# **Space Charge Limited Currents from a Strip Shaped Emitter in a Parallel Plate Geometry**

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#### Abstract

Microscopic electrical defects in the photoreceptor termed Charge Deficient Spots (CDSs) can give rise to objectionable print defects in xerographic systems such as laser printers. Theoretical calculations of Space Charge Limited Currents (SCLC) between concentric cylindrical and spherical electrodes show that a finite SCLC can be sustained, for a finite applied voltage, even as the radius of the emitting electrode goes to zero. Numerical simulations<sup>1</sup> of the SCLC in a model system consisting of a ground plane with a finite circular emitting region and a parallel collector kept at constant applied potential have shown that the SCLC varies *linearly* with emitter radius for small radii. In this paper we consider the emitting region to be an infinitely long strip of finite width, which may be termed a Charge Deficient Line (CDL). Using the same numerical technique, we address the following questions: i) does the current vanish as the width of the CDL vanishes and ii) what is dependence of the current on the width? Finite element methods were used to calculate the SCLC consistent with Maxwell's equations and the correct boundary conditions. Our results show that, for a fixed applied voltage, the SCLC varies linearly with emitter width for large width and becomes independent of width for small width.

## Introduction

The phenomenon of Space Charge Limited Current (SCLC) has been of considerable scientific and engineering interest for sometime. SCLC is observed in a wide variety of low conductivity materials: inorganic and organic solids<sup>2,3</sup>, liquids<sup>4,5</sup> and gases<sup>6</sup>. SCLC has been employed as a tool for determining the mobility of injected charges<sup>2-6</sup> and, in the case of solids, a considerable amount of theoretical work has been done to understand the role of traps in the samples used in experimental studies<sup>2,7</sup> on the steady state and transient SCLC responses. Most of the theoretical work on SCLC has focused on infinite electrodes in the plane and cylindrical geometries. In ideal SCLC the electric field at the emitter is zero, by definition, and this leads to infinite charge densities at that electrode. The steady state and transient responses for the case of non-ideal SCLC, where there is a non-zero field at the emitter has been studied<sup>8,9</sup> in order to better approximate experimental conditions. The finite field at the emitter is usually associated with the threshold field for injection at the emitter electrode-sample interface.

While the theoretical work is largely confined to regular, infinite geometries many practical devices (e.g. solid state semiconductor chips and xerographic photoreceptors) which have finite domains and complex electrode geometries can operate in the SCLC regime. Calculations of the steady state (i. e. current-voltage or I-V characteristics) and transient responses of the SCLC in such cases require computational methods. Such methods are essentially simulations of the behaviour of the device that is subjected to some well specified external driving forces such as, for example, a voltage. In a previous paper<sup>1</sup>, we carried out such an analysis for the case of a circular, finite sized emitting region and studied the dependence of the SCLC on the radius of the region. It was shown that, for a fixed applied voltage, the SCLC varies *linearly* with emitter radius for small radii and goes over to a quadratic dependence for large radii. A plausible scaling argument was presented to explain the linear dependence for small radii. However, it must be borne in mind that theoretical analysis of SCLC in 2 and 3 dimensional geometries (between, respectively, infinitely long concentric cylinders and between concentric spheres) shows that a finite SCLC can be sustained by finite potentials even as the dimension of the emitting inner electrode is reduced to zero<sup>2</sup>.

In this paper we present the results of a simulation of unipolar charge transport in the SCLC limit in the usual parallel plate geometry but where the emitting region is a strip of finite width, but of infinite length. The fundamental question we address is the dependence of the current on the width of the emitting region, especially for the case of widths small compared to the electrode spacing. From the practical point of view, our work has bearing on two technologies: 1) xerographic photoreceptors and 2) semiconductor chips. If the SCLC can be sustained in the planar geometry by a vanishingly thin linear emitter, then even microscopic linear sources in xerographic photoreceptors can cause defects in the final print<sup>10</sup>. In the case of semiconductor chips, the linear interconnects are sources of current into the surrounding insulating medium and so the existence of a SCLC from even infinitely thin connects may provide a limit to how close to each other they can be before crosstalk sets in.

Modern xerographic photoreceptors<sup>11</sup> (PRs) are planar, layered devices with a ground plane on one interface. The functional layers consist of a charge generation layer next to the ground plane and a charge transporting layer on top. The total thickness of typical devices is about 25 µm. Typical transport layer materials have mobilities of the order of  $10^{-5}$  cm<sup>2</sup>/(V-s) are hence readily reach SCLC conditions at applied voltages of a few hundred Volts. In a typical xerographic process<sup>12,13</sup>, the free surface of the charge transport layer is first uniformly charged and then the device is exposed to light in an image wise fashion<sup>12</sup>. Thus, charges are generated in the charge generation layer in an image wise fashion which are then transported to the surface. Upon reaching the surface these charges neutralise the original uniform charge image wise and thus form an electrostatic latent image. This latent image is subsequently developed with toner and the developed toner is then transferred to paper and fused to give the final print. Each step in this process is subject to noise, which results in some loss of quality or the appearance of artifacts in the final print. As most present day printers discharge the areas that have to be developed with toner (Discharged Area Development), any spurious, unintended discharge can give rise to dark areas in what should be plain paper. As the human visual system readily picks up these defects, they have a very adverse effect on print quality. The usual cause of these defects<sup>10</sup> is a localized discharge of the photoreceptor due to sources (often called Charge Deficient Spots - CDSs) either in the charge generation layer or its interface with the ground plane. Due to the increasing output print quality requirements, it is useful to gain an understanding of the nature of these sources. The CDSs observed in practice are better modeled as being circular in shape but linear defects (which we may term Charge Deficient Lines – CDLs) can occur in principle in xerographic printing systems that employ belt PRs. Such belts move over supporting bars and are operated under tension; hence long scratches on the back of the belt are common. Internal stresses caused by such scratches could lead to mechanical failures in the functional layers of the PR that then behave as long thin charge emitting regions. In addition, the belts are wrapped around rollers and the stresses due to the constant flexing could also cause mechanical failures that lead to CDLs. In semiconductor chips the interconnects are sources or sinks of charge and it is vital that the crosstalk between them is minimised. The trend towards miniaturisation has resulted in interconnects getting continually thinner – about 0.18  $\mu$ m at present. It is interesting to explore how the leakage currents depend on the thickness of the interconnects and see if thinner interconnects do result in less leakage and hence less crosstalk.

### **Model Description and Calculations**

The geometry used for our calculations is shown in Fig. 1. The basic electrode structure corresponds to that of two infinite parallel plates separated by a distance L. A voltage

V is applied across the gap which is taken to be filled with a homogenous material characterized by a dielectric constant  $\varepsilon$  and a mobility  $\mu$ . Without loss of generality, we may take the negative electrode to correspond to ground and also assume that the emitter lies in this electrode. The actual emitter consists of a strip of halfwidth a, as shown in Fig. 1, and is of infinite length along the y axis which is taken to be perpendicular to the page. Due to translational symmetry along the y axis the problem is two dimensional. In a practical situation, the emitting region would correspond to an ideal Ohmic contact -- a contact that can supply whatever current necessary to have enough space charge in the bulk to drive the electric field just above it to zero. In addition to this, the electrostatic boundary conditions have to be satisfied: the collection electrode must be at a potential V and the ground at zero.



Figure 1. The geometry of the model system. The electrodes are maintained at a potential difference of V. The emitter is a region of halfwidth a in the negative electrode, which is taken to correspond to ground.

The CDL discharge is a space charge limited discharge problem and is governed by the solution to the charge transport equations:

div (grad 
$$\phi$$
) = -  $\rho/\varepsilon$  (1)

$$\delta \rho / \delta t + \operatorname{div} \mathbf{J} = 0 \tag{2}$$

where  $\phi$  is the electrostatic potential,  $\rho$  is the space charge density. **J** is the current density given by

$$\mathbf{J} = -\rho\mu \operatorname{grad} \phi \tag{3}$$

where  $\mu$  is the mobility and  $\varepsilon = K\varepsilon_0$  is the permittivity of the media.

The boundary conditions are: at z = 0,  $\phi = 0$ , and at z = L,  $\phi = V$ . Additionally, on the emitter surface: z = 0,  $|x| \le a$ ,  $E_z = E_{th}$  where  $E_{th}$  is the threshold field ( $\approx 0$ ) at which the space charge limited discharge takes place. The initial condition is that at t = 0,  $\rho = 0$  everywhere in the domain. The simulation of the time evolution of the system is performed until steady state is reached in a manner identical to the case of the CDS calculation<sup>1</sup>. The details of the algorithm are available in Refs. 14 and 15.

The actual numerical calculations are carried out employing scaled, dimensionless variables. The scaling used here preserves the fundamental scaling law for all SCLC:

$$I \propto V^2$$
 (4)

Because the simulation builds in this scaling into the calculation, we do not explicitly show the I-V characteristics though calculations were carried out to verify the integrity of the model. Also, the same scaling laws show that the total current per unit length is of the form

$$I = \varepsilon \mu V^2 / L^2 \bullet 2f(a/L) \tag{5}$$

where the function f() depends only on the ratio a/L. However, for the sake of clarity, we will discuss only the total current per unit length *I* for the case of  $L = 25\mu$ m in what follows. Using the scaling laws our results can be easily used to calculate currents expected for any photoreceptor thickness, emitter width, and applied voltage. We would like to emphasise that the scaling law given in (5) is different in form from that obtained for the axisymmetric case<sup>1</sup> (i.e. the CDS). This the origin of the difference in the dependence of the SCLC on the CDS radius for small radii and small widths in the case of the CDL.

#### **Results and Discussion**

The simulations were run for a PR thickness of 25  $\mu$ m and a dielectric constant of 3. Figure 2 shows the dependence of the total current as a function of the CDL full width, 2a. The dashed lines represent the current perunit length that would have been emitted from a strip of the width as the CDL in the one dimensional parallel plane case, i.e.

$$I_{Id} = 2aJ_{Id} \tag{6}$$

where  $J_{1d} = (9/8) \epsilon \mu V^2/L^3$  is the one dimensional SCLC<sup>2</sup>. Thus in the limit a >>L, f(a/L) is equal to (9/8)a/L and the current per unit length depends *linearly* on the width of the CDL. As the width of the CDL shrinks to the order the PR thickness, the calculated current/length shows an increasing sublinear dependence on the width of the strip. For CDL widths much smaller than the PR thickness, the current becomes more and more independent of the width and approaches a finite value as the width of the CDL goes to zero.

#### Scaling Arguments for Small Width Behaviour

We will now present a general scaling argument which shows that a sublinear relationship is expected between strip width and total current per unit length for small widths and that a finite current persists even as the width of the CDL goes to zero. Let us assume that a flat plate capacitor of thickness L has a localized injection strip of width 2a, which is much smaller than L (Fig. 1). Under the voltage V applied to the capacitor plates, current per unit length I will flow through the circuit. We can therefore describe the system with four characteristic parameters:

$$L, a, V, I \tag{7}$$



Figure 2. The current per unit length of a CDL as a function of its full width for three fixed applied voltages. Simulation results are denoted by the diamonds (100 V), asterisks (300 V) and pluses (500 V). The dashed lines correspond to the current per unit length in the planar case from **a** strip of the same width as the CDL, i.e.  $2aJ_{id}$  for the above voltages. The PR thickness is 25  $\mu$  m and its dielectric constant is 3. The planar current density is given by  $J_{id} = 4.78 \times 10^5 V^2 (2a/L) \mu A/m$ .

If we increase linear dimensions of the system by a factor  $\beta$  Equation (5) describing the space charge limited current for a finite width emitter leads to the following scaling of characteristic lengths, voltage and current:

$$\beta L, \beta a, \beta V, I$$
 (8)

If the charge emitter width, 2a, is very small (i.e. a << L), the space charge will be also very small and therefore the electric field inside the capacitor will be approximately constant everywhere, except in the immediate vicinity of the injection spot. Therefore, if we increase the distance between plates by a factor  $\alpha$  and the voltage on the capacitor by the same factor, the electric field will remain unchanged and the perturbation on the injection spot, the size of which has remained the same, will be negligible. Consequently the current should also remain the same and the four characteristic parameters of the system for a << L will be:

$$\alpha L, a, \alpha V, I \tag{9}$$

If we now scale this last set of system parameters by a factor  $1/\alpha$  using Eq.(8) we arrive at:

$$L, a/\alpha, V, I \tag{10}$$

A comparison of (7) and (10) shows that in the limit of small emitter sizes ( $a \ll L$ ) the current remains *unchanged* as the emitter size is decreased by a factor of  $\alpha$  with all

other parameters staying the same. In other words, I will be independent of a for  $a \ll L$  and f(a/L) will approach a constant value. This is the scaling law obtained by numerical simulation.

#### Conclusion

In this paper we have carried out a numerical simulation of the SCLC from a strip of finite width and infinite length in the parallel plate geometry. The behavior of the SCLC when the width is large compared to the plate separation approaches that of the planar one dimensional case, with linear dependence on emitter width. When the width of the emitter becomes much smaller than the plate separation, the SCLC shows a sublinear dependence on the width and becomes independent of the width of the CDL as the width approaches zero. This behavior can be explained by a scaling argument. The scaling argument implies that the SCLC should become independent of width as the emitter width approaches zero and approach a constant value. These facts are borne out by the numerical simulations. Charge injecting defects in PRs are typically spots (CDSs) and they cause readily detectable print quality defects. CDLs are not seen in practice, but if they were to occur, they would have far more catastrophic impact on print quality as the long, linear streaks on the print would be far more easily perceived. The fact that the SCLC is finite for vanishingly thin emitters has consequences in the semiconductor chip industry. As the trend towards miniaturisation continues and the density of transistors and interconnects increases, there has been an effort to find material packages that reduce the crosstalk between the interconnects. Our results show that because the sublinear SCLC becomes independent of width for very thin emitters the problem is more serious than simple density arguments may indicate.

#### References

1. S. Jeyadev, P. Ramesh, Z. D. Popovic and S. Maitra, Space Charge Limited Currents from a Finite Sized Emitter in a Parallel Plate Geometry, Proc. 13th International Conf. On Digital Printing Tech. (NIP 13), pg. 241 (1997).

- 2. M. A. Lampert and P. Mark, *Current Injection in Solids*, Academic Press, New York, 1970.
- 3. M. Pope and C. E. Swenberg, *Electronic Processes in Organic Crystals*, Oxford University Press, Oxford, 1982.
- 4. I. Adamczewski, *Ionization, Conductivity and Breakdown in Dielectric Liquids*, Taylor and Francis, London, 1969.
- 5. Z. Croiteru, *Progress in Dielectrics*, Vol. **IV**, Academic Press, New York, 1962, pg. 103.
- 6. J. D. Cobine, *Gaseous Conductors: Theory and Engineering Applications*, Dover, New York, (1958).
- 7. A. Many and G. Rakavy, Phys. Rev. 126, 1980 (1962).
- 8. M. Zahn, C. F. Tsang and S.-C. Pao, J. Appl. Phys. 45, 2432 (1974).
- 9. M. Zahn and H. Chatelon, J. Appl. Phys. 48, 1797 (1977).
- Z. D. Popovic, P. Iglesias, D. Parco and S. Robinette, J. Imaging Tech. 17, 71 (1991).
- 11. P. Borsenberger, Organic Photoreceptors for Imaging Systems, Marcel Dekker, Inc., New York, 1993.
- E. M. Williams, *The Physics and Technology of Xerographic Processes*, Krieger Pub. Co., Malabar, FL, 1993.
- M. E. Scharfe, *Electrophotography Principles and Optimization*, Electronic & Electrical Engineering Research Studies, Electrostatics and Electrostatic Application Series, Research Studies Press Ltd., John Wiley & Sons Inc., 1984.
- 14. S. Maitra, P. Ramesh and S. Jeyadev, *SPIE Proc.*, **2658**, pg. 112, (1996).
- 15. M. H. Lean and P. Ramesh, *Boundary Elements XIII*, pg. 385, (1991).

### **Biography**

Palghat Ramesh recieved his Ph.D in Mechanical Engineering from Cornell University in 1988. Since 1990 he has worked at the Wilson Center for Research and Technology at Xerox Corporation in Webster NY. His work is focused on modeling and simulation of xerographic processes.