Improved Dot Diffusion For Image Halftoning

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Abstract

The dot diffusion method for digital halftoning has the advantage of parallelism unlike the error diffusion method. The method was recently improved by optimization of the so-called class matrix so that the resulting halftones are comparable to the error diffused halftones. In this paper we will first review the dot diffusion method. Previously, 8^2 class matrices were used for dot diffusion method. A problem with this size of class matrix is that enhancement of images is necessary before halftoning. However, enhancement may not be desirable in some applications. In order to eliminate the enhancement step, we increase the size of the class matrix to 16^2 and optimize the class matrix for a set of gray levels. In the optimization, the Human Visual System is used in the cost function. The optimization is done with the pairwise exchange algorithm. Since we increase the size of the class matrix, we are compromising the parallelism, i.e., the algorithm will terminate in 16^2 steps rather than 8×8 steps. This is the price paid for avoiding the enhancement step.

Keywords: Halftoning, dot diffusion, parallelism.

1. Introduction

Digital halftoning is the rendition of continuous-tone pictures on displays that are capable of producing only two levels. There are many good methods for digital halftoning[5]. Ordered dithering is a thresholding of the continuous-tone image with a spatially periodic screen. In error diffusion, the error is 'diffused' to the unprocessed neighbor points. The dot diffusion method for halftoning introduced by Knuth[4] is an attractive method which attempts to retain the good features of error diffusion while offering substantial parallelism. The method was improved by optimization of the so-called class matrix[5] and inverse halftoning algorithms for dot diffused images is proposed in [6]. A mathematical description of dot diffusion algorithm is also given in [7]. In this paper, the description of dot diffusion is reviewed in Sec. 2. The optimization without the enhancement step is discussed in Sec. 3. Then the problems with the enhancement step will be pointed out and the 16×16 class matrix is optimized for dot diffusion without enhancement in Sec. 3.1.

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2. Dot Diffusion

The dot diffusion method for halftoning has only one design parameter, called class matrix **C**. It determines the order in which the pixels are halftoned. Thus, the pixel positions (n_1, n_2) of an image are divided into IJ classes according to $(n_1 \bmod I, n_2 \bmod J)$ where I and J are constant integers. Let $x(n_1, n_2)$ be the contone image with pixel values in the normalized range $[0, 1]$. Starting from class $k = 1$, we process the pixels for increasing values of k. For a fixed k, we take all pixel locations (n_1, n_2) belonging to class k and define the halftone pixels to be

$$h(n_1, n_2) = \begin{cases} 1 & \text{if } x(n_1, n_2) \geq 0.5 \\ 0 & \text{if } x(n_1, n_2) < 0.5 \end{cases} \quad (1)$$

We also define the error $e(n_1, n_2) = x(n_1, n_2) - h(n_1, n_2)$. We then look at the eight neighbors of (n_1, n_2) and replace the contone pixel with an adjusted version for those neighbors which have a higher class number (i.e., those neighbors that have not been halftoned yet). To be specific, neighbors with higher class numbers are replaced with $x(i, j) + 2e(n_1, n_2)/w$ for orthogonal neighbors and $x(i, j) + e(n_1, n_2)/w$ for diagonal neighbors where w is such that the sum of errors added to all the neighbors is exactly $e(n_1, n_2)$. The extra factor of two for orthogonal neighbors (i.e., vertically and horizontally adjacent neighbors) is because vertically or horizontally oriented error patterns are more perceptible than diagonal patterns. The contone pixels $x(n_1, n_2)$ which have the next class number $k+1$ are then similarly processed. The pixel values $x(n_1, n_2)$ are of course not the original contone values but the adjusted values according to earlier diffusion steps (2).

When the algorithm terminates, the signal $h(n_1, n_2)$ is the desired halftone.

Usually an image is enhanced [4] before dot diffusion is applied. For this the continuous image pixels $C(i,j)$ are replaced by $C'(i,j) = \frac{C(i,j) - \alpha \bar{C}(i,j)}{1-\alpha}$ where $\bar{C}(i,j) = \frac{\sum_{u=i-1}^{i+1} \sum_{v=j-1}^{j+1} C(u,v)}{9}$. Here the parameter α determines the degree of enhancement. If $\alpha = 0$, there is no enhancement, and the enhancement increases as α increases. If $\alpha = 0.9$ then the enhancement filter can be further simplified[4].

3. Optimization of the Class Matrix

Knuth introduced the notion of barons and near-baron in the selection of his class matrix. A baron has only low-class neighbors, and a near-baron has one high class neighbor. The quantization error at a baron cannot be distributed to neighbors, and the error at a near-baron can be distributed to only one neighbor. Knuth's idea was that the number of barons and near-barons should therefore be minimized. He exhibited a class matrix with two barons and two near-barons. The quality of the resulting halftones still exhibits periodic patterns similar to ordered dither methods (See Fig. 1). Knuth has also produced a class matrix with one baron and near-baron, but unfortunately these were vertically lined up to produce objectionable visual artifacts. In our experience, the baron/near-baron criterion does not appear to be the right choice for optimization.

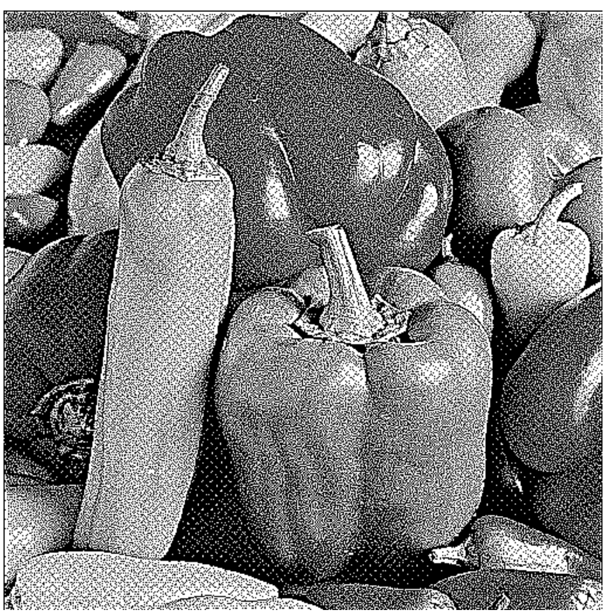

Figure 1: Dot Diffusion with Knuth's class matrix (8×8 class matrix).

We used a different cost function for differentiating the good halftones from other halftones [5]. In this cost function, the Human Visual System (HVS) is taken into account. The images are passed through the HVS function imitating the Human Visual System. Since our model is linear, we apply the HVS function to the difference image between the original and halftone image. Energy of the resulting image is defined to be the cost of the halftone image.

The HVS function has been derived in [8] and in [2] experimentally. In the frequency domain the HVS function is approximated well by:

$$H(u,v) = aL^b e^{-\frac{1}{s(\phi)} \frac{\sqrt{u^2+v^2}}{c\log(L)+d}}$$

where $a = 131.6$, $b = 0.3188$, $c = 0.525$, $d = 3.91$. We used $L = 0.091$ in our experiments where L is the average luminance. Furthermore, the phase dependent function $s(\phi)$ is defined as $s(\phi) = \frac{1-w}{2} cos(4\phi) + \frac{1+w}{2}$ where $w = 0.7$ and $\phi = atan(\frac{u}{v})$. With $h(x,y)$ denoting the inverse Fourier transform of $H(u,v)$, the discretized version $h[m,n] = h(Tm, Tn)$ is used in the calculations. In Fig. 2, the HVS function is shown for T=0.2.

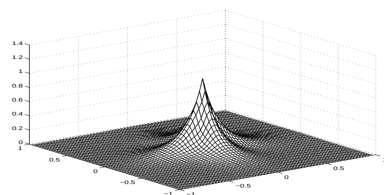

Figure 2: HVS function H(u,v) for T=0.2. The axes are $\frac{u}{\pi}$ and $\frac{v}{\pi}$.

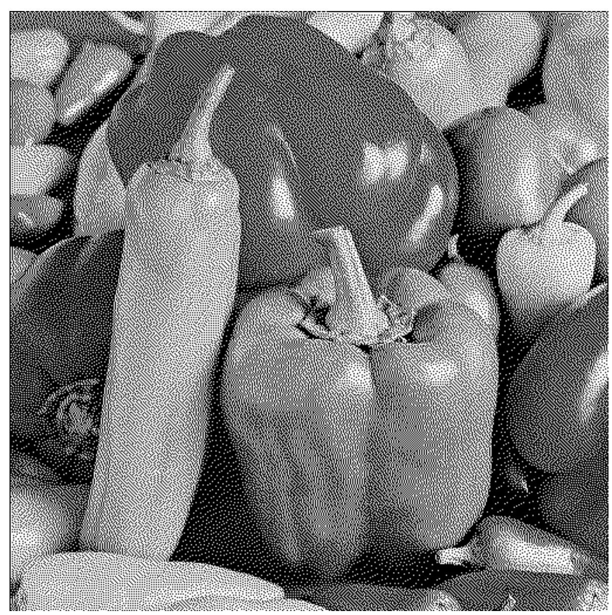

Figure 3: Floyd-Steinberg error diffusion.

In the optimization we are looking for a class matrix which minimizes the cost function. Notice that the optimization is equivalent to finding a combination of

numbers from 1 to IJ such that the related cost is minimized. Since the cost function depends nonlinearly on its parameters, we will use an optimization procedure to get the desired class matrix. The choice of the class matrix that minimizes this cost function was performed using the pairwise exchange algorithm [1] described below:
1) Randomly order the numbers in the class matrix.
2) List all possible exchanges of class numbers.
3) If an exchange does not reduce cost, restore the pair to original positions and proceed to the next pair.
4) If an exchange does reduce cost, keep it and restart the enumeration from the beginning.
5) Stop searching if no further exchanges reduce cost.
6) Repeat the above steps a fixed number of times and keep the best class matrix. [1]

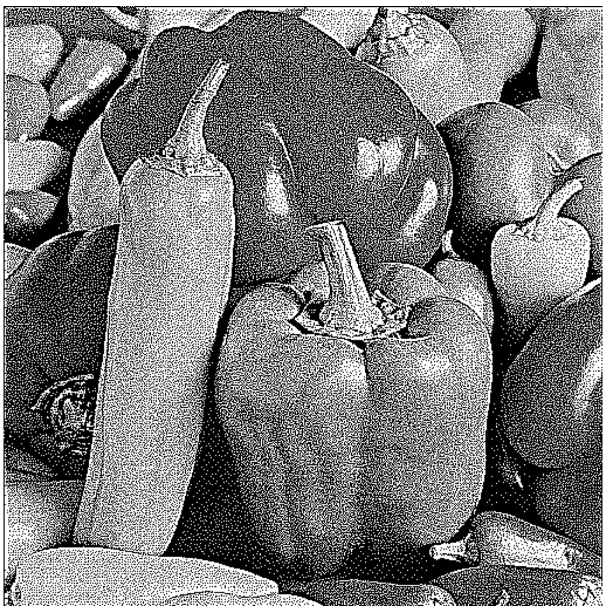

Figure 4: Dot Diffusion with the optimized class matrix and with enhancement (8×8 class matrix)

Example: The 512×512 continuous tone peppers image was halftoned by using Knuth's class matrix (Fig. 1), and by the optimized 8×8 class matrix (Fig. 4). It is clear that the dot diffusion method with the optimized 8×8 class matrix is visually superior to dot diffusion method with Knuth's class matrix. In fact, dot diffusion with the optimized 8×8 class matrix offers a quality comparable to Floyd-Steinberg error diffusion method (Fig. 3). Error diffused images suffer from worm-like patterns which are not in the original image, whereas dot diffused halftones do not contain these artifacts. Notice that the artificial periodic patterns in Fig. 1 are absent in Fig. 3 and in the dot diffusion with the optimized 8×8 class matrix (Fig.4).

[1] Note that pairwise exchange algorithm yields a local minimum of the cost function. We start the pairwise exchange with random class matrices and take the class matrix having the least local minimum in order to get closer to the global minimum. Global minimum is not guaranteed.

3.1. Dot Diffusion without Enhancement

If we compare the halftone images obtained with enhancement (Fig. 4) and without enhancement (Fig. 5) , we can conclude that the enhancement step reduces halftoning noise, but it might be objectionable in some applications because of its very visible sharpening effect (e.g., see Fig. 4).

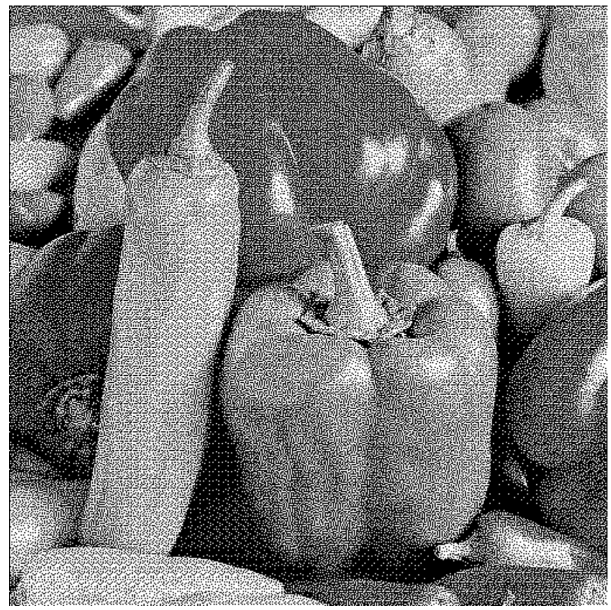

Figure 5: Dot Diffusion with the optimized class matrix and no enhancement (8×8 class matrix).

It turns out that we can get good halftones without use of the enhancement step provided we make the class matrix larger than the standard 8×8 size. The price paid for the larger class matrix is that the parallelism of the algorithm is compromised. We found that if a 16×16 matrix is used, the halftone images resulting from the optimization of this matrix are very good even without the enhancement step. (For comparison we note here that whenever enhancement is used, the class matrix can be as small as 5×5 without creating noticeable periodicity patterns.) Such optimization was carried out using a gray scale ramp as the training image. The HVS function was used in the optimization, and the associated cost was optimized using the pairwise exchange algorithm. The 16×16 optimized class matrix is shown in Table 1.

The peppers image halftoned with the resulting class matrix is shown in Fig. 6. There are no periodic artifacts in this result. While the overall visible noise level appears to be higher than for error diffusion, the problematic halftone patterns of error diffusion in the

Table 1: 16x16 Class Matrix

208	1	14	18	29	56	19	103	82	98	75	145	150	170	171	173
4	7	24	37	57	51	66	88	146	131	138	159	183	185	196	222
8	15	25	38	68	70	87	6	107	153	151	166	184	193	225	2
16	27	44	54	52	102	116	132	140	137	167	120	209	224	227	5
23	40	53	72	85	104	165	136	158	174	114	191	223	226	228	17
41	86	73	84	105	118	168	134	169	181	201	220	232	229	13	22
48	121	55	106	124	133	147	177	180	203	221	231	246	3	21	42
77	74	128	110	139	135	179	182	207	197	230	245	247	20	43	50
81	100	113	148	143	172	178	204	219	233	244	249	248	34	49	69
109	108	141	144	186	164	205	218	234	243	250	256	45	46	71	80
111	142	89	76	176	206	215	235	242	251	255	39	47	78	117	101
112	149	161	175	202	216	236	241	252	253	254	62	63	94	95	126
152	160	190	200	198	217	237	240	26	32	61	83	93	96	125	115
157	189	192	210	214	238	239	30	33	60	65	92	119	79	129	156
188	195	199	213	10	11	31	36	59	64	91	97	123	130	155	162
194	211	212	9	12	28	35	58	67	90	99	122	127	154	163	187

mid gray level are eliminated here. (Examine the body of the middle pepper in Fig. 3). By comparing Fig. 1 and 6 we see that 16×16 dot diffusion without enhancement is also superior to 8×8 enhanced dot diffusion using Knuth's matrix because there are no noticeable periodic patterns any more, and there are no enhancement artifacts.

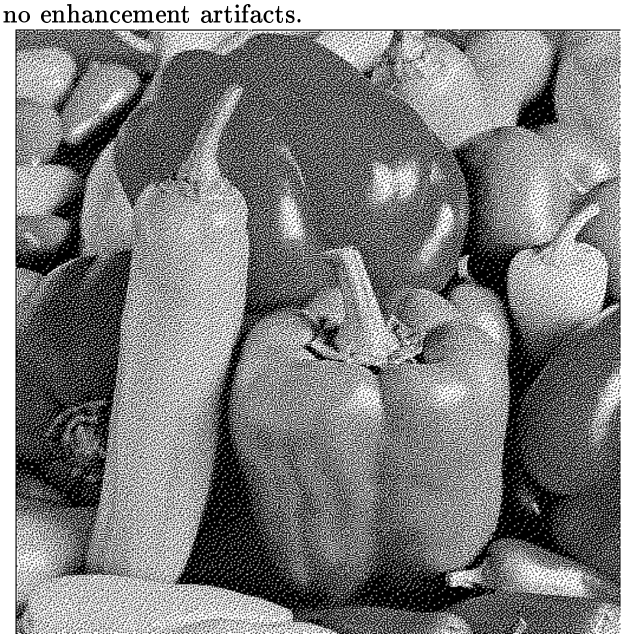

Figure 6: Dot Diffusion with HVS optimized 16x16 class matrix and no enhancement.

4. Conclusion

Dot Diffusion offers more parallelism than error diffusion and the method has been optimized in order to get rid of the periodic artifacts. The enhancement step prior to dot diffusion was preserved in previous optimizations. Since the enhancement can be objectionable in some cases, the method has been improved by optimizing a bigger class matrix.

5. References

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6. Biography

Murat Meşe was born in Turkey. He received the B.S. degree from Bilkent University, Ankara, Turkey in 1996, and M.S. degree from California Institute of Technology in 1997 both in electrical engineering. He is currently pursuing the Ph.D. degree in California Institute of Technology. His research interests are in digital halftoning, multirate DSP and applications to communications.

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