Halftone Color: Diffusion of Light and Dot Shape

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Abstract

The diffusion of light within paper has a significant affect on the color of a halftone printed image. This effect, known as optical dot gain, or the Yule-Nielsen effect, depends on the shape and size of the halftone dots, on the locations of dots with respect to each other, as well as upon the degree of light diffusion within the paper. The current work reports on a theoretical investigation of the influence that the shape of the halftone dot has on optical dot gain. A comparison is made between the colors produced by two types of halftone screens – a circular dot halftone screen and a line halftone screen. These are also compared to the colors produced when there is no diffusion of light within the paper. The comparisons are made for a monochromatic (one ink) halftone print, and for a bichromatic (two inks) halftone print. It is shown that there is a slight difference in halftone color between the circular dots and the lines, but that this difference is much less than the case of no diffusion.

Introduction

With the increasing quality of digital imaging technologies, there has been a renewed interest in modeling the microstructure of the halftone print in order to better control the colors produced in the halftone image [1, 2, 3, 4]. Diffusion of light within the paper has a significant effect on halftone color – an effect known as optical dot gain or the Yule-Nielsen effect[5]. To investigate how the shape of the halftone dot affects the shift in color due to light diffusion, a comparison is made of three different cases: no diffusion, diffusion with a circular dot halftone screen, and diffusion with a line halftone screen. Such a comparison has been made for monochromatic halftones[2, 6]. Here, a theoretical comparison for bichromatic halftones as well as monochromatic halftones, with conventional AM halftone screens, is reported.

The diffusion of light within paper can be characterized by a point-spread function, or equivalently, a line-spread function. In the following, it is assumed that the diffusion can be characterized by an exponential line-spread function:

$$L(x) = \frac{1}{2\bar{\rho}} \exp\Big[- |x| / \bar{\rho} \Big]$$

where $\bar{\rho}$ is the scattering length and is proportional to the first moment of the point-spread function.

Monochromatic Halftone

The halftone reflectance, R, of a monochromatic print is given by[4]:

$$R = R_p \Big[1 - 2\mu(1-T) + \mu P (1-T)^2 \Big], \qquad (1)$$

where R_p is the reflectance of the paper, μ is the fractional ink coverage, T is the ink transmittance, and P is the ink-ink probability: $P \equiv P(i|i)$, the conditional probability that if a photon enters the paper through an inked region then it exits the paper through an inked region. This expression for the reflectance is correct for any dot shape. The value of P is different, however, for different dot shapes. The quantity P depends on dot size, dot shape, dot locations, and on the degree of scattering within the paper.

For the exponential line-spread function, the ink-ink probability for circular dots is[7]:

$$P(\text{dots}) = R_p \begin{cases} 1 - \xi(\mu), & 0 \le \mu \le \pi/4 \\ 1 - \Big[(1-\mu)/(1-\mu_0) \Big] \xi(\mu_0), & \pi/4 \le \mu \le 1 \end{cases}$$

where $\mu_0 = \pi/4$ and

$$\xi(\mu) = 2 I_1(\alpha\sqrt{\mu/\pi}) \Big[K_1(\alpha\sqrt{\mu/\pi}) - I_1(\alpha\sqrt{\mu/\pi}) S(\bar{\rho}) \Big]$$

where I_1 and K_1 are modified Bessel functions of the first and second kind, α is the ratio between the screen period r and the scattering length: $\alpha = r/\bar{\rho}$, and

$$S(\bar{\rho}) = \sum_{k=1}^{\infty} p_k K_0(\alpha\sqrt{k}),$$

with p_k the number of combinations[8] of n and m such that $n^2 + m^2 = k$.

For a line halftone, one constructs a model to calculate P in a way similar to that for circlular dot halftones[7]: a uniform stream of photons is incident on the paper within the area of one dot, and one calculates the fraction of the photons that exit through this dot and all other dots. This fraction is P. One finds that:

$$P(\text{lines}) = R_p\left\{1 - \frac{1}{\alpha\mu}\left[1 - \exp(-\alpha\mu)\right]\right.$$
$$\left.\times\left[1 - \left[\exp(\alpha\mu) - 1\right]/\left[\exp(\alpha) - 1\right]\right]\right\} \quad (2)$$

where again $\alpha = r/\bar{\rho}$ with r the screen period (the distance between line centers).

Figure 1 shows the ink-ink probability for the two cases with a scattering length of $\bar{\rho} = 0.15r$.

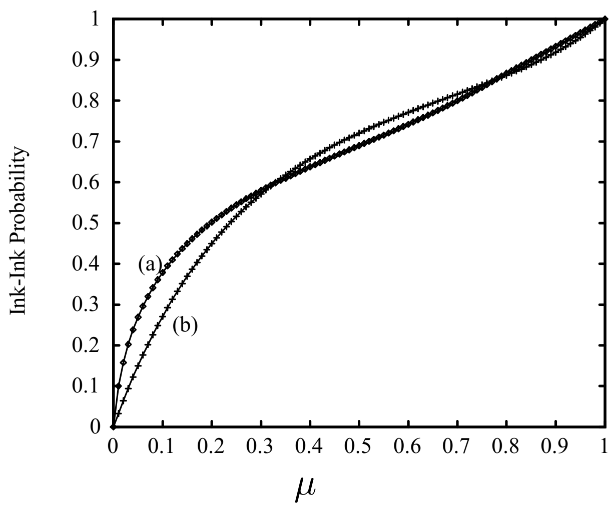

Figure 1. The ink-ink probability, P, as a function of fractional ink coverage. The scattering length is: $\bar{\rho} = 0.15r$. (a) circular dots, and (b) lines

In both cases, if $\bar{\rho} \to 0$ i.e. no scattering, then $P \to 1$, so that the reflectance is identical for the two cases; dot shape has no effect in the absence of diffusion.

Figure 2 shows the reflectance, Eq. (1), as a function of ink coverage μ, for the three cases: no diffusion, circular dot halftone and line halftone, both with $\bar{\rho} = 0.15$. In all three curves, $T = 0.10$. One sees that there is very little difference in the reflectance between the circular dot and line halftones, consistent with experiment[6].

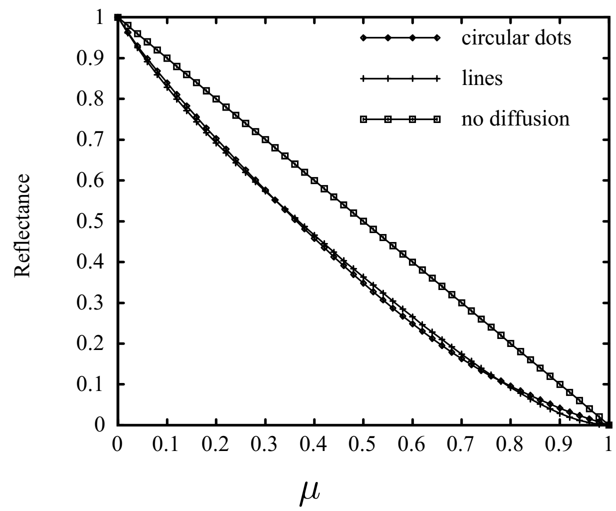

Figure 2. Reflectance as a function of fractional ink coverage for a monochromatic halftone. The scattering length is: $\bar{\rho} = 0.15r$.

Bichromatic Halftone

A halftone color is a partitive mixture of the halftone micro-colors – the colors of the inks, the various ink combinations, and the white paper. The tristimulus values X, Y, Z of the partitive mixture are given by[3]:

$$X = \text{Tr}[\mathbf{P}\,\mathbf{X}]$$
$$Y = \text{Tr}[\mathbf{P}\,\mathbf{Y}]$$
$$Z = \text{Tr}[\mathbf{P}\,\mathbf{Z}] \quad (3)$$

where **P**, **X**, **Y**, **Z** are symmetric matrices, and Tr[] indicates the trace of the matrix product. The size of the matrices depends on the number of inks in the halftone; for one ink they are 2×2, for two inks, 4×4, and for n inks, $2^n \times 2^n$. The elements of the micro-color matrices **X**, **Y**, and **Z** are the tristimulus values of the micro-colors contributing to the partitive mixture. The elements of the matrix **P** give the *amount* of each of the micro-colors that contribute to the mixture. If there is *no* diffusion of light, then **P** is diagonal and only the diagonal elements of the micro-color matrices contribute. In this case, Eqs. (3) reduce identically to the Neugebauer equations: the amount of each color is equal to the fractional area covered by that color. The diagonal elements of the micro-color matrices are the only colors that contribute to the partitive mixture in the absence of diffusion – the off-diagonal elements are "new" colors that arise due to diffusion.

Because of diffusion a photon may exit the paper from a different region of the halftone microstructure than the region into which it entered the paper. The diagonal elements are the colors that result when the photon exits the *same* region as it entered the paper, and the off-diagonal elements are the colors that result when the photon exits from a *different* region than that through which it entered the paper. For a monochromatic halftone, there are two regions: ink and bare paper (white); for a bichromatic halftone there are four regions: ink1, ink2, overlap of ink1 and ink2, and bare paper (white).

The elements of the matrix **P** give the probabilities that a photon diffuses from one region of the halftone microstructure to another region. The element $\mathbf{P}_{ij}$ is the joint probability that a photon enters the paper through the region i and then exits the paper through the region j. These scattering probabilities depend on the size and shape of the dots, the statistics of dot locations, and on the degree of light diffusion within the paper. These probabilities can be expressed[3] in terms of the fractional coverages of the inks, μ_n, and the ink-ink probability P_n: the conditional probability that if a photon enters the paper through a dot of ink n then it exits the paper through a dot of ink n. These ink-ink probabilities are identically the probabilities used in the previous section.

A signicant difference between a bichromatic dot screen and a line screen is the percent area of ink overlap. For dot screens it can be shown[9] that for most screen angles, the dots of one screen are effectively random with respect to the dots of the other screen. In this case the fractional ink overlap is simply the product of the fractional coverage of the two inks. The bichromatic line halftone consists of two screens of different color inks, with the lines of both screens having the same orientation but the line centers of one screen displaced with respect to the line centers of the other screen. In this case the fractional overlap depends not only on the fractional ink coverages, but also on the positions of the lines: the positions of the lines of one screen (the offset) are not random with respect to the lines of the other screen but are fixed. Therefore to make a comparison between colors obtained from a circular dot halftone and a line halftone, we consider an "ensemble" of line halftones. Each member of the ensemble has a different, random offset between screens, and a comparison is made between the ensemble average color with the circular dot halftone color.

In this case, the forms of the expressions for the line $\mathbf{P}_{ij}$'s are identical to those for circular dots. The only difference lies in the values of the ink-ink probabilities for a given μ. In the following, we use the expressions for the $\mathbf{P}_{ij}$ as derived in reference [3], with the P_n(lines) and P_n(dots) used in the previous section.

A comparison is made between the colors obtained with the circular dot screen, the line screen, and colors obtained with no diffusion. The inks are magenta and cyan. The cyan ink fractional coverage is held constant, $\mu_{\text{cyan}} = 0.5$, and the magenta ink fractional coverage, μ_{magenta} is varied between 0 and 1. Figure 3 shows the (no diffusion) gamut of the magenta and cyan inks, and the locus of points as μ_{magenta} is varied for the three cases. One sees that, as in the monochromatic case, there is little difference between colors produced by the circular dot and the line halftones, particularly when compared to the difference between either one and the colors obtained when there is no diffusion.

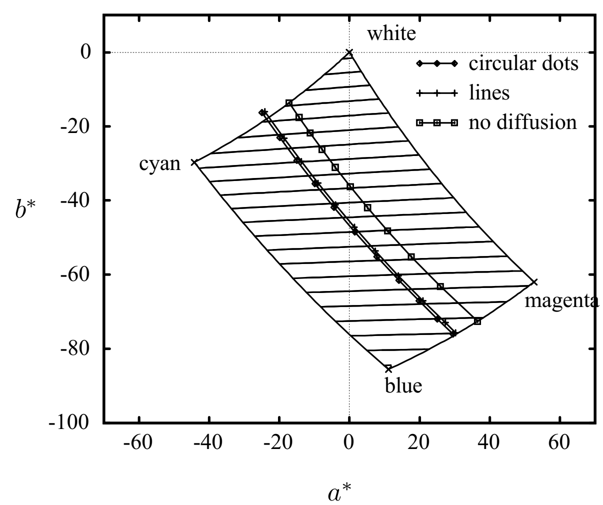

Figure 3. Lab gamut of the cyan and magenta inks and the colors obtained with* $\mu_{\text{cyan}} = 0.5$ *and* μ_{magenta} *varied from 0 to 1. Contour lines are lines of constant L* spaced 2.5 units apart, and* $\bar{\rho} = 0.15r$.

Figure 4 shows the specific color differences between the circular dot and the line halftones, and Figure 5 shows the color differences between the circular dots and the color obtained when there is no diffusion. Note the difference in scales in Figures 4 and 5.

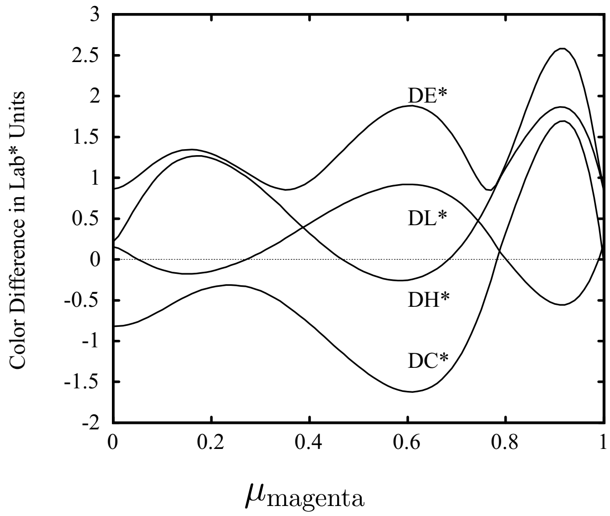

Figure 4. Color differences between bichromatic circular dot screens and line screens. $\mu_{\text{cyan}} = 0.5$ *and* μ_{magenta} *varied from 0 to 1, and* $\bar{\rho} = 0.15r$.

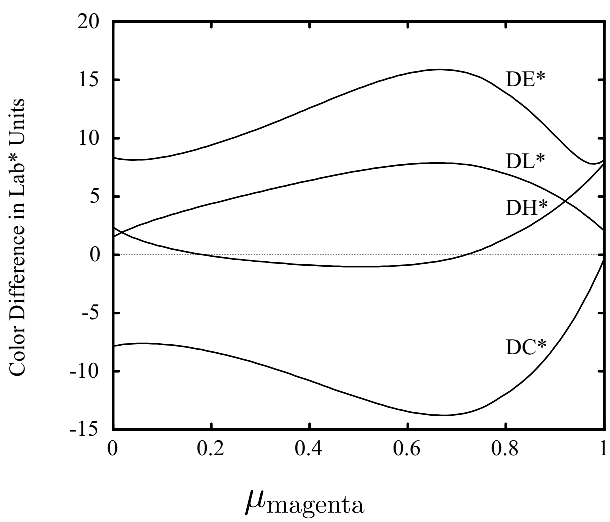

Figure 5. Color differences between bichromatic circular dot screens with diffusion and when there is no diffusion. $\mu_{\text{cyan}} = 0.5$ *and* μ_{magenta} *varied from 0 to 1, and* $\bar{\rho} = 0.15r$.

Conclusion

A theoretical investigation of the effect of dot shape on optical dot gain has been reported. It is shown that dot shape has very little affect on the colors produced, for both monochromatic and bichromatic halftone prints. These results are consistent with experimental findings.

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Biography

Geoffrey Rogers received his Ph.D. degree in Physics from New York University in 1992. Dr. Rogers teaches at the Fashion Institute of Technology in New York City where he directs the program in Color Science, and he also does consulting work through his company Matrix Color. Dr. Rogers' current research interests include the optics of paper, and the effect that diffusion of light within paper has on halftone color.