

Momentum Fluctuations in and Break-up of an Ink-Jet

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Abstract

The break-off of an ink-jet is considered in terms of momentum fluctuations in the jet. The indications were that non-linear minima and peaks in the break-off length, with respect to modulation level, were due to the jet relaxation and interaction between the jet boundary layer and modulation wave. The momentum fluctuations in the jet were measured with a sensitive resonant force transducer.

A commercial CFD free-surface modeling package was also used to obtain a deeper insight into the jet dynamics, than is available by experimental and analytical methods.

Nomenclature

A	Nozzle, or jet, cross section area
dm/dt	Mass flow rate
D, R	Diameter and radius of nozzle or jet, $D = 2.R$
F	Momentum force
L	Nozzle length of parallel section
Le	Nozzle length parameter, $(L/D)/Re$
M	Jet momentum
P	Nozzle pressure
Re	Reynolds number, $\rho.V.D / \mu$
V	Velocity of jet, or mean nozzle flow
η	Efficiency of jet, or nozzle
λ	Modulation wavelength in jet
μ,ρ	Fluid viscosity and density respectively

Introduction

For printing with a continuous ink-jet, the drop break-off must be precisely controlled. Available technical papers do not appear to adequately explain this. To obtain optimum printer performance these effects need to be understood.

Break-off length depends on the frequency and magnitude of acoustic pressure, together with the mean pressure, applied to the nozzle entry. This causes a periodic fluctuation of jet velocity that initiates a surface wave, which then grows, due to surface tension, to drop break-off.

For this study the acoustic pressure was created by applying an alternating voltage to piezoelectric crystals of an off-tune vibrating rod, a quarter wavelength behind the nozzle.

In conjunction with the jet velocity, the frequency was selected to achieve the most favorable wave number, $\pi.D/\lambda$, of about 0.7, for maximum wave growth.[1]

Nonlinear Break-Up Response

It is well known that as the modulation is increased, the jet break-off length decreases to a minimum.[1] Increasing modulation further gives a peak in break-off length, followed by a reduction, possibly with further minima and peaks.

Analysis has been developed to explain the decrease in break-off length with increasing modulation,[5] but there appears to be no adequate explanation for the minima and peaks in the break-off length. The understanding was that this was due to non-linear wave growth.

Jet Velocity Distribution

Bousfield[5] appears to assume that that the velocity distribution is uniform at a given section. This allows a "thin-filament" model of jet instability. However it is known that at the nozzle exit the velocity distribution is not uniform. This is because of a nozzle boundary layer due to no-slip conditions at the wall. At the jet free-surface there is minimal friction, so the jet can relax to a uniform velocity.

To conserve flow and momentum the jet increases in velocity and decreases in diameter, as shown by Middleman[2, 7] for fully developed lamina flow. As nozzle diameter may be the reference dimension used, such as in the wave number, this can explain differences in results.[1, fig 8]

It seems to be assumed that the modulation fluctuation of jet velocity are retained when it relaxes to a uniform distribution. This investigation found that the fluctuations can be greatly inhibited by the jet relaxation.

Jet Momentum

Various authors have concentrated on the shape of the jet surface wave, and its harmonic components.[3,6] In this investigation we consider the momentum fluctuation within the jet and how this relates to the jet break-up. The advantage is that momentum includes the internal velocity fluctuations, as well as the visible surface wave, and the transmutation between them near the nozzle exit.

Background

Computational Fluid Dynamic, CFD, analysis of nozzle flow showed that the boundary layer distribution and thickness is mostly related to the "Length parameter", *Le*. The nozzle entry shape seemed to only introduce a small length correction and close to the entry any deviation from the expected boundary layer profile is quickly dispersed.

This indicated that the nozzle entry shape has much less effect on the jet velocity distribution and drop break-off than may be expected.[1] It is considered that differences attributed to nozzle geometry may be more related to the flexibility and vibration of the nozzle plate.

The conjection of this paper is that the jet boundary layer near the nozzle exit interacts with the jet velocity modulation to create the minima and peaks in jet break-off length relative to modulation level.

Experimental Measurements

The standard technique for studying wave instability uses the obstruction of a flat laser beam by the opaque ink-jet,[1, 3, 6] but this only measures the surface waves. In this investigation the method used was to impinge the modulated jet, or drops, onto a sensitive, high frequency, force transducer. This measured momentum force fluctuations.

Assuming the jet momentum is dissipated on impact with the probe, the resulting force equals the loss of momentum. For uniform velocity at a section the jet force is given by;

$$F = M = V.dm/dt = \rho.A.V^2 = \pi.\rho.R^2.\mathbf{V}^2 \quad (1)$$

$$dF / F = 2.(dR / R + dV / V) \quad (2)$$

Hence the fractional change in momentum force is twice the sum of the fractional change in the jet radius and jet velocity. Thus the momentum detector responds to both the jet surface wave and the unseen internal velocity wave.

The present probe only measures the first harmonic. The piezoelectric transducer was tuned, mechanically and electrically, to the modulation frequency for maximum signal. Force fluctuations cause this to vibrate and induce a signal in the piezoelectric crystals. With a 50% water/glycerol mixture, a 75μm nozzle at 3 bar and 64kHz the maximum signal was about 1v p-p. Signal averaging reduced noise.

Figure 1 shows a modulated water/glycerol jet, under stroboscopic light, with shortest break-off length, impinging on the momentum sensor.

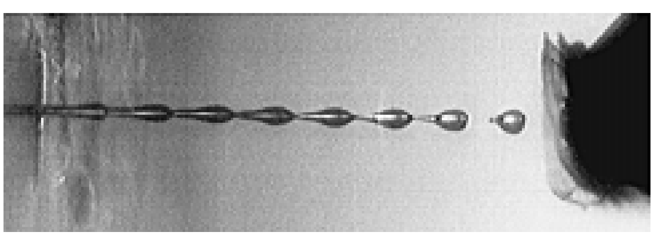

Figure 1. Modulated Jet impinging on momentum sensor

At the nozzle exit the jet can only have a velocity wave, and cannot have a surface wave. Further from the nozzle the velocity wave diminishes and is transmuted to a surface wave, which then grows due to the unstable surface tension forces. The momentum probe can follow the transition, from velocity modulation to surface wave, and subsequently into drops, with no disturbance to the probe response.

The probe signal phase can also be related to that of the driving voltage. For a fixed frequency the vibrating rod and probe produce a constant phase shift. Moving the probe along the jet changes the phase, so the distance for one phase cycle gives the wavelength. When the nozzle distance is plotted against phase cycles this produces a straight line from close to the nozzle to well into the drop stream. This shows that the jet and drop velocities are, effectively, constant, even for high modulation levels. Jet velocity is calculated from the wavelength and frequency.

From a nozzle study it was found that the nozzle boundary layer thickness, and hence the jet velocity distribution, could be determined from the jet efficiency. This is the ratio of the jet kinetic energy to the nozzle pressure energy;

$$\eta = \rho.V^2 / 2.P \quad (3)$$

Figure 2 shows the jet efficiency for the nozzle theory, from CFD results, and experimental measurements.

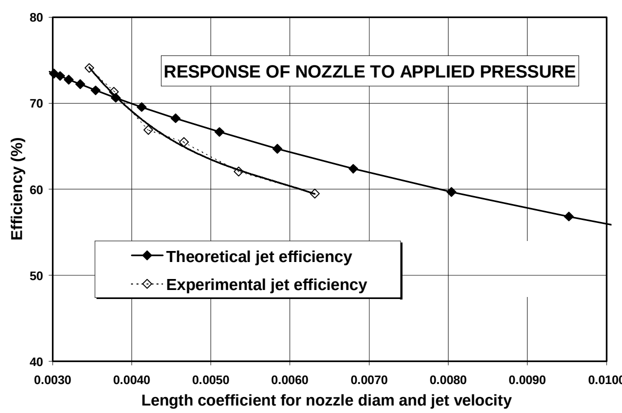

Figure 2. Jet efficiency of nozzle

Hence from a simple measurement of the jet velocity, the jet efficiency, nozzle flow and jet properties, such as boundary layer thickness, can be determined. Also knowing the Newtonian nozzle characteristics, it is also possible to determine some non-Newtonian ink properties.

Effect of Modulation on Jet Velocity

It was found that the mean jet velocity increased, by up to 10%, as modulation level was increased to its maximum. The effect was, approximately, proportional to the square of modulation, so at low modulation the effect was negligible. This has a substantial effect on the nozzle flow and jet break-off, hence to measure the steady-state behaviour of the nozzle and jet the modulation level must be small.

A model was developed to explain this increase in jet velocity relative to the acoustic modulation pressure at the nozzle entry. It used the nozzle analysis results, Figure 2, and assumed the nozzle was rigid and responded instantaneously to pressure changes. Conservation of flow and momentum was assumed between the nozzle exit and the drops over one modulation cycle.

The relationship between the drive voltage and nozzle acoustic pressure is difficult to measure, or analyse. The increase in jet velocity due modulation provided a means of quantifying this, since the predicted velocity increase due to acoustic pressure can be related to the experimental increase produced by the drive-rod voltage. This indicated that, for normal conditions, the acoustic pressure amplitude was a substantial proportion of the mean nozzle pressure.

Momentum Measurement of Wave Growth

The probe signal at different distances from the nozzle shows the growth of momentum fluctuations along the jet.

The results were for three modulation levels, minimum break-off length, first peak in break-off length and a lower modulation with the same break-off length as the peak. The results are shown in figure 3 with Log of probe signal plotted against distance from the nozzle.

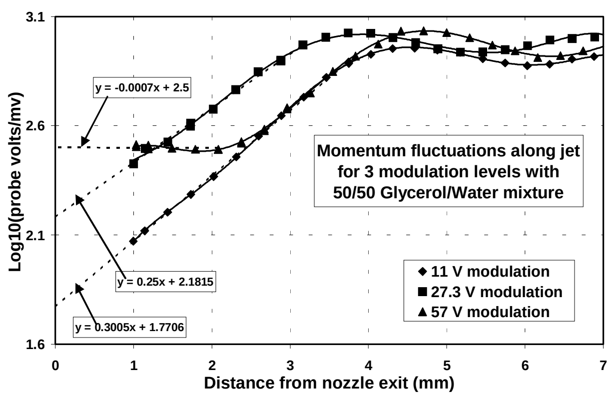

Figure 3. Modulation fluctuations along jet

For the lowest modulation the response was as predicted by the linear models, with exponential wave growth until break-off at 4mm, when the response "leveled-off".

At 57v modulation and up to 2mm from the nozzle there was no wave growth, indicating the applied modulation was dominant. Between 2.7mm and 3.5mm the wave growth was at the same rate as for 11 volts, with the same break-off length. The modulation at 27.3v, for the shortest break-off, gave the highest probe signal, beyond 1.5mm. When the three curves were extrapolated to the nozzle exit the probe signal were proportional to the input voltage.

The overshoot and oscillation is attributed to drop vibration after break-off. The frequency and damping can be used to evaluate dynamic fluid properties. The higher overshoot at 57 volts is attributed to the higher jet velocity.

In an alternative experiment the momentum probe was held at a fixed distance, 1.5mm, from the nozzle exit, while the modulation was increased. For the same conditions the effect of modulation on the break-off length was measured.

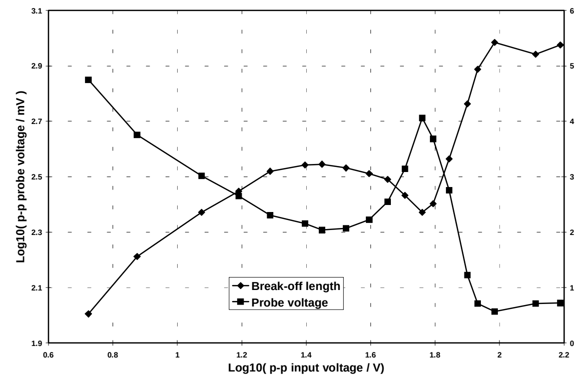

Figure 4. Probe voltage vs modulation voltage

Figure 4 shows how as the modulation was increased, the probe signal increased, reached a maximum, then reduced to a minimum. It can be seen that this is a "mirror-image" of the break-off curve. The minimum and maximum break-off occur at about the same modulation as the maximum and minimum of the momentum fluctuations.

We believe the nozzle exit jet velocity fluctuations directly respond to the acoustic pressure at the entry. This was indicated by the linear relationship between the probe signal, extrapolated to the nozzle exit, and the applied voltage.

The conclusion is that the non-linear break-off response is due to the jet relaxation, close to the nozzle and not due to non-linearity in the drop formation. This is attributed to a complex interaction between the boundary layer and modulation wave. Within this region surface tension has insufficient time to cause any substantial effect.

CFD Modeling of a Modulated Jet

With the complex non-linear transients and free-surface it is difficult to develop adequate analysis for a modulated jet and only limited measurement can be made. To overcome this Computational Fluid Dynamics, CFD, has been used.

Until recently free-surface CFD was not readily available and few industrial companies had such facilities, but the availability of commercial packages and advances in personal computers now makes such analysis feasible.

Domino uses an integrated Finite Element Analysis, FEA, package, with fluid-acoustics, electrostatics and fluid/-structure interaction. The ALE free-surface algorithm is used, which, at present, is restricted to continuous jets.

Typical CFD Results

Figure 5 shows the axial velocity for a nozzle with a no-slip wall. This is at the axis of the nozzle exit, also at the jet axis and jet surface 1mm from the nozzle. At the nozzle exit jet surface the velocity is zero. One can see that 1mm from the nozzle the velocity modulation is greatly reduced and the velocity distribution is near uniform.

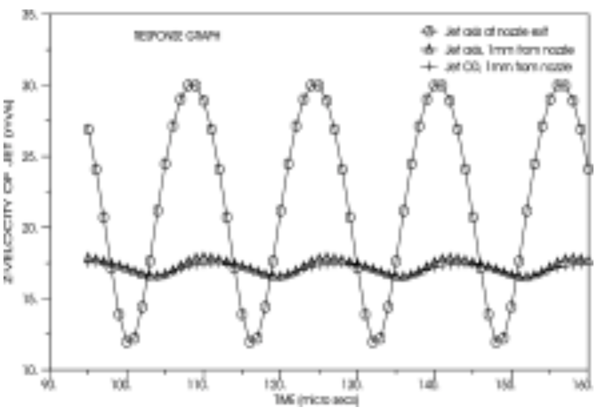

Figure 5. Velocity fluctuations in a modulated jet

The axial velocity distribution is shown as a colored band plot in fig 6. The upper plot is for a no-slip nozzle and the lower for a frictionless nozzle. With the no-slip nozzle there are high velocity gradients in the jet and the surface wave is skewed forward, as in fig 1. In comparison, for the frictionless nozzle the surface wave is symmetrical and radial velocity gradients are much less.

In both cases the velocity fluctuations rapidly diminish away from the nozzle and are transmuted to surface waves.

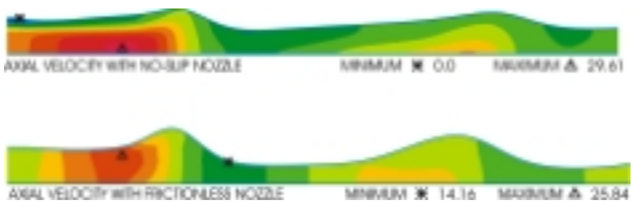

Figure 6. Jet velocity for a no-slip and frictionless nozzle.

Fig 7 shows the maximum shear stress for the no-slip nozzle. This indicates the regions of high velocity gradient, at the OD of the nozzle exit. It to be noted that the velocity gradients are mostly dissipated in the relaxation region, within a wavelength, about 0.3mm, from the nozzle exit.

Figure 7. Shear stress for jet of a no-slip nozzle

For the frictionless nozzle the shear stress was negligible compared to that in fig 7, hence the velocity gradients were minimal compared to a no-slip nozzle.

It was deduced that there would be little, if any, jet relaxation with a frictionless nozzle and so it would not show the minimum and peak in the break-off response.

It is concluded that the minimum and peak in the break-off curve for a no-slip nozzle is caused by the nozzle boundary layer. Work is continuing to investigate this. It has, for now, not been realistically possible to evaluate jet momentum fluctuation from the CFD results because there is, currently, no line integral function in the package. It is anticipated that this will soon be implemented.

Conclusions

The development of the instability wave of a free-surface fluid jet can be measured by the momentum force fluctuations of the jet impinging onto the rigid probe surface.

Such a probe can determine the jet velocity fluctuations in conjunction with the surface wave effects. This enables the modulation of the jet to be determined near the nozzle exit.

For low jet modulation the probe response and wave growth, relative to distance from the nozzle, has the exponential growth predicted by linear theory. For high modulation the wave is initially dominated by inertial forces and wave growth is inhibited. As the surface tension effects develop the probe response, relative to distance, is at a similar rate to that for a much lower level of modulation.

With the probe a fixed distance from the nozzle exit, preferably less than break-off length and more than the relaxation length, the probe output, with respect to modulation level, was a “mirror-image” of break-off length. This indicates that the nonlinear break-off curve is related to jet relaxation close to the nozzle.

Nozzle boundary layer thickness was found to be related to the nozzle length parameter and jet efficiency. Jet efficiency measurement can be used to determine boundary layer thickness and to assess some non-Newtonian effects. Nozzle entry shape effects appeared to be small.

It was found that transient CFD analysis could now be applied to free-surface jets. Results were obtained, on a PC, in an acceptable time and adequately represented experimental results. The CFD package used can, for now, only analyse continuous jets and did not have a line integration facility for evaluating momentum fluctuations. Other packages may not have these restrictions, but may not have the integrated FEA capability with fluid-structure interaction.

The CFD analysis showed very high velocity gradients and high shear rates in the nozzle boundary layer. These could produce rheological effects in non-Newtonian fluids. These high shear rates quickly decayed in the jet relaxation close to the nozzle exit.

For an equivalent jet from a frictionless nozzle it appeared that jet relaxation was not likely to produce such a nonlinear break-off response, with respect to modulation level. It was concluded that this nonlinear response was caused by the nozzle boundary layer and the subsequent interaction of this with the jet velocity modulation that occurred in the jet relaxation near the nozzle exit.

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References

1. B. Lopez, A. Soucemarianadin, and P. Attane, Break-up of Continuous Liquid Jets: Effect of Nozzle geometry, *J. Imaging Science and Tech.*, vol. **43**, no. 2, March/April 1999, p. 145
2. S. Middleman, Modeling Axisymmetric Flows – Dynamic Films, Jets and Drops, Academic Press, 1995.
3. A. Soucemarianadin, J, Xing, P. Attane, A. Dunand, Experimental Study of the Capillary Instability of Jets, *7th Proc. Non Impact Printing*, Oct 91, P 367-375.
4. D. W. Bousfield, I. H. Stockel and C. K. Nanivadekar, The breakup of viscous jets with large velocity modulations, *J. Fluid Mech.* (1990) vol. **218**, p 601-617
5. D. W. Bousfield, Keunings, Marrucci and Denn; Nonlinear Analysis of the Surface Tension Driven Breakup of Viscoelastic Filaments; *J Non-Newt Fluid Mech.*, **21** (1986) 79-97
6. H.H. Taub; Investigation of nonlinear waves on Liquid Jets; *Phys Fluids*, v **19**, n 8, Aug 1976
7. S. Middleman and J. Gavis; Expansions and Contraction of Capillary Jets of Viscoelastic Liquids: *Phys Fluids*, vol. **4**, no. 8, p 963, Aug 1961