Modeling of Fluid Dynamics in Drop-on-Demand Printing

K. Addi, P. Pierron, M. Normandin, A. Soucemarianadin
J.R. Clermont and P. Attané
Laboratory of Rheology, University of Grenoble
BP 53X, 38041 Grenoble, France

Abstract

In this paper a numerical simulation of the drop formation in drop-on-demand ink-jet print head devices is described. Different pressures histories are considered and a finite difference technique is used to obtain the velocity profile at the exit of the nozzle. For the computation of the unknown free surface shape, we propose an efficient technique based on the stream-tube method, which has been used elsewhere essentially for simulating extrusion problems for which the effect of surface tension is negligible.

The distinguishing features of the proposed method in contrast to other codes are the easiness in introducing elaborate rheological constitutive equations in the model and the limited amount of computer resources, which is necessary. The results obtained simulate drop formation from a nozzle for different specified driving pressures and cover both the inviscid and the viscous cases.

They reveal the essential features namely the transient evolution of the velocities and the pressures inside the filament that lead to the pinch-up of the drop. Finally, the numerical results are discussed in the light of drop formation experiments.

Introduction

Drop-on-demand (DOD) ink-jet print heads in which the controlled production of ink drops is achieved through the action of a sudden pressure pulse produced by thermal means[1] or by a piezoelectric device[2] are becoming the preferred method of ink-jet printing[3]. Indeed they are widely used for various purposes and cover the whole spectrum from office printing to large width industrial printing[4].

In order to improve the fundamental understanding of the drop-on-demand technique from applied pressure history to drop pinch off, numerical calculations need to be performed at every stage of the process. If standard commercial codes have proved to be a comprehensive tool for the development of print heads[5], there is still room for improvement in terms of modeling of the non-linear dynamics of drop ejection.

The purpose of this paper is to combine the calculations performed in the print-head and of the free surface shape in order to allow prediction of the drop behavior. In section 2, we describe the numerical simulation of the flow in the nozzle channel for different pressure histories. In section 3, we will first develop shortly the stream-tube method[6] and give the initial and boundary conditions for the problem at hand. As a result of this, we will describe the free surface evolution of a filament as a function of time.

Numerical Simulation of Channel Flow

Overview

In contrast with the modeling of a continuous liquid jet break-up process where it is sufficient to give the initial velocity field[7] to start the computation process, the numerical simulation of the DOD device (see Figure 1), requires the knowledge of the transient start-up of the liquid in response to the onset of a pressure pulse[8].

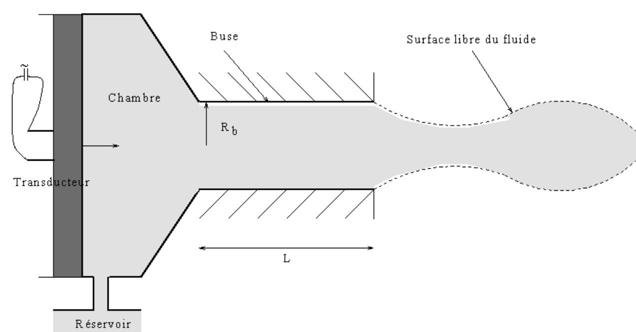

Figure 1. Schematic representation of a DOD print head

According to this requirement, we have first developed a model able to compute the velocity fields at the nozzle outlet corresponding to different drive pressures. The pressure transients leading to drop ejection can be generated by various means as emphasized in the introduction.

The modeling of the fluid flowing in the nozzle channel can typically be performed like a transient Poiseuille flow in a pipe. Following Middleman, we assume that the flow is laminar and that the non-linear inertial terms are small compared to viscous effects. Moreover, we assume that the axial velocity is independent of the axial coordinate. When radial velocity is neglected and since the problem is axisymmetric the momentum equation reads:

$$\rho \frac{\partial w}{\partial t} = -\frac{\partial P(t)}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r \frac{\partial w}{\partial r} \right) \quad (1)$$

where μ is the constant viscosity of the fluid, ρ the density of the fluid, P(t) the transient pressure pulse and *w* the axial velocity.

Initial and Boundary Conditions for Nozzle Flow

At initial time, the fluid is quiescent and then suddenly accelerated. The boundary conditions need to take into account the no-slip wall condition (2a), the axial symmetry consideration (2b) and the homogeneous Neumann condition (2c) on the axis of the nozzle.

$$\begin{cases} w(R_C,t)=0 \\ w(0,t) \textit{ finite value} \\ \left[\dfrac{\partial w}{\partial r}\right]_{r=0}=0 \end{cases} \quad (2)$$

Constant Pressure Drive

In the existing literature[8,9], the drive pressure history for the DOD print head is generally taken to be of a rectangular form comparable to the driver signal pulse. The choice of a rectangular form is sufficient for preliminary studies, but in the future, we would have to consider more realistic pumping chamber pressures[10]. The functional relationship for a simple rectangular pulse is given by the following set of equations:

$$\begin{cases} P(t)=0 \;\; for \;\; t<0 \\ P(t)=P \;\; for \;\; 0\le t\le T_P \\ P(t)=0 \;\; for \;\; t>T_P \end{cases} \quad (3)$$

From a numerical point of view, we use a finite difference method and we apply an explicit scheme for the resolution of equation (1). Then for each node, we can write:

$$w_k^{i+1}=w_{k+1}^{i}+\frac{\Delta t P^{i+1}}{\rho L}+\frac{\Delta t \mu}{\rho(\Delta r)^2}\left(w_{k+1}^{i}-2w_k^{i}+w_{k-1}^{i}\right) \quad (4)$$

In figure 2, we give the velocity on the axis for a given pressure profile. We notice that the velocity is the highest at the time when the pressure is set to zero. There is no lag between pressure and velocity. two typical velocity fields due to different drive pressures.

Figure 3 depicts the velocity field at the nozzle for different times and for the same pressure pulse as for figure 2. We have compared the computed drop volume with the analytical result given by Middleman[8]. The agreement is very good and this gives some confidence in the numerical simulation of the channel flow .

Figure 4 shows again the velocity on the axis for a different pressure profile. This pressure profile is close to actual signal drives and includes a positive and a negative pulse. In contrast to the above case, no analytical results are available for this complex pressure pulse.

In figure 5 we show the velocity profiles obtained at different times for the pressure pulse represented in figure 4. We can note that the velocities become negative near the nozzle walls. The expected result of such an evolution in the velocities is an enhancement of the pinch off process.

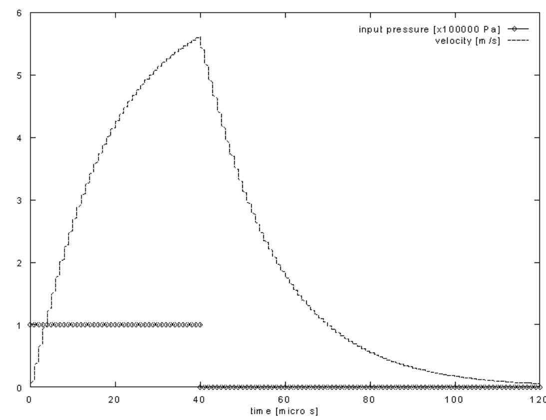

Figure 2. Pressure pulse and axial velocity

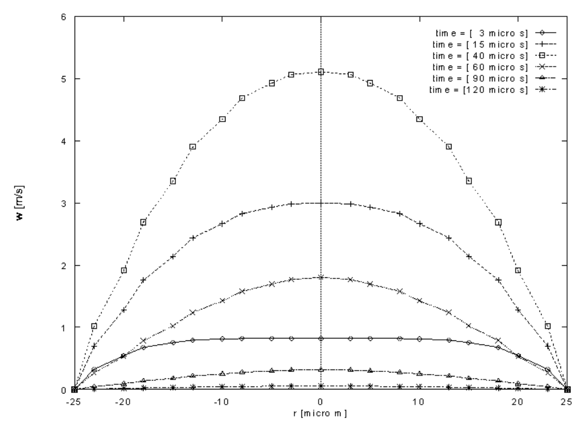

Figure 3. Velocity profiles for different times

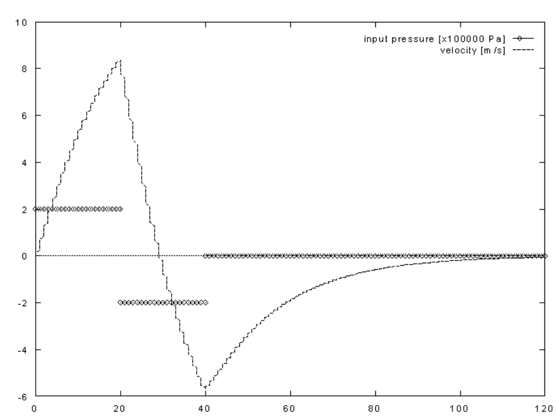

Figure 4. Pressure pulse and velocity on the axis

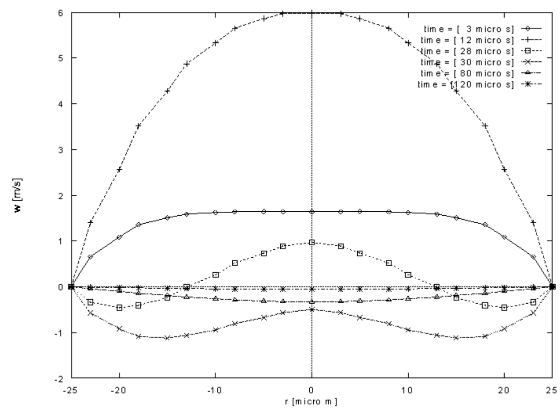

Figure 5. Velocity profiles at different times for the above pressure pulse

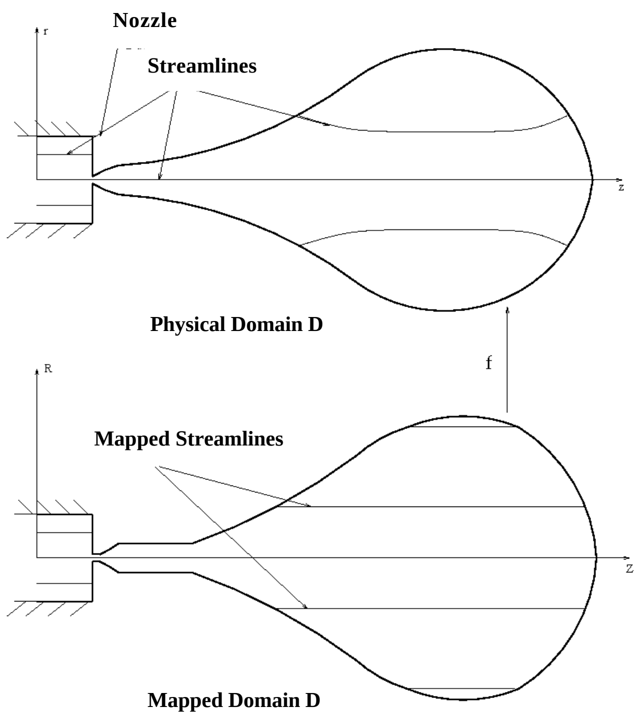

Figure 6. Representation of physical and mapped domains

Stream Tube Method

The basic elements of the stream tube method have been discussed in an exhaustive manner elsewhere[11] and therefore only the main features necessary for the understanding of the results given hereinafter will be presented in this section.

Transformation of the Physical Domain

In contrast to classical analyses of flow simulation, the approach given here involves a transformation function f between a physical domain D and its mapped domain D* where the transformed streamlines are parallel and straight as shown in figure 6.

The function f is an unknown of the problem to be solved in the mapped domain which is geometrically much simpler. An another requirement of the stream tube method is to define reference sections. In this problem, we have to consider two reference sections, one at the nozzle and the other one on the free surface as shown in figure 7.

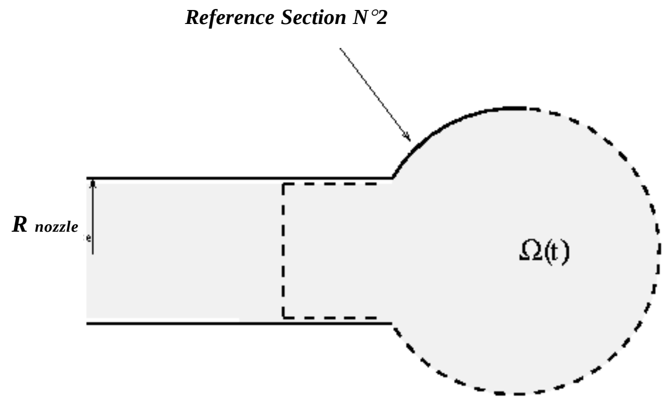

Figure 7. Reference section on the free surface

The Constitutive Equations

In this sub-section, we define the governing equations for our problem. Taking into account the incompressibility and the axisymmetric conditions, the momentum equation can be cast as shown below:

$$\rho \frac{D\vec{V}}{Dt} + div\,\bar{\sigma} = \vec{f} \qquad (5)$$

where $\bar{\sigma}$ is the total stress tensor.

The different assumptions given above lead to the following equations written in cylindrical coordinates:

$$\begin{cases} \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) + \left(-\frac{\partial P}{\partial r} + \frac{\partial T_{rr}}{\partial r} + \frac{\partial T_{rz}}{\partial z} + \frac{T_{rr} - T_{\theta\theta}}{r}\right) = 0 \\ \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) + \left(-\frac{\partial P}{\partial z} + \frac{\partial T_{rz}}{\partial r} + \frac{\partial T_{zz}}{\partial z} + \frac{T_{rz}}{r}\right) = 0 \end{cases} \qquad (6)$$

Since the components of the extra-stress tensor are written in an implicit manner, the introduction of elaborate constitutive equations can be performed in a straightforward way. This will allow in the future to account for complex fluid behavior.

Initial and Boundary Conditions for Free Surface Flow

At time t equal to t_0, the fluid is assumed to be in a quiescent state. For all times, the boundary conditions are the following:

- On Γ_1, the velocities are known
- On Γ_2, the velocities are equal to zero.
- On Γ_3, which is the free surface, the effect of the surface tension is considered.

The boundary evolutions are shown in figures 8 and 9 for initial and subsequent times.

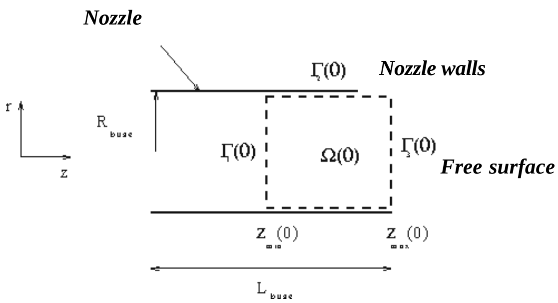

Figure 8. Boundaries at initial time

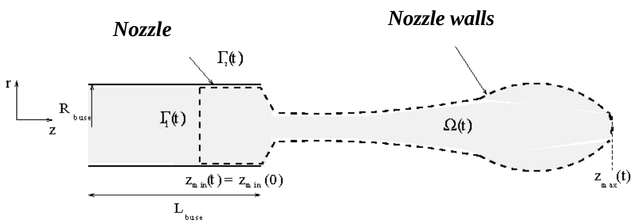

Figure 9. Boundaries at later time

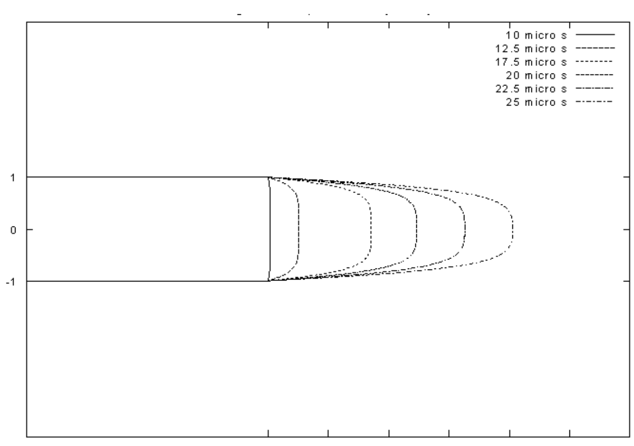

Figure 10. Free surface shapes at different times

Preliminary Results

In figure 10, we show the evolution of the free surface as a function of time. The axis have been made dimensionless by dividing by the nozzle radius. At the latest time shown here, there is a beginning of filament pinching. This result is encouraging in view of the velocity and pressure fields which have been obtained. Nevertheless, at that time we have to re-mesh finely near the free surface at the expense of additional computational time.

Conclusion

In this paper, we have detailed the procedure for the modeling of the drop formation in a DOD device. We have proposed a numerical method for the calculation of the velocity profiles at the nozzle exit for different pressure histories. Concerning the drop formation problem, we have introduced a stream-tube formulation of the governing equations together with appropriate initial and boundary conditions. The results obtained at the initial stages of filament pinching are encouraging. Presently, we are in the process of reducing the computational time in order to account for re-meshing and to add further sophistication to the numerical simulation of drop formation behavior. Additional features are needed to obtain closer agreement between experimental and computed profiles.

References

1. A. V. Bazilevsky, J. D. Meyer and A. Rozhkov, IS&T, *NIP 14*, pg 15 (1998).
2. G. Beurer and J. Kretschmer, IS&T, *NIP 13*, pg 621 (1997).
3. R. Beeson., IS&T, *NIP 14*, pg 27 (1998).
4. P. Giraud and E. Auboussier, IS&T, *NIP 14*, pg 643 (1999).
5. Y. Zhou, IS&T, *NIP 13*, 809-812 (1997).
6. M. Normandin and J.R. Clermont*: Intl. Journal for Num. Method in Fluids* **23**, pg 937 (1996).
7. P. Pierron, M. Normandin, A. Soucemarianadin, J.R. Clermont and P. Attané IS&T, *NIP 14,* pg 62 (1998).
8. S. Middleman, Modeling Axisymmetric Flows, (Academic Press) 1995.
9. T.W. Shield, D.B. Bogy and F.E. Talke, *IBM J. Res. Develop*., **31**, 96 (1987).
10. M. McDonald, IS&T, *NIP 12*, pg 53 (1996).
11. J.R. Clermont, *C. R. Acad. Sci. Paris*, 297, (1983).

Acknowledgements

The authors wish to thank the Région Rhône-Alpes for partial financial support of this work.

Biography

Arthur Soucemarianadin is Professor at the University Joseph Fourier in Grenoble, France. His current research interests include the study of instabilities and of rheologically complex fluids submitted to acoustic, electrical and/or thermal fields. He is the author of numerous scientific and technical papers and holds several patents related to petroleum and printing engineering processes. He is a member of various international scientific societies and has been recently appointed as Associate Editor of the Journal of Imaging Science and Technology. E-mail : souce@ujf-grenoble.fr