 

# CIE Fundamentals for Color Measurements

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## Abstract

The paper first overviews the CIE system of colorimetry, covering CIE 1931 color matching functions, *XYZ* tristimulus values, *x, y* diagram, 1976 *u', v'* diagram, and evolvement of CIELUV and CIELAB color spaces and color difference formulae. The paper then reviews the measurement of object colors introducing CIE standard illuminants and the CIE terminology for color and reflectance measurements, then the measurement of light-source colors (including displays) with calculation of correlated color temperature and color rendering indices. The paper also discusses practical aspects of color measurements for imaging applications using spectroreflectometers, spectroradiometers, and tristimulus colorimeters. Overview is given for calibration and verification of instruments' accuracy, spectral irradiance and reflec-tance standards (available from national laboratories), and uncertainty components.

## Introduction

The term *color* is used with different meanings in different technologies. To lamp engineers, color refers to a property of light sources. To graphics art engineers, color is a property of an object's surface (under a given illumin-ation). In each case, color must be physically measured in order to record it and reproduce the same color. The perception of color is a psychophysical phenomenon, and the measurement of color must be defined in such a way that the results correlate accurately with what the visual sensation of color is to a normal human observer. *Colorimetry* is the science and technology used to quantify and describe physically the human color perception. The basis for colorimetry was established by CIE (Commission Internationale de l'Éclairage) in 1931 based on visual experiments. Even though limitations are sometimes discussed, the CIE system of colorimetry remains the only internationally agreed metric for color measurement. All the official color-related international standards and speci-fications use the CIE System. The CIE system works well in most cases, but one should know the assumptions and limitations in visual conditions where the CIE system is defined. In this paper, the CIE system of colorimetry is briefly overviewed, and then practical aspects of color measurements and instruments – spectroreflectometers, spectroradiometers, and colorimeters – are discussed, with a focus on the calibration methods and standards. Uncer-tainty components and correction for errors are also discussed. For further details in colorimetry and color science, refer to official CIE publications [1-3] and many other appropriate references.[4]

## CIE System of Colorimetry

### History and Basis

By the early 19th century before the colorimetry system was defined, it became known that there were three types of cones in the eyes to sense colors. It was also known that two light stimuli having different spectra could produce the same color (*metamerism*). It was inferred that each cone had spectral sensitivities corresponding to R, G, B (*Trichromatic Theory*, Young, 1800's) or sensitivities corresponding to opponent colors, W/Bk, R/G, and Y/B (*Opponent Theory*, Hering, late 1800's). The spectral sensitivities of the cones were yet to be known at that time, but a color could be matched by combination of three primaries, which could be used to specify color.

Around 1930, Wright and Guild made independent visual experiments to derive color matching functions using three R/G/B primaries, the results of which became the basis of the CIE colorimetry system. Observers viewed a 2° circular split field and their task was to adjust the three primaries so that their mixture visually matched the visible spectrum presented sequentially. Fig. 1 shows the results of this experiment using a set of primaries at 435.8 nm, 546.1 nm, and 700 nm. This is the plot of the relative intensities of R,G,B primaries (white-balanced to equal energy white) that matched monochromatic stimulus at each wavelength. The minus value means that one of the primary colors had to be added to the monochromatic stimulus to make the match. In 1931, CIE adopted these results as the standard-ized RGB color matching functions. Then still in 1931, for practical convenience, CIE transformed the RGB color matching functions to a new set of primaries, XYZ, to eliminate negative values and with the G function to be equal to the 1924 CIE spectral luminous efficiency func-tion, $V(\lambda)$. These are called the *CIE 1931 XYZ color matching functions*, shown in Fig. 2, each function denoted as $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$. This is simply a linear transfor-mation from the RGB color matching functions. There are two important assumptions in these color matching functions:

First, rod intrusion is excluded, thus it applies to only narrow field of view (2°). Second, additivity of light stimuli (*Grassmann's Law*) is assumed. The ideal observer whose color-matching properties correspond to the color matching functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ with the 2° field of view and satisfying the Grassmann's Law is called the *CIE 1931 standard colorimetric observer*. Practically, this observer can be used for a field of view-of-view of up to 4°. In 1964, the CIE defined a second set of standard color matching functions for a 10° field-of-view, denoted as $\bar{x}_{10}(\lambda), \bar{y}_{10}(\lambda), \bar{z}_{10}(\lambda)$, to supplement those of the 1931 standard observer. This is called the *CIE 1964 supplementary standard colorimetric observer*, and can be used for a field of view greater than 4°.

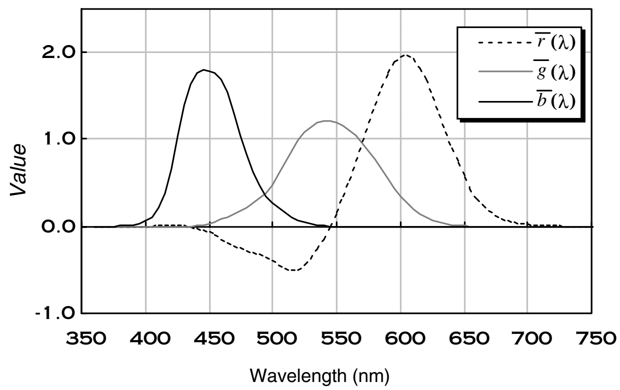


*Figure 1. CIE 1931 RGB color matching functions using primaries at 435.8 nm, 546.1 nm, and 700 nm.*

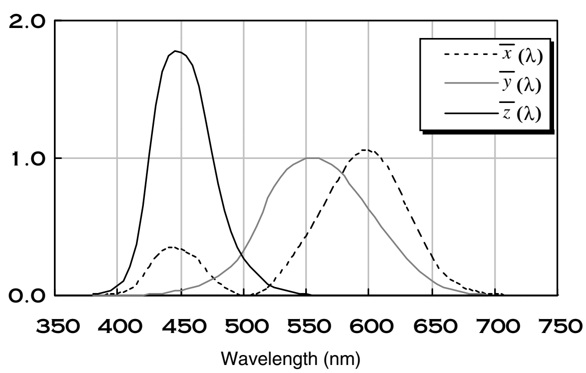


*Figure 2. CIE 1931 XYZ color matching functions.*

### *Tristimulus Value*

By using the color matching functions, light stimuli having any spectral power distribution can be specified for color by three values:

$$X = k\int_{\lambda} \Phi(\lambda)\bar{x}(\lambda)\mathrm{d}\lambda \qquad Y = k\int_{\lambda} \Phi(\lambda)\bar{y}(\lambda)\mathrm{d}\lambda \quad (1)$$

$$Z = k\int_{\lambda} \Phi(\lambda)\bar{z}(\lambda)\mathrm{d}\lambda$$

where $\Phi(\lambda)$ is the spectral distribution of light stimulus and $k$ is a normalizing constant. These integrated values are called *Tristimulus values*. For light sources and displays, $\Phi(\lambda)$ is given in quantities such as spectral irradiance and spectral radiance. If $\Phi(\lambda)$ is given in an absolute unit and $k$=683 lm/W is chosen, $Y$ yields an absolute photometric quantity such as illuminance or luminance.

For object colors, $\Phi(\lambda)$ should be

$$\Phi(\lambda) = E(\lambda) \cdot R(\lambda) \qquad (2)$$

where $R(\lambda)$ is the spectral reflectance or radiance factor of the object, $E(\lambda)$ is the (relative) spectral irradiance of the illumination, and

$$k = 100 / \int_{\lambda} E(\lambda)\bar{y}(\lambda)\mathrm{d}\lambda. \qquad (3)$$

Actual integration can be carried out by numerical summation of spectral data.

### *Chromaticity Diagrams*

By projecting the tristimulus values on to the unit plane ($X$+$Y$+$Z$=1), color can be expressed in a two dimensional plane. Such a unit plane is known as the chromaticity diagram. The color can be specified by the chromaticity coordinates ($x$, $y$) as given by

$$x = \frac{X}{X+Y+Z}; \quad y = \frac{Y}{X+Y+Z} \qquad (4)$$

The diagram using the chromaticity coordinates ($x$, $y$) is referred to as the *CIE 1931 chromaticity diagram*, or the *CIE (x, y) chromaticity diagram*.

The chromaticity scale in the (x, y) chromaticity diagram is very non-uniform in terms of color difference. The minimum perceivable color differences in the CIE (x, y) diagram, known as MacAdam ellipses, are shown in Fig. 3(a). To improve this, in 1960, CIE defined an improved diagram – CIE 1960 (u, v) chromaticity diagram (now deprecated), and in 1976, a further improved diagram – CIE 1976 uniform chromaticity scale (UCS) diagram, with its chromaticity coordinate (u', v') given by

$$u' = \frac{4X}{X+15Y+3Z}; v' = \frac{9Y}{X+15Y+3Z} \qquad (5)$$

The 1976 (*u', v'*) chromaticity diagram is a significant improvement from the (*x, y*) diagram, yet it is still far from perfect as shown in Fig. 3 (b).

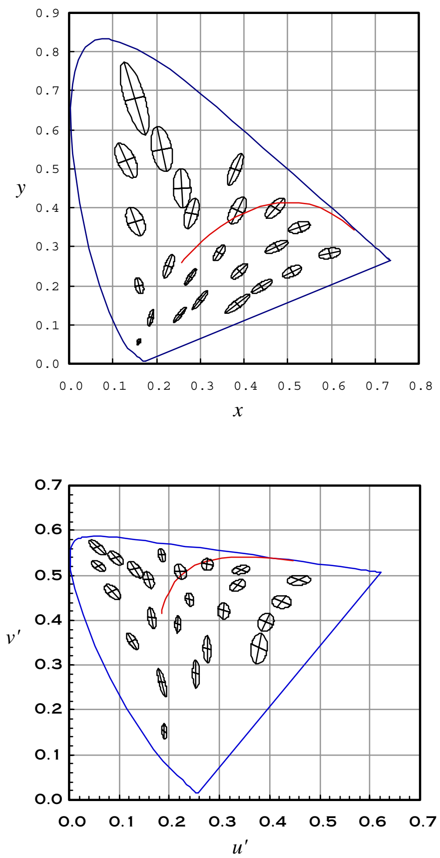


*Figure 3 – MacAdam Ellipses in CIE 1931(x, y) diagram and the CIE 1976 (u'v') diagram. The ellipses are plotted 10 times their actual size.*

### *Uniform Color Spaces and color difference formulae*

Three attributes of color, are hue, chroma (saturation), and lightness, and are expressed in a three dimensional space. In the chromaticity diagrams as mentioned above, lightness is missing, and the hue and chroma are laid out very nonlinearly. To allow accurate specification of object colors and color differences, CIE recommended three-dimensional uniform color spaces – CIELAB and CIELUV in 1976. Since the equations are long, they are omitted here. These are called the *CIE 1976* ($L^*a^*b^*$) *color space* or *CIELAB color space*, and the other, *CIE 1976* ($L^*u^*v^*$) *color space* or *CIELUV color spac*e, and have similar structures as the Munsell color solid. In imaging applica-tions, CIELAB space is commonly used. In CIELAB space, $L^*$ shows the lightness, and ($a^*$, $b^*$) the color as shown in Fig. 4, The coordinate ($L^*$,$a^*$,$b^*$) is calculated from the ($X,Y,Z$) of the given light stimulus and ($X_n$,$Y_n$,$Z_n$) of the white point. Therefore, the CIELAB space has a function of correcting for chromatic adaptation to the white point, and is intended for object color and displays. The color difference in the CIELAB space is calculated as the Euclidean distance between the points in this three-dimensional space, and is given by,

$$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]. \quad (6)$$

This equation is called the *CIE 1976* ($L^*a^*b^*$) *color difference formula*. The chroma $C^*_{ab}$ and the hue angle $h_{ab}$ are also calculated from ($L^*a^*b^*$) by

$$C^*_{ab} = (a^{*2} + b^{*2})^{1/2} \quad (7)$$

$$h_{ab} = tan^{-1} (b^*/a^*). \quad (8)$$

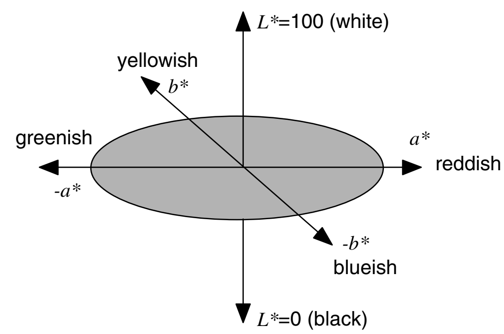


*Figure 4. CIELAB color space*

The CIELUV space is defined in a similar manner, and the coordinate ($L^*$,$u^*$,$v^*$) is calculated from the $Y$ and ($u'$,$v'$) of the given light stimulus and the white point. Refer to appropriate references [1,4] for the details.

While the color difference $\Delta E^*_{ab}$ is widely used, its chroma scale is known to be fairly nonlinear. For more accurate color difference evaluations, CIE recommended an improved industrial color difference formula in 1994 - *CIE94 Formula* [5]. The color difference $\Delta E^*_{94}$ is calculated from $\Delta L^*$, $\Delta C^*_{ab}$, and $\Delta H^*_{ab}$ of the CIELAB formula. Another improved formula, *the CMC Colour Difference Formula*, is mainly used in textile industry.[6] Further improved color difference formulae are being investigated by CIE (TC1-55).

### *Correlated Color Temperature*

The color of light sources are measured and expressed by the resultant chromaticity coordinates ($x$,$y$) or ($u'$,$v'$). However, it is difficult to imagine immediately what colors they are from these numbers. For such practical purposes, the color of "white light" can be expressed by *correlated color temperature (CCT)* in the unit Kelvin [K]. For example, 2800 K is immediately associated with the warm color of incandescent lamps, and 9000 K the bluish white from a CRT. CCT is defined as the temperature of the Planckian radiator whose perceived color most closely resembles that of a given stimulus at the same brightness and under specified viewing conditions.[7] According to this definition, CCT can be calculated using one of the chromaticity diagrams. Due to the long tradition, CIE still recommends to calculate CCT using the 1960 ($u$,$v$)

chromaticity diagram (now deprecated).[1] From $(u',v')$ coordinates, $(u,v)$ can be obtained by $u=u'$, $v=2v'/3$. On the $(u,v)$ diagram, find the point on the Planckian locus that is at the shortest distance from the given chromaticity point. CCT is the temperature of the Planck's radiation at that point. A practical way of computing CCT is available.[8]

### *Color Rendering Index*

For lamps in lighting applications, it is important to evaluate how well a given illumination can render colors of objects in the illuminated scene. CIE defined the *color rendering index (CRI)* first time in 1965. Going through minor revisions, the recommendation[9] has been in wide use mainly by lighting industry. The procedure of calculation is first to calculate the color differences $\Delta E_i$ (on the 1964 $W^*U^*V^*$ uniform color space – now obsolete) of 14 selected Munsell samples between the conditions when illuminated by a reference illuminant and when illuminated by the given illumination. The process incorporates the von Kries chromatic adaptation transformation. Then the *Special Color Rendering Index* $R_i$ for each color sample is calculated by

$$R_i = 100 - 4.6\Delta E_i. \quad (9)$$

This gives an indication of color rendering for each particular color. The *General Color Rendering Index*, $R_a$, is given as the average of the first eight color samples (medium saturation). With the maximum value being 100, $R_a$ gives a scale that matches well with the visual impression of color rendering. For example, lamps having $R_a$ values greater than 80 may be considered suitable for interior lighting, and $R_a$ greater than 90 for visual inspection purposes.

### *Standard Illuminants*

The colors of objects change depending on the spectrum of illumination. Thus, there is a need to specify the illumination for any object color specification. For this purpose, colorimetric illuminants are standardized by CIE and ISO.[1,2] *CIE Standard Illuminant A* (representative of tungsten-filament lighting with a color temperature of 2856 K) and *CIE Standard Illuminant D65* (representative of average daylight with a CCT of 6500 K) are the two primary standard illuminants.[2] It is recommended that either of these illuminants be used in all applications. However, other phases of daylight illuminant are already widely used in specific application areas, and CIE also defines D50, D55 and D75.[1] Equations are available to obtain the data table for Illuminant A and any phase of D illuminant. Even though no longer recommended for use, Illuminant B was intended to represent direct sun light with a CCT of ~4900 K, and Illuminant C to represent average daylight with a CCT of ~6800 K and to be realized by a tungsten source combined with a prescribed liquid filter.

## Measurement of Object Color

### Terminology of Reflectance Measurement

Object color, in most cases, is determined by spectral reflectance measurements. The terminology for reflectance measurements is often confused and misused by the imaging community. Some important terms are reviewed here according to Ref. 7.

*Reflectance* is the ratio of the reflected radiant or luminous flux to the incident flux in the given conditions of spectral composition, polarization, and geometrical distribution. The geometrical conditions are very important for correctly describing and measuring reflectance, and can lead to confusion regarding reflectance measurements.

*Perfect Reflecting Diffuser* is an ideal isotropic diffuser with a reflectance equal to 1.

*Reflectance Factor* is the ratio of the radiant or luminous flux reflected in the direction delimited by the given cone to that reflected in the same direction by a perfect reflecting diffuser identically irradiated or illuminated.

*Radiance Factor* is the ratio of the radiance of a surface element in a given direction to that of a perfect reflecting diffuser identically irradiated.

*Radiance Coefficient* is the ratio of the radiance of the surface element in the given direction to the irradiance on the medium.

There are several important implications that follow from the above definitions. "Factor" in these terms means with respect to a perfect reflecting diffuser, and therefore can be greater than one. Reflectance, on the other hand, can never be greater than one, and is often used descriptively to represent all of these reflectance-related quantities. Reflectance factor is defined in terms of a cone, while radiance factor is defined only in terms of a direction. Therefore, if the solid angle of the cone approaches zero, the reflectance factor approaches the radiance factor for the same conditions of irradiation. If the solid angle of the cone approaches $2\pi$ sr, the reflectance factor approaches the reflectance for the same conditions of irradiation. Finally, radiance coefficient is similar to the *bi-directional reflectance distribution function (BRDF)* except that the latter is defined for directional incident flux.

### *Illuminating and viewing conditions*

For the colorimetry of objects, CIE recommends the use of one of four standard geometries – 45º/normal (45/0), normal/45º (0/45), diffuse/normal (d/0), and normal/diffuse (0/d). Geometry is one of the most important conditions to specify in reflectance colorimetry. The details on this subject are covered by the paper by Danny Rich[10].

### *Reflectance standards*

Most spectroreflectometers are calibrated using white reflectance standards for one of the geometries listed above. *Spectral radiance factor standards* are needed for the 45/0, 0/45, and d/0 geometries, while *diffuse spectral reflectance standards* are needed for the 0/d geometry. Highly diffuse white materials such as pressed or sintered polytetrafluoroethylene (PTFE) are used for such standards. Because absolute measurements of radiance or reflectance factors are very difficult, calibrated standards are provided by national metrology laboratories,[11, 12] and industrial measurements are normally made with traceability to these standards. Since a perfect reflecting diffuser does not exist, the radiance factor is calibrated by absolute measurements of the radiance coefficient. The radiance factor is then obtained from the radiance coefficient by multiplying by the constant $\pi$.

The reflectance characteristics of even the most diffuse materials are sensitive to the illumination and viewing angles. An example for the measured spectral radiance factor of a pressed PTFE sample is shown in Fig. 5.

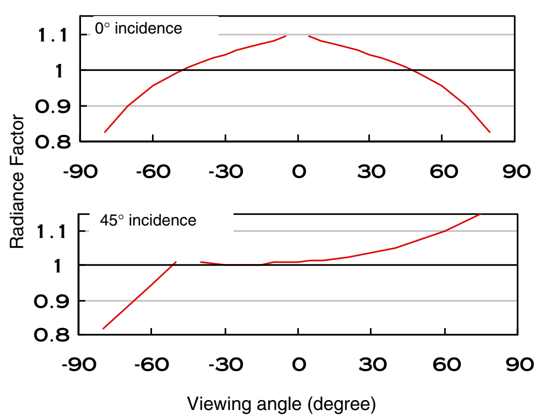


*Figure 5. Radiance factor of pressed PTFE as a function of viewing angle at a wavelength of 555 nm.*

***Measurement instruments for object color***

Spectroreflectometers are commonly used for object color measurements. These instruments measure the spectral reflectance of a test sample under a given geometrical condition, and most are calibrated by a reference standard traceable to a national metrology laboratory. Thus, their measurement uncertainty first depends upon the uncertainty of the reference standard. Uncertainties also arise from the characteristics of the spectroreflectometer. Effects contributing to the uncertainty include wavelength error, detector nonlinearity, stray light, bandwidth, the geometrical conditions for both illumination and viewing, and measurement noise. The effect of bandwidth can be serious for bandwidths greater than 10 nm. For example, a 20 nm bandwidth can cause errors of as much as two to three CIELAB units for saturated colors. An effective correction method is available.[13] Recommendations for standard methods to characterize spectroreflectometers are being developed by the CIE TC2-28 committee. Finally, the uncertainty can also depend upon the characteristics of the test sample. For example, saturated color samples tend to have larger errors.

In order to verify stated measurement uncertainties for spectrophotometers, calibrated color standards are used. Ceramic tiles of various colors manufactured by the British Ceramic Research Association (BCRA) are available from the National Physical Laboratory (NPL), UK and will be available from the National Institute of Standards and Technology (NIST), USA.

For measurements of small color differences, tristimulus colorimeters are used because of their benefits of high speed and low cost. The uncertainty of tristimulus colorimeters is limited, however, due to the mismatch of the illumination to the CIE illuminants and of the spectral response of the detectors to the CIE color matching functions. Thus, they are not suitable for absolute color measurements over a wide range of colors.[14]

A number of recommendations on spectral reflectance and color measurements are available from the American Society for Testing and Materials (ASTM).[15]

## Measurement of Light Source Color

The measurement of light source color is represented by measurement of lamps, LEDs, and displays. Both spectroradiometers and tristimulus colorimeters are widely used.

***Measurement instruments for light source color***

Spectroradiometers are normally designed to measure either *spectral irradiance* (unit: W $m^{-2}$ $nm^{-1}$) or *spectral radiance* (unit: W $sr^{-1}$ $m^{-2}$ $nm^{-1}$). The former is equipped with a diffuser or a small integrating sphere as input optics, and the latter equipped with an imaging optics. For example, lamps are normally measured for spectral irra-diance and displays are measured for spectral radiance to obtain colors. There are two types of spectroradiometers; mechanical scanning type and diode-array type. Generally, the former is more accurate but slow, and the latter is fast but less accurate. Spectroradiometers are calibrated against spectral irradiance or radiance standards traceable to national standards.[16] Thus, their measurement uncertainty first depends on that of the reference standard. Then, like spectroreflectometers, there are many other uncertainty components including wavelength error, detector nonline-arity, stray light of monochromator, bandwidth, measure-ment noise, etc. The errors vary depending on the spectrum of the source measured. Even if the instrument's specifi-cation shows a low uncertainty for a tungsten source (normally a calibration source), the instrument's uncertain-ty for other colors can be much larger. For example, typical diode-array spectroradiometers exhibit errors of up to 0.005 in *x, y* for various display colors while they are specified for an uncertainty of ~0.001 in *x, y* for CIE Illuminant A. For applications where highest accuracy is required, it is necessary to calibrate the instruments for various actual colors to be measured. For color measuring instruments for displays, such a calibration facility and services are available.[17]

Tristimulus colorimeters are also widely used for colorimetry of lamps and displays. While they have benefits of low cost and high speed, errors due to spectral mismatch are inevitable and their uncertainty for measurement tend to be higher than spectroradiometers. To improve accuracy in display measurements, effective correction techniques are available.[18,19]

## Uncertainty of Measurements

*Uncertainty of measurement* is an estimate of the range of values within which the true value lies. When making colorimetric and photometric measurements, it is important to know the uncertainty of the results. Uncertainty of measurements depends not only on that of the measurement instruments but also measurement conditions. The uncertainty of measurement must be stated for official exchange of measurement results, and it must follow the international recommendation [20]. The term *accuracy* is no longer recommended for use to specify the values of uncertainty. For industrial measurement, it is recom-mended to use an *expanded uncertainty* with a coverage factor $k$=2. See Ref. 20 for the details.

## Conclusion

An overview has been given for the fundamentals of the CIE colorimetry system and practical issues in measurements of object color and light-source color. When making measurements, one should be aware of the uncertainty of the measurement instruments and uncertainty components arising from the measurement conditions. Refer to the given references for the details in the subjects covered in this paper.

## Acknowledgement

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## Biography

Yoshi Ohno received a Ph.D. in engineering from Kyoto University, Kyoto, Japan in 1993. He started his career in photometry and colorimetry at Matsushita Electric Ind. Co. in Japan in early 1980s, and immigrated to the U.S. in 1992 to be employed by NIST. He is currently the project leader for Photometry at Optical Technology Division, NIST, and recently led a project for colorimetry of displays. He serves as the Secretary of CIE Division 2, and chairs two CIE technical committees in photometry. He is a member of CIE TC1-48 (revision of CIE 15.2) and IEC TC100/TA2. Email: ohno@nist.gov