

Analysis of the Magnetic Force Acting on Magnetic Toner in Magnetography with Longitudinal Recording

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Abstract

The magnetic force acting on magnetic toner from the magnetic latent image recorded by longitudinal recording is theoretically analyzed. Heretofore analysis using sinusoidal approximation of magnetization has been reported wherein the force acting on toner can be several hundred times the force of gravity and it is not much smaller than the force used in electrostatic printing. This paper employs arctangent approximation of magnetization for digital longitudinal recording. A new approximate equation of the force attracting toner is presented which is applied to the magnetic printer using metal thin film recording medium and dry toner. Typically, the recording medium comprises Co-Ni-P plated metal thin film with the thickness of 1 micrometer. The toner comprises soft magnetic toner with the diameter of 10 micrometers. The resulting force acting on toner can be about three orders larger than the force of gravity. It is almost the same as the force used in electro-photography.

Introduction

It is one of the most important issue to know the magnetic force acting on magnetic toner from the magnetic latent image. A few studies have been made regarding the analysis of the magnetic force. Schloeman analyzed the magnetic force using sinusoidal approximation of the magnetization and he reported that the force acting on toner can be several hundred times the force of gravity and it is not much smaller than the force used in electro-photography[1]. Eltgen analyzed the magnetic force in the case of perpendicular recording in magnetography[2]. The author have developed the magnetic printer using digital longitudinal recording[3]. Here the magnetic force obtained in this case will be discussed.

Equations of Magnetic Force and Magnetic Field

When a small magnetically soft particle is placed in the magnetic field, the magnetic force acting on the particle is generally expressed as follows:

$$F = M \times dH/dr = \chi H \times dH/dr \tag{1}$$

Where F: magnetic force acting on the magnetic particle per unit volume, H: magnetic field at the place of the magnetic particle, r: location of the magnetic particle, M: magnetic moment of the magnetic particle which is induced in the magnetic particle and $M=\chi H$, χ: effective susceptibility of the particle including demagnetization factor; F, H and r have the same or opposite direction respectively.

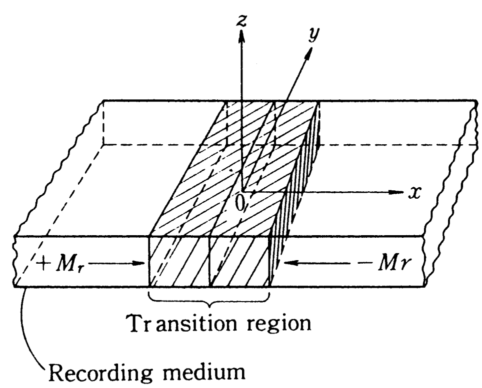

Figure 1. Coordinate system of the transition region

Fig.1 shows the coordinate system of the transition region. For the simplification, it is assumed that the recording medium is magnetized only in the x-direction and that the dimension in the y-direction is sufficiently larger than the thickness of the recording medium.

Fig. 2 shows the magnetization function of the transition region.?In this study the arctangent function is adopted for the magnetization function.

The magnetization in the medium is approximated by the following equations[4]:

$$M_x(x) = -2/\pi \times M_r \times \tan^{-1}(x/a) \tag{2}$$

$$M_y = M_z = 0 \tag{3}$$

where M_x, M_y, M_z: magnetization in the recording medium in the x, y, and z direction, respectively, M_r: residual magnetization in the recording medium, a: transition constant $a=l/\pi$.

Then the volume magnetic charge density is expressed by Eq.(4)[4]:

$$\rho v=-dM_x/dx=2M_r/\pi\times a/(x^2+a^2) \qquad (4)$$

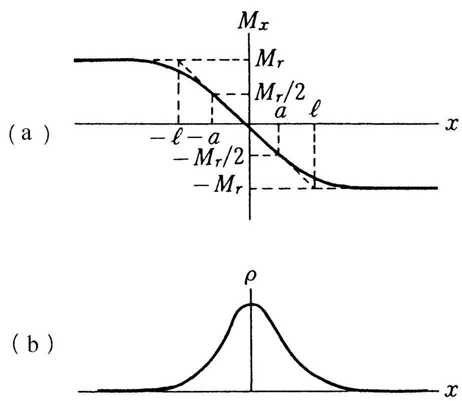

Figure 2. Magnetization function of the transition region (a) magnetization function (b)magnetic charge density

Then the magnetic field is expressed by the following equations[(4)]:

$$H_x(x,y,z)=4M_r\times [\tan^{-1}\{(z+\delta/2+a)/x\}-\tan^{-1}\{(z-\delta/2+a)/x\}] \qquad (5)$$

$$H_z(x,y,z)=2M_r\times \ln[\{x^2+(z+\delta/2+a)^2\}/\{x^2+(z-\delta/2+a)^2\}] \qquad (6)$$

where δ: medium thickness, $z>=\delta/2$ (that means Eqs. (5) and (6) are useful in the outside of the recording medium.), unit system: *cgs*..

New Approximate Equations of the Magnetic Force and the Magnetic Field

It would be not easy to understand the distribution of the magnetic field and/or the physical meaning from Eqs. (5) and (6). In addition, if we try to change Eq.(1) using Eqs.(5) and (6), the resulting equation would be complicated and it would be inconvenient to understand the physical meaning.

Therefore the new approximate equations will be introduced. The following assumptions used to deriving the new approximate equations satisfy the conditions for the metal-plated recording medium and dry toner which are employed in our magnetic printer[(3)].

Since there is no change in the *y*-direction, the Eqs. will be expressed by 2 dimensions, *x*-direction and *z*-direction, for the purpose of simplification. Using the following formula Eq.(7), Eq.(5) is changed to Eq. (8).

$$\tan(\theta_1-\theta_2)=(\tan\theta_1-\tan\theta_2)/(1+\tan\theta_1\times\tan\theta_2) \qquad (7)$$

$$H_x(x,z)=4M_r\times\tan^{-1}[\delta x/\{x^2+(z+a)^2-(\delta/2)^2\}] \qquad (8)$$

If the numerator is much smaller than the denominator in Eq.(8), using the formula $\tan^{-1}\theta=\theta$, the equation (8) is changed to Eq.(9):

$$H_x(x,z)=4M_r\times[\delta/\{x^2+(z+a)^2-(\delta/2)^2\}] \qquad (9)$$

In addition, if $\{x^2+(z+a)^2\}$ is much larger than $(\delta/2)^2$, the Eq.(10) is obtained:

$$H_x(x,z)=4M_r\times[\delta x/\{x^2+(z+a)^2\}]=4M_r\delta x/r^2 \qquad (10)$$

where *r* is the distance from the point *(0,-a)* to the point *(x, z)* and expressed by Eq.(11):

$$r=\{x^2+(z+a)^2\}^{1/2} \qquad (11)$$

In the same way, Eq.(6) is changed to Eq.(12), using the formula $\ln(1+x)=x$ ($|x|$ is much smaller than *1*):

$$H_z(x,z)=4M_r[\delta(z+a)/\{x^2+(z+a)^2\}]=4M_r\delta(z+a)/r^2 \qquad (12)$$

Using Eqs.(11) and (12), the magnetic field *H* is expressed by Eq.(13):

$$H(x,z)=+,-\{H_x(x,z)+H_z(x,z)\}^{1/2}=+.-4M_r\delta/\{x^2+(z+a)^2\}^{1/2}=+,-4M_r\delta/r \qquad (13)$$

Using Eqs.(1) and (13), the magnetic force F is expressed by Eq.(14):

$$F(x,z)=-\chi(4M_r\delta)^2/r^3=-\chi(4M_r\delta)^2/\{x^2+(z+a)^2\}^{3/2} \qquad (14)$$

From the Eq.(14), the *x* and *z* components of *F* are expressed by the Eqs.(15) and (16), respectively:

$$F_x(x,z)=F\cos\theta=-\chi(4M_r\delta)^2x/r^4 \qquad (15)$$

$$F_z(x,z)=F\sin\theta=-\chi(4M_r\delta)^2(z+a)/r^4 \qquad (16)$$

where

$$\theta=\tan^{-1}\{(z+a)/x\} \qquad (17)$$

Now assume that an infinitely long linear magnetic charge extends parallel to the *y*-axis through the point (*0*,-*a*) and that the linear magnetic charge density is expressed by Eq.(18):

$$\rho_1=2M_r\delta \qquad (18)$$

Then using Gauss's theorem, the magnetic field of the point *(x, z)*, which is apart from the assumed magnetic charge center(*0*, -*a*) by the distance *r*, is expressed by the Eq.(19).

$$H(x,z)=+,-4M_r\delta/r=+,-4M_r\delta/\{x^2+(z+a)^2\}^{1/2} \qquad (19)$$

The Eq.(19) is coincident with Eq.(13). This fact means that the model for the volume magnetic charge density profile expressed by Eq.(4) is equivalent to the model for the linear magnetic charge density expressed by the Eq.(18). Assuming a cylindrical magnetic charge which has the assumed linear magnetic charge center line as the axis of the cylinder, the same result is obtained.

The Result of the Calculations and Studies

Calculations are effected using the above new approximate Eqs.(10)-(17) and table 1. The values of table 1 are the typical values employed in the magnetic printer with longitudinal recording[(3)]. The transition constant *a* is approximated by Eq.(20), if the square ratio of the recording medium is high and $4M_r$ *is* much larger than H_c.

$$a=2M_r\delta/H_c \qquad (20)$$

Table 1. Values of the Parameters used for Calculation of the Magnetic Force

Recording medium	Co-Ni-P
Thickness	$\delta=1\mu m$
residual magnetization	$M_r=600$ emu/cc
coecive force	$H_c=400$ Oe
transition constant	$a=3\mu m$
Toner susceptibility	$\chi=0.1$

The result of the calculations are shown in Fig.3-5. The result calculated by the new approximate equations Eqs.(10)-(17) are coincident with the result of numerical calculations by computer using the Eqs.(1), (5) and (6). The errors between them are less than 1 %.

Fig.3 shows the magnetic field distribution at $z=10\mu m$. When we use the toner which average diameter is $10\mu m$, the thickness of the toner layer deposited on the recording medium is about $20\mu m$. Then this value of z is about half ???? the thickness of the toner layer. According to Fig.3, the magnetic field near the magnetic charge center is a few hundreds oersted. Considering the experimental fact that the developing magnetic field of a few hundreds oersted influences the image quality, this calculated value is reasonable.

Fig. 4 shows the magnetic force distribution at $z=10\mu m$. Since F_x is negative in the region of $x>0$, the toner is attracted toward the negative direction, that is to the point $x=0$. Since F_x is positive in the region of $x<0$, the toner is attracted toward the positive direction, that is to the point $x=0$. This means that the F_x moves the toner to the point $x=0$, or the center of the magnetic charge. F_x is maximum near the edge of the magnetic transition region. This indicates that the toner near the edge of the transition region is attracted very strongly to the center of the magnetic charge. At the point $x=0$, F_x is 0. This means that the toner on this place will be moved nowhere.

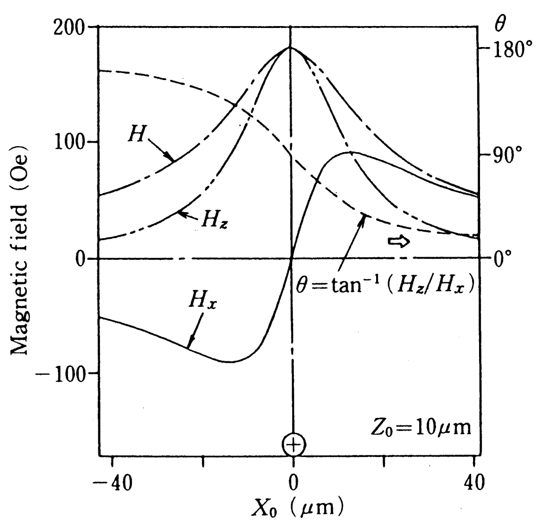

Figure 3. Magnetic field distribution

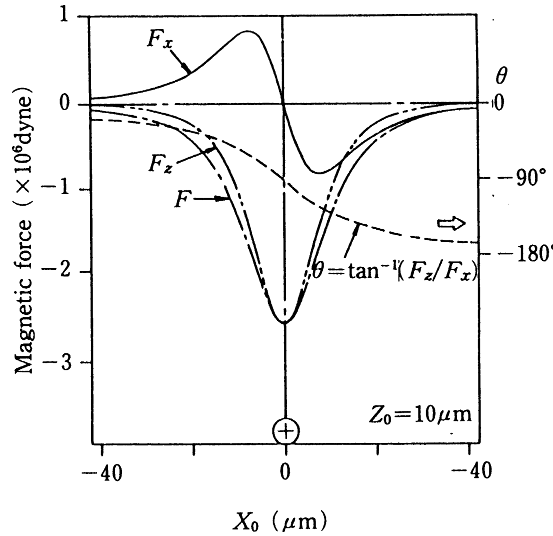

Figure 4. Magnetic force distribution

F_z is always negative. This means that the toner is attracted toward the negative direction, that is toward $z=0$.

F is always negative. This means that F has the opposite direction to the direction of r which is the distance from the assumed magnetic charge center $(0, -a)$ and that the toner is always attracted to the assumed magnetic charge center.

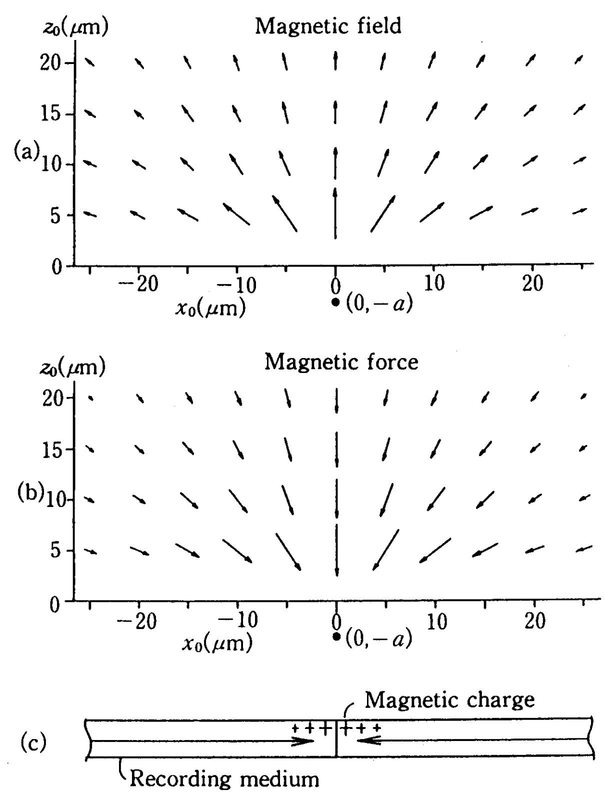

Figure 5. Vector expression of the magnetic field and the magnetic force distributions

Fig. 5 shows vector expression of the magnetic field distribution and the magnetic force distribution. The magnetic field diverges from the assumed magnetic charge center, while the magnetic force focuses to the assumed magnetic charge center.

Examples of the magnetic force are as follows:

$$F(0, 10)=2.6 \times 10^{6} dyne/cm^{3}=2.6\times 10^{7} N/m^{3}$$

$$= 2.6 \times 10^{3} gf/cm^{3} =2.6 \times 10^{6} kgf/m^{3} \quad (21)$$

$$F(0, 5)=1.1 \times 10^{7} dyne/cm^{3}=1.1\times 10^{8} N/m^{3}$$

$$= 1.1\times 10^{4} gf/cm^{3} =1.1 \times 10^{7} kgf/m^{3} \quad (22)$$

where z=5μm is nearly at the center of the first toner layer. These values are several times larger than those calculated by Schloemann(1). Since the mass density of the magnetic toner is $2.1g/cm^3$ (=$2,1\times 10^3 kg/m^3$), the magnetic force acting on the toner is larger than the gravitational force acting on the toner by about three orders. This is almost the same in the case of the electrostatic force between the photosensitive medium and the toner in electro-photography(5) . Since the image density of the print by the magnetic printer is substantially the same as that of electro-photographic printer, these values are reasonable.

The magnetic force acting on a toner sphere with the diameter of $10\mu m$ is as follows:

$$f(0, 10)=1.4 \times 10^{-3} dyne=1.4 \times 10^{-8} N$$

$$=1.4 \times 10^{-6} gf =1.4 \times 10^{-9} kgf \quad (23)$$

These values are almost the same as those calculated by Eltgen in the case of perpendicular recording in magnetography(2).

Conclusion

The magnetic force acting on magnetic toner in magnetography with digital longitudinal recording is theoretically analyzed. New approximate equations are introduced. They are applied to the magnetic printer which comprises metal plated recording medium and dry toner. The results of calculations are coincident with the experimental facts.

1. The magnetic field and the magnetic force can be approximated by the following equations:

$$H = +,-4M_r\delta /r$$

$$F=-\chi (4M_r\delta)^2/r^3$$

2. The magnetic field diverges from the assumed magnetic charge center *(0, y, -a)*, and the magnetic force focuses to the assumed magnetic charge center
3. The magnetic force acting on the toner is larger than the gravitational force on the toner by about three orders. This is comparable to the electrostatic force in electro-photography.

References

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Biography

Norio Kokaji received the B.E. and Ph.D. degrees from Tohoku University, Japan in 1965 and 1991, respectively. He joined Hitachi Koki Co., Ltd., Iwatsu Electric Co., Ltd., and Meisei University in 1965, 1969 and 1997, respectively. At present he belongs to Department of Electrical Engineering of Meisei University as a professor. He has been engaged in R&D of printing technology, especially magnetography. His works include almost the whole areas of magnetography using longitudinal recording.

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