

Ryoichi Saito received his BS degree in Image Science from Chiba University in 1983. Since 1983, he has been working on direct plate making, digital image processing and color reproduction at Chiba University.