

On the Relation between DBS and Void and Cluster*

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Abstract

In this paper we compare and contrast two well known approaches for designing screen functions: direct binary search (DBS) and void and cluster (V&C). They are both iterative, search based methods that minimize a metric of error between the perceived halftone and the perceived continuous-tone original image. Despite the many differences between these two approaches, these is a close correspondence between them. In this paper, we will explore these links in detail, and examine their implications for the performance of both algorithms.

1 Introduction

Halftoning algorithms are used to transform continuous-tone grayscale images into binary images. This process is necessary to render the image by a laser or inkjet printer. Due to the lowpass characteristics of the human visual system (HVS), these rendered images are perceived as exhibiting grayscale. Thus, the objective of every halftoning algorithm is to minimize some measure of the difference between the perceived halftone and perceived continuous-tone image. An efficient halftoning method is to employ an array of thresholds to screen the continuous-tone image. Such methods require only one threshold comparison per halftone pixel. In this paper we compare and contrast two well known approaches for designing screen functions: Direct binary search (DBS) and void and cluster (V&C).

V&C consists of two stages. During the first stage, the prototype binary pattern is designed. Starting with the prototype binary pattern, the remaining levels in the dot profile function are designed during the second stage. Although different strategies may be employed using DBS [1], to compare and contrast DBS and V&C it is convenient to follow the same approach. We will show that under certain conditions, DBS and V&C very nearly coincide throughout each stage. Note, the dither array and prototype binary pattern must exhibit the periodic wrap-arround property [2] whereby the top and bottom edges as well as the left and right edges *wrap* together. Therefore, the screen is topologically equivalent to the surface of a donut.

1.1 Brief Description of DBS

Direct binary search is a recursive search heuristic. The algorithm evaluates the effect of trial halftone changes on a measure of perceived error E. DBS *processes* a pixel when it considers the effect on E of changing (*toggling*) the pixel's binary state or exchanging (*swapping*) the states of two pixels with different binary states. Only a change which reduces E may be accepted. The processing of every halftone pixel is referred to as an *iteration*. To design the prototype binary pattern and the subsequent levels of the dot profile function, one iteration will not guarantee convergence; several iterations are necessary. In general, the exact number can not be determined in advance. When no changes that reduce E are found throughout an entire iteration, the algorithm has converged. In this way, the prototype binary pattern and each level of the dot profile function is designed. Subject to the constraints of a valid screen design, each pixel at each level of the dot profile function will converge to a local minimum of the cost function.

1.2 Brief Description of V&C

Void and cluster only uses a recursive search heuristic during the design of the prototype binary pattern. The filered halftone global peak and trough represent clusters and voids of dots where the absolute perceived errors are at a maximum. To design the prototype binary pattern, pairs of pixels are exchanged in an attempt to minimize these errors. We refer to this V&C procedure as the *processing* of a pixel swap. This procedure is repeated recursively until the prototype binary pattern design converges. To design subsequent higher (lower) levels of the dot profile function, dots are toggled on (off) within the maximum void (cluster); the *rank* of a pixel is determined by the order in which dots are added or deleted. Rank increases by 1 with each added dot, decreases by 1 for each removed dot; and the dither array thresholds increase with pixel rank.

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2 Preliminaries

Throughout this paper, we use $[\mathbf{m}] = [m,n]^T$ and $(\mathbf{x}) = (x,y)^T$ to represent discrete and continuous spatial coordinates respectively. In addition, the details of Sec. 2 apply exclusively to DBS except where explicitly specified using the subscript ”V&C”.

2.1 Visual Models and Scale Parameter

We model the lowpass characteristics of the HVS with a linear shift-invariant impulse response. The perceived DBS halftone $\tilde{g}(\mathbf{x})$ is defined by interpolating halftone $g[\mathbf{m}]$ with a HVS point spread function (PSF) $\tilde{p}(\mathbf{x})$; so

$$\tilde{g}(\mathbf{x}) = \sum_{m} g[\mathbf{m}]\tilde{p}(\mathbf{x} - \mathbf{X}\mathbf{m}), \qquad (1)$$

where $\mathbf{X}$ is the periodicity matrix whose columns comprise the basis for the lattice of printer addressable dots.

The kernel $\tilde{p}(\mathbf{x})$ is based on the contrast sensitivity function (CSF) of the human visual system. In Sec. 2.3, we shall see that the mean-squared error only depends on $\tilde{p}(\mathbf{x})$ through its autocorrelation function $c_{\tilde{p}\tilde{p}}(\mathbf{x}) = \int \tilde{p}(\mathbf{y})\tilde{p}(\mathbf{y}+\mathbf{x})d\mathbf{y}$, evaluated on the printer lattice, i.e. $c_{\tilde{p}\tilde{e}}[\mathbf{m}] = c_{\tilde{p}\tilde{e}}(\mathbf{X}\mathbf{m})$. Assuming a printer with resolution R dots/inch (dpi), an expected viewing distance D, and a rectangular lattice of addressable points, we find that

$$c_{\tilde{p}\tilde{p}}[\mathbf{m}] = D^4 \int h(\mathbf{y})h(\mathbf{y} + \frac{\mathbf{m}}{RD})d\mathbf{y}, \qquad (2)$$

where h is the inverse Fourier transform of the CSF evaluated at the retina in units of radians, and $S \equiv RD$ is the *scale parameter*.

V&C uses a simpler visual model. The perceptual filter kernel is assumed to be a truncated 2D Gaussian function

$$\tilde{p}_{V\&C}[\mathbf{m}] = \mathbf{W}[\mathbf{m}]\mathbf{e}^{-\mathbf{m}^2/2\sigma^2}, \qquad (3)$$

where $\mathbf{W}[\mathbf{m}]$ is a 2D rectangular function which extends over the region of significant support of the Gaussian kernel and σ=1.5. The parameter σ is used to control the *scale* of the visual model. For convenience, the windowed kernel may be normalized to sum to one. However, all changes accepted by DBS and V&C are based on relative magnitudes; thus, normalizing is unnecessary. The perceived V&C halftone $\tilde{g}_{V\&C}[\mathbf{m}]$ is defined by

$$\tilde{g}_{V\&C}[\mathbf{m}] = \sum_{\mathbf{n}} g[\mathbf{n}]\tilde{p}_{V\&C}[\mathbf{m} - \mathbf{n}]. \qquad (4)$$

2.2 Error Metrics

We use $\overline{g}$ to denote the average absorptance value of $g[\mathbf{m}]$. $\overline{g}$ is a constant between 0 and 1, whereas the pixels of halftone $g[\mathbf{m}]$ take on values 0 (white) or 1 (black). Our goal is to compute $g[\mathbf{m}]$ to minimize a measure of a perceptually filtered version of the error image $e[\mathbf{m}] = g[\mathbf{m}] - \overline{g}$. We model the continuous-space perceived error as

$$\tilde{e}(\mathbf{x}) = \sum_{\mathbf{m}} e[\mathbf{m}]\tilde{p}(\mathbf{x} - \mathbf{X}\mathbf{m}). \qquad (5)$$

The global error measure minimized by DBS is the total squared perceived error given by

$$E = \int |\tilde{e}(\mathbf{x})|^2 d\mathbf{x}. \qquad (6)$$

V&C focuses its attention on the maximum absolute perceived error. The perceived error image is defined as

$$\tilde{e}_{V\&C}[\mathbf{m}] = \sum_{\mathbf{n}} e[\mathbf{n}]\tilde{p}_{V\&C}[\mathbf{m} - \mathbf{n}]. \qquad (7)$$

The V&C heuristic attempts to eliminate the largest dot clusters and voids. As dots are added or removed, V&C attempts to maximize $E^v_{V\&C}$ or minimizes $E^c_{V\&C}$, respectively. These error metrics are defined as

$$E^v_{V\&C} = |min(\tilde{e}_{V\&C}[\mathbf{m}])|, \qquad (8)$$

$$E^c_{V\&C} = |max(\tilde{e}_{V\&C}[\mathbf{m}])|. \qquad (9)$$

Using these error metrics, V&C may search for the index of the next threshold assignment efficiently. In DBS, computing the effect of trial change on the mean-squared error can be more computationally intensive. Next, we describe an efficient technique for computing the effect of trial changes.

2.3 Efficient Evaluation of the Effect of Trial Halftone Changes with DBS

To reduce complexity, we employ an efficient procedure for evaluating the effect of trial halftone changes [3]. We begin by defining an additional correlation function

$$c_{\tilde{p}\tilde{e}}(\mathbf{x}) = \int \tilde{p}(\mathbf{y})\tilde{e}(\mathbf{y}+\mathbf{x})d\mathbf{y}. \qquad (10)$$

Then, by substituting (5) into (6), the error E may be expressed as

$$E = \int \sum_{\mathbf{m}} \sum_{\mathbf{n}} e[\mathbf{m}]e[\mathbf{n}]\tilde{p}(\mathbf{x} - \mathbf{X}\mathbf{m})\tilde{p}(\mathbf{x} - \mathbf{X}\mathbf{n})d\mathbf{x}$$
$$= \sum_{\mathbf{m}} \sum_{\mathbf{n}} e[\mathbf{m}]e[\mathbf{n}]c_{\tilde{p}\tilde{p}}[\mathbf{n} - \mathbf{m}]. \qquad (11)$$

Substituting (5) into (10), we also observe that $c_{\tilde{p}\tilde{e}}[\mathbf{m}] = c_{\tilde{p}\tilde{e}}(\mathbf{X}\mathbf{m}) = \sum_{\mathbf{n}} e[\mathbf{n}]c_{\tilde{p}\tilde{p}}[\mathbf{n} - \mathbf{m}]$. Although our error metric operates upon a continuous-space version of the perceived error image $\tilde{e}(\mathbf{x})$, we may evaluate this measure by computing $c_{\tilde{p}\tilde{p}}[\mathbf{m}] = c_{\tilde{p}\tilde{p}}(\mathbf{X}\mathbf{m})$ only. Consider the effect on $\tilde{e}(\mathbf{x})$ of a trial change in the states of pixels at indices $\mathbf{m}_0$ and $\mathbf{m}_1$. The new perceived error image $\tilde{e}'(\mathbf{x})$ is

$$\tilde{e}'(\mathbf{x}) = \tilde{e}(\mathbf{x}) + a_0\tilde{p}(\mathbf{x} - \mathbf{X}\mathbf{m}_0) + a_1\tilde{p}(\mathbf{x} - \mathbf{X}\mathbf{m}_1), \qquad (12)$$

where $a_0 = -1$ if $g[\mathbf{m}_0] = 1$, $a_0 = 1$ if $g[\mathbf{m}_0] = 0$, $a_1 = 0$ to toggle one pixel, and $a_1 = -a_0$ to swap the binary states of two pixels. We substitute (12) into (6), to express the change in error $\triangle E$ as

$$\triangle E = (a_0^2 + a_1^2)c_{\tilde{p}\tilde{p}}[\mathbf{0}] + 2a_0 c_{\tilde{p}\tilde{e}}[\mathbf{m}_0] + 2a_1 c_{\tilde{p}\tilde{e}}[\mathbf{m}_1] + 2a_0 a_1 c_{\tilde{p}\tilde{p}}[\mathbf{m}_1 - \mathbf{m}_0]. \qquad (13)$$

$\triangle E$ can be evaluated with a few table lookups and additions. If a trial change is accepted, $c_{\tilde{p}\tilde{e}}[\mathbf{m}]$ must be updated to $c'_{\tilde{p}\tilde{e}}[\mathbf{m}]$ as follows

$$c'_{\tilde{p}\tilde{e}}[\mathbf{m}] = c_{\tilde{p}\tilde{e}}[\mathbf{m}] + a_0 c_{\tilde{p}\tilde{p}}[\mathbf{m}-\mathbf{m}_0] + a_1 c_{\tilde{p}\tilde{p}}[\mathbf{m}-\mathbf{m}_1]. \quad (14)$$

The approach described above can be used to reveal a correspondence between the DBS and V&C error metrics.

3 Relating the DBS and V&C Error Metrics and Visual Models

Notice the similarty between lowpass filters $c_{\tilde{p}\tilde{p}}[\mathbf{m}]$ and $\tilde{p}_{V\&C}[\mathbf{m}]$ as illustrated in Fig. 1. However, $c_{\tilde{p}\tilde{p}}[\mathbf{m}]$ is used

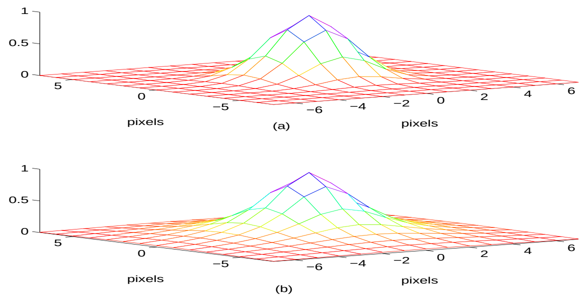

Figure 1: $c_{\tilde{p}\tilde{p}}[\mathbf{m}]$ (a) and $\tilde{p}_{V\&C}[\mathbf{m}]$ (b). The DBS and V&C perceptual filter kernels are similar.

to compute the $c_{\tilde{p}\tilde{e}}[\mathbf{m}]$ LUT to efficiently implement DBS whereas $\tilde{p}_{V\&C}[\mathbf{m}]$ is used to compute the perceived error image $\tilde{e}_{V\&C}[\mathbf{m}]$. According to the dual interpretation of DBS [3], $c_{\tilde{p}\tilde{e}}[\mathbf{m}]$ is another lowpass correlate of perceived error using the same visual model used to define $\tilde{e}(\mathbf{x})$, but from twice the viewing distance. Therefore, throughout the remainder of this paper we will refer to $c_{\tilde{p}\tilde{e}}[\mathbf{m}]$ as another correlate of perceived error. To relate DBS and V&C, we will show that the procedure used in Sec. 2.3 has the same effect on $c_{\tilde{p}\tilde{e}}[\mathbf{m}]$ as the V&C heuristic has on $\tilde{e}_{V\&C}[\mathbf{m}]$; each have their maximum error minimized pointwise absolutely.

Consider the impact of toggling one halftone pixel $g[\mathbf{m}_0]$. If the toggle reduces error E, then $\triangle E < 0$ and (13) reduces to

$$\frac{1}{2} c_{\tilde{p}\tilde{p}}[\mathbf{0}] < -a_0 c_{\tilde{p}\tilde{e}}[\mathbf{m}_0], \quad (15)$$

where $a_0 = -1$ or $+1$ depending on whether $g[\mathbf{m}_0]$ is initially set to 1 or 0 respectively. This implies that a toggle reduces E only when $|c_{\tilde{p}\tilde{e}}[\mathbf{m}_0]|$ exceeds a threshold of $\frac{1}{2} c_{\tilde{p}\tilde{p}}[\mathbf{0}]$. For a toggle, the update (14) simplifies to

$$c'_{\tilde{p}\tilde{e}}[\mathbf{m}] = c_{\tilde{p}\tilde{e}}[\mathbf{m}] + a_0 c_{\tilde{p}\tilde{p}}[\mathbf{m}-\mathbf{m}_0]. \quad (16)$$

If $g[\mathbf{m}_0]$ is toggled from 1 to 0, $c_{\tilde{p}\tilde{e}}[\mathbf{m}_0]$ must have been greater than $\frac{1}{2} c_{\tilde{p}\tilde{p}}[\mathbf{0}]$ before the toggle. After the toggle is accepted, the update (14) decreases $c_{\tilde{p}\tilde{e}}[\mathbf{m}_0]$ by $c_{\tilde{p}\tilde{p}}[\mathbf{0}]$. Similarly, if $g[\mathbf{m}_0]$ is toggled from 0 to 1, $c_{\tilde{p}\tilde{e}}[\mathbf{m}_0]$ must have been less than $-\frac{1}{2} c_{\tilde{p}\tilde{p}}[\mathbf{0}]$ before the toggle. After the toggle is accepted, the update (14) increases $c_{\tilde{p}\tilde{e}}[\mathbf{m}_0]$ by $c_{\tilde{p}\tilde{p}}[\mathbf{0}]$. In either case, the update decreases $|c_{\tilde{p}\tilde{e}}[\mathbf{m}_0]|$. Thus, DBS only accepts toggles which locally reduce the perceived error image $c_{\tilde{p}\tilde{e}}[\mathbf{m}]$ in the *pointwise absolute sense*. It can also be shown [3] that accepted trial swaps also reduce $|c_{\tilde{p}\tilde{e}}[\mathbf{m}]|$.

3.1 Relating DBS and V&C during the Prototype Binary Pattern Design

V&C adjusts the prototype binary pattern in an attempt to minimize the maximum absolute filtered error $\tilde{e}_{V\&C}[\mathbf{m}]$. First, V&C searches for the pixel set to 1 (black) which exhibits the largest positive filtered absorptance error and begins to process this pixel by toggling it to 0 (white). $\tilde{e}_{V\&C}[\mathbf{m}]$ is then updated to account for the effect of this toggle. After this update, V&C searches for the pixel set to 0 which exhibits the largest negative filtered absorptance error. If this is the same pixel that was toggled to 0 in the previous step, then the V&C stopping criterion is satisified; and the prototype binary pattern design has converged. Otherwise this pixel is toggled to 1 to complete the swap. This procedure is repeated iteratively until the stopping criterion is satisified.

Using our block based optimization strategy [4] with a block size equal to the screen size, the DBS and V&C strategies are similar. We also restrict the changes performed upon the prototype binary pattern to the swapping of pairs of pixels with different binary states. However, the methodology used by DBS differs from that of V&C in two ways. First, the DBS and V&C stopping criteria described in Sec. 1.1 and 1.2 differ. Furthermore, each swap accepted by V&C *must* involve the pixel set to 1 which exhibits the largest positive filtered absorptance error. This aspect of the V&C search is based exclusively on the error before the change is made. Alternatively, the changes that DBS accepts are only based on the difference between the error before and what the error will be after the change is made. These are important distinctions. Knowing which pixels to process and when to terminate processing will significantly effect the quality of the final pattern as illustrated in Fig. 2. A local minimum of both the DBS and

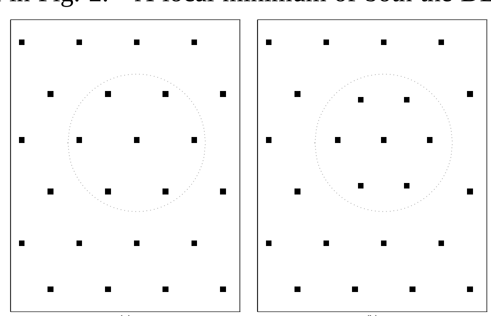

Figure 2: Prototype binary pattern with hexagonal lattice (a) and perturbed lattice (b).

V&C cost functions is illustrated in Fig. 2(a); this pattern resembles a hexagonal packing pattern. DBS and V&C each use a circircularly symmetric kernel which decreases monotonically as a function of Euclidean distance from the kernel center. Thus, neither DBS nor V&C will accept any changes due to the symmetry of this pattern. The nearest black neighbors of each minority black pixel are a constellation of six roughly equidistant pixels. Referring to the pixel in the middle of the circle as the *center* pixel, its six black nearest neighbors reside in an annulus close to the circle boundry. Refer to these six dots as the *surrounding* dots. Now modify this pattern by moving each of the surrounding dots slightly closer to the center dot to generate the pattern of Fig. 2(b). Clearly, the center pixel is the black pixel with maximum filtered absorptance and V&C will attempt to swap it. Since the surrounding dots have only moved slightly, toggling the center pixel creates the largest void; so the center pixel is restored and V&C terminates processing. DBS will find opportunities for reducing function cost at each of the surrounding dots; these dots will be swapped into positions further from the center pixel and thus, reduces the maximum absolute error. When DBS converges, the original pattern or a similar one will have been generated.

We have established that DBS and V&C accept a sequence of swaps in an attempt to decrease the maximum absolute filtered error. However, it is not obvious that updating the filtered pattern following an accepted toggle at $\mathbf{m}_0$ may not tend to increase the maximum absolute filtered error for some $\mathbf{m} \neq \mathbf{m}_0$ within the region of support of the perceptual filter kernel. For DBS, this subtle issue has been resolved with a proof [3]. This proof shows that DBS is *guaranteed* to achieve a local minimum pointwise absolutely. However, the V&C perceptual filter fails to satisify the conditions of this proof, and V&C does not actually confirm that a reduction in error results from a swap. Occasionally, V&C accepts swaps that increase function cost.

3.2 Relating DBS and V&C during the Design of the Remaining Levels of the Dither Array

Consider the design of a screen function intended to binarize 8-bit graylevel imagery. Assume the screen function has height H, width W where $HW \gg 255$. Assuming an ideal printing device, the dither array is expected to preserve tone if each level of gray (except 0) matches $HW/255$ of the threshold values. V&C designs the dither array by ranking pixels from 1 to HW whereas DBS ranks pixels in groups of size $HW/255$.

Consider the design of the higher levels of the dot profile function. V&C ranks the pixels of the dither array by adding dots to the prototype binary pattern *one-at-a-time*. The added dot is placed into the largest void. This is repeated until no zeros remain. As described in Sec. 3, DBS could begin by following a similar strategy. The largest void within $c_{\tilde{p}\tilde{e}}[\mathbf{m}]$ can determine the first dot placement. Then $c_{\tilde{p}\tilde{e}}[\mathbf{m}]$ is updated; and the next largest void is found. In this way, the placement of all $HW/255$ consecutive toggles may be determined. After these toggles are accepted, DBS then allows these dots to be swapped around in order to jointly optimize their placement. In general, several DBS iterations are required during which over $HW/255$ swaps are accepted. Clearly, V&C uses a greedy strategy for each threshold assignment whereas DBS jointly optimizes for a series of threshold assignments corresponding to each graylevel. Since the joint optimization is performed after the dots are first added (using the same method as does V&C), each accepted swap reduces function cost below the level attained without joint optimization. So, the DBS strategy attains better quality at each graylevel.

4 Conclusions

By design, DBS accepts halftone changes to generate a halftone $g[\mathbf{n}]$ which locally minimizes $\tilde{e}(\mathbf{x})$ in the mean square sense. These changes also locally reduce another perceptual error correlate $c_{\tilde{p}\tilde{e}}[\mathbf{n}]$ in the minimax absolute sense [3]. This latter metric resembles that of V&C.

We have shown that V&C uses a premature stopping criterion compared to DBS. Therefore, DBS is likely to design a more optimal prototype binary pattern. In addition, throughout the second stage V&C uses a greedy strategy for each threshold assignment. Using our block based strategy [4], DBS jointly optimizes over a series of simultaneous threshold assignments. In particular, the threshold selection process used in the second stage of V&C has been shown to be a proper subset of the procedures used by DBS. Thus using DBS will result in a lower cost at each gray level.

5 Biography

David Lieberman received his B.S. in Electrical Engineering from the University of Wisconsin at Madison in 1986 and his M.S in Electrical Engineering from Purdue University in 1996. He is a Ph.D. candidate in Electrical Engineering at Purdue University where he is employed as a research assistant. His research interests include model based halftoning, screen design, and image enhancement.

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