Two-by-Two Centering Printer Model with Yule-Nielsen Equation

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Abstract

A binary printer can generate only two solid colors, black and white. Any other gray levels between them are simulated by halftone techniques and shown as the result of spatially averaging neighbor pixels. In the recently proposed 2x2 centering printer model, the conventional coordinates, which specify the location of all pixels, are redefined by shifting the entire grid by half-pixel distance in both the horizontal and the vertical directions. So, we claim that any binary printer can generate seven, instead of only two, "solid" colors. Here, the word "solid" means that all pixels within a large patch of the printout are microscopically identical, at least in the statistical sense. The seven "solid" colors, or gray levels, can be directly measured macroscopically. The 2x2 centering modeling can interpret any output of a binary printer as a seven-level gray image with the same spatial resolution as the binary input. To accurately predict the appearance of any combination of the seven gray levels, we use a modified Yule-Nielsen equation for non-linear spatial averaging. Applications of this 2x2 centering model to halftone screen design and calibration are also presented.

Introduction

Few hard-copy devices generate square-shape non-overlapping outputs as shown in Fig.1. Instead, overlapping between adjacent dots is quite significant from most printers. A circular-dot printer model, shown in Fig. 2, is perhaps still too idealized in reality. The combination of geometric overlapping and optical scattering in the paper creates many difficulties in modeling a simple B/W printer. Recently, we proposed a novel 2x2 centering concept, which greatly simplifies the estimation of the actual ink area and provides direct applications to screen design and error diffusion in B/W and color halftoning[1, 2].

In this paper, we combine the 2x2 centering concept and the Yule-Nielsen equation for a monochrome printer model and illustrate some applications of this new modeling to different halftone algorithms.

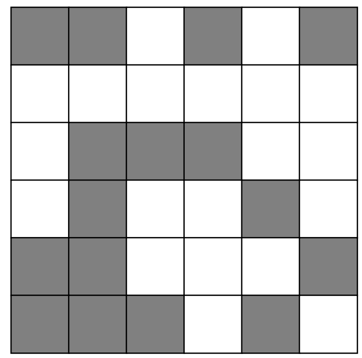

Figure 1. Idealized Non-Overlapping Printer Model

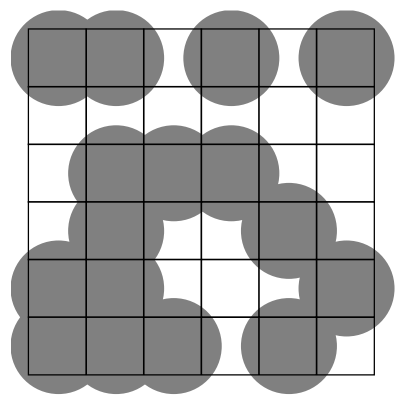

Figure 2. Circular-Dot Printer Model

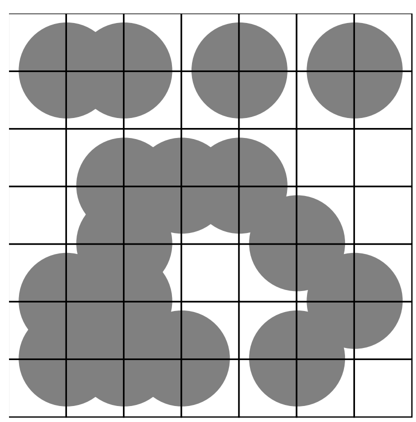

Figure 3. Two-by-Two Centering Printer Model

Two-by-Two Centering Concept

The difference between the 2x2 centering concept, shown by Fig. 3, and the conventional approach, shown by Fig. 2, is the definition of the output pixels. In Fig. 2, the position of each output pixel, represented by a rectangle, is coincident with the circular dot, representing the physical output by the printer. Obviously, the grid defining the output pixels is a conceptual coordinate for modeling purpose only. Any change on the grid, or the coordinate, will not affect the actual physical output of the printer at all. Therefore, we can shift the grid to the position shown in Fig. 3, so that each dot, representing the physical output, is coincident with one cross point of the grid.

Defined by the shifted coordinate, an output pixel is distinguished from the physical output dot, which appearance is directly controlled by the binary signal sent to the printer. The output pixel is located in the center of two-by-two overlapped dots, therefore, the overlapping, or the ink coverage, inside the output pixel is determined by the binary status of the four dots and has only $2^4 = 16$ possibilities. If we can further assume that all dots, the physical outputs of the printer, are identical and symmetric about both the vertical and the horizontal axis, the 16 overlapping possibilities can be categorized into seven groups represented by seven 2x2 patterns, G0 to G6, shown in Fig. 4. Each group consists of one 2x2 pattern and the corresponding mirror images, which are identical in terms of shape and area of the ink coverage. The assumption made above is true for most output devices at least in the statistical sense. Hence, the seven groups also represent seven different gray levels and any output form a binary monochromatic printer can be interpreted as a seven-level gray image. For example, the binary output shown by Fig. 3 can be interpreted as a seven-level image shown in Fig. 5.

Figure 4. Seven 2x2 Patterns Representing Seven Gray Levels

G1	G2	G1	G1	G1	G1	G1
G1	G2	G1	G1	G1	G1	G1
G0	G1	G2	G2	G1	G0	G0
G0	G3	G5	G2	G4	G1	G0
G1	G5	G3	G0	G1	G4	G1
G2	G6	G5	G1	G1	G4	G1
G1	G2	G2	G1	G1	G1	G0

Figure 5. Representation of the Binary Output Shown by Fig. 3 in Seven 2x2 Patterns

Furthermore, the seven gray levels are not only defined in the micro level but also appear in the macro scope. For each 2x2 pattern there exists a unique combination such that a large area with thousands pixels can be printed in a "solid" color, or gray level, which is composed of one and only one 2x2 pattern. Fig. 6 shows the seven "solid" colors with the grid used for the 2x2 centering. Thus, the actual reflectance of each of the seven gray levels can be macroscopically measured by a densitometer directly from a patch printed with the corresponding pattern shown in Fig. 6.

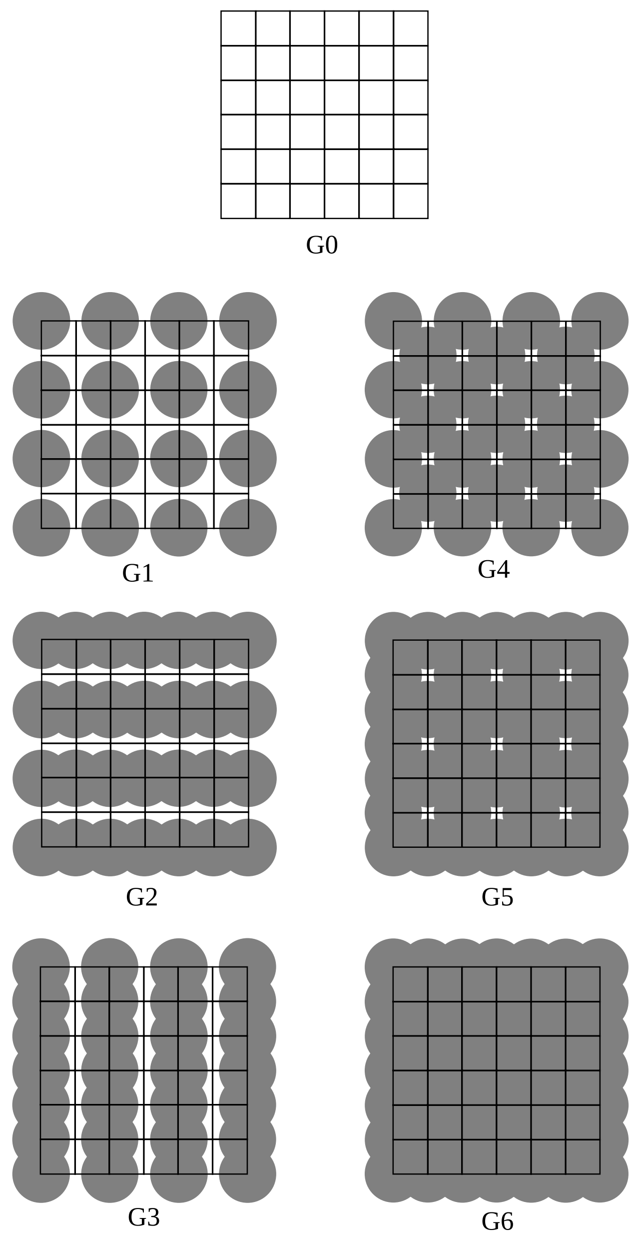

Figure 6. Seven Calibration Patches Consisting of "Solid"2x2 Gray Levels, Respectively

Yule-Nielsen Equation

In 1950s, Yule and Nielsen worked out equations representing propagation of light in the paper with halftone prints[3]. For single-color images, the density of the halftone print is given by the following Yule-Nielsen equation:

$$D = -n \log\left[1 - a\left(1 - 10^{-D_s/n}\right)\right] \tag{1}$$

a is the dot area, D_s is the solid ink density, and n is a factor to allow for the amount of light diffusion in the paper, depending on the halftone screen and the type of paper. When n = 1, corresponding to no penetration of light into the paper, the Yule-Nielsen equation reduces to the Murray-Davies equation:

$$D = -\log\left[1 - a\left(1 - 10^{-D_s}\right)\right] \tag{2}$$

Using R to represent the reflectance of the halftone print, where $D = -\log(R)$, we can rewrite the Yule-Nielsen equation in term of the reflectance of the solid ink, R_s:

$$R^{1/n} = aR_s^{1/n} + (1-a) \tag{3}$$

After introducing the paper area, $a_p = 1 - a$, the ink area, $a_s = a$, and the reflectance of paper, $R_p = 1$, Eq. 3 becomes a symmetric form:

$$R^{1/n} = a_s R_s^{1/n} + a_p R_p^{1/n} \tag{4}$$

2x2 Printer Model with Yule-Nielsen Equation

Since the Yule-Nielsen equation is quite successful in modeling binary halftone printing, we may extend this approach to the case with multi-level halftone. Assume that there are M gray levels including the solid-ink level and the paper level, a_m and R_m are the area and the reflectance of level m, respectively, where

$$\sum_{m=0}^{M-1} a_m = 1, \qquad m = 0,1,\ldots, M-1 \tag{5}$$

The reflectance of a halftone print with M gray levels is given by

$$R^{1/n} = \sum_{m=0}^{M-1} a_m \cdot R_m^{1/n} \tag{6}$$

If am = 1, or the entire area is printed in a single gray level, Eq. 6 is reduced to R = Rm. In other words, the reflectance of each gray level can be directly measured macroscopically from a patch printed with a “solid” color, or a “solid” gray level. To calculate the reflectance of any combination of multiple gray levels, the difficulty comes from estimation of the actual ink area. Usually, the estimation can be done only at the average level with plural pixels and is based on the calibration of a particular halftone algorithm.

Since the 2x2 centering concept interprets any output from a binary printer as a multi-level gray image and every pixel is specified in one of the seven gray levels, we can rewrite the modified Yule-Nielsen equation, Eq. 6, explicitly for the 2x2 printer model as

$$R^{1/n} = \sum_{m=0}^{6} i_m \cdot R_m^{1/n} \Big/ \sum_{m=0}^{6} i_m \tag{7}$$

where i_m is the number of output pixels with m-th 2x2 gray-level.

Although the validity of the 2x2 centering concept is limited by the maximal size of overlapped dots, the corresponding printer model is measurement-based and should be appropriate for most binary printers. The measurement on the seven calibration patches reflects all linear or non-linear effects beyond the 2x2 overlapping. The modified Yule-Nielsen equation above provides an optimal estimation of the actual output for an arbitrary binary input.

Applications

To apply the new 2x2 printer model to most existing halftone algorithms is not difficult. For a given binary image, the output image presented in seven 2x2 gray levels can be easily obtained by a 16-element look-up-table. Since the modified Yule-Nielsen equation, Eq. 7, shows a linear relation between $R^{1/n}$ and numbers of output pixels at each 2x2 gray level, most multi-level halftone algorithms can be directly employed for the seven-level gray images with a *1/n* gamma-correction on the reflectance input *R*.

An immediate application of the 2x2 printer model is calibrating halftone screens. For a given halftone screen and a constant input gray level, the halftone output is a determined binary pattern. Using the 2x2 printer model and the measurement of the seven patches shown in Fig. 6 from a selected printer, we can quickly calculate the expected average reflectance of the determined binary pattern.

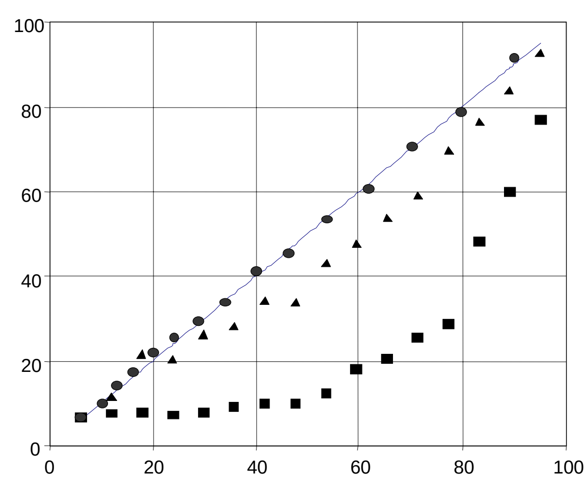

Figure 7. Measured Output Reflectance Verses Desired Input:
■: Idealized Non-Overlapping Model;
▲: 2x2 Printer Model with n = 1;
●: 2x2 Printer Model with n = 3.

The method to apply the 2x2 printer model to the error-diffusion halftoning without considering the Yule-Nielsen equation was described in our previous publication[1]. Actually, it is equivalent to the current model described by Eq. 7 with the Yule-Nielsen coefficient n = 1. As an experiment, we printed a 16-step gray-level wedge using a 400 dpi B/W laser printer, which was calibrated by measuring the reflectance from seven calibration patches shown in Fig. 6. Three different error-diffusion algorithms were used to generate the binary files for printing the test images. The first one is the standard Floyd-Steinberg method without overlapping correction; the second one is based on the 2x2 printer model with n = 1; and the third one is with n = 3. We measured the actual reflectance from the outputs and normalized both the desired input and the measured output in a scale from 0 to 100. Fig. 7 shows the comparison of three different printer-modeling methods.

Conclusion

The combination of the 2x2 centering concept and the Yule-Nielsen equation provides an efficient dot-overlapping model. This measurement-based printer model interprets the output from a binary printer as a seven-level gray image and accurately predicts the effective reflectance at the resolution level. The simplicity in the implementation of this novel approach certainly will find many applications to B/W and color halftoning, device calibration, as well as process control.

References

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Biography

Shen-ge Wang is currently a principal scientist with Xerox Corporation. He received a BS degree in Instrumental Mechanics from Changchun Institute of Optics, China, in 1970 and a Ph.D. degree in Optics from University of Rochester in 1986, respectively. His current research includes image processing, halftoning and printer modeling.

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